Beam-Beam effect

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Scope

• Beam-beam effect happens when two charged beam collides
  – Strong/Weak interaction ----Goal of the Collider
  – Electric-Magnetic interaction ----parasitic, hard to avoid

• We are discussing the unwanted electromagnetic interaction of two colliding beam.

• Main limit from achieving high luminosity.
Beam-beam and Space charge force

• Charged particle not only feel the force from the opposing beam, but also from the bunch which it resides.
  – Beam-beam force, from the opposing beam
  – Space-charge force, from the its own bunch.
Determine the field

- Usually it is easier to calculate in the rest frame:
  \[ \vec{E}' \neq 0; \quad \vec{B}' = 0 \]
  
- From the charge distribution, both the scalar potential (Poisson equation) and electric field (Gauss Law):
  \[
  \nabla^2 U = -\rho(x, y, z)/\varepsilon_0 \\
  \vec{E}' = -\vec{\nabla}U
  \]
Transform to lab frame

- The electric and magnetic field in the lab frame can be obtained:
  
  \[ E_{\parallel} = E'_{\parallel}; \quad E_{\perp} = \gamma E'_{\perp} \]

  \[ \vec{B} = \vec{\beta} \times \vec{E}/c \]

- In lab frame, the perpendicular field is much larger than the longitudinal component.
- Beam-beam effect is mainly transverse effect.
Beam-beam force

- Lorentz force gives:

\[
\vec{F} = q \left( \vec{E} + \vec{v} \times \vec{B} \right)
\]

\[
= q \left( \vec{E} - \vec{B} \times \vec{\beta} \times \vec{E} \right)
\]

- In ultra-relativistic case, the reads:

\[
F_r = q E_\perp \left( 1 + \beta^2 \right)
\]
Simple Example

- Let’s assume a 2D round Gaussian beam:

\[
\rho(x, y) = \frac{n}{2\pi \sigma_r^2} \exp\left(-\frac{x^2 + y^2}{\sigma_r^2}\right)
\]

- The beam-beam force reads

\[
F_r = \frac{n(z) e^2 (1 + \beta^2)}{2\pi \epsilon_0 r^2} \left[ 1 - \exp\left(-\frac{r^2}{2\sigma^2}\right) \right] \vec{r}
\]
Beam-beam kick

- What is more important is the transverse kick of the test particle when traveling through the opposing beam.

\[ \Delta r' = \frac{\Delta P}{P_0} = \frac{1}{mc\beta\gamma} \int_{-\infty}^{\infty} F_r(r, z = s + vt) dt \]

\[ \Delta r' = \frac{2N r_0}{\gamma} \frac{r}{r_0^2} \left[ 1 - \exp\left(-\frac{r^2}{2\sigma_r^2}\right) \right] \]

With classical radius:

\[ r_0 = \frac{e^2}{4\pi\varepsilon_0 mc^2} \]
Beam-beam parameter

• For small amplitude test particle, the only effect is tune shift.
• Linear focusing in both transverse direction.
• Beam-beam parameter $\xi$ is the linear tune shift.

$$\xi = \frac{\beta^*}{4\pi} \frac{1}{f} = \frac{Nr_0\beta^*}{4\pi\gamma\sigma^2}$$
BEAM-BEAM EFFECT IN RING-RING COLLIDER
Beam-beam parameter limitation

<table>
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<th>Type</th>
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<th>RHIC</th>
<th>Tevatron</th>
<th>B-factory</th>
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<td>980</td>
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<td>Beam-Beam Parameter</td>
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- Beam-beam effect is the major limitation factor from achieving higher luminosity.
- Reason is complicate
- Let’s start from linear consideration
Linear Model

Linear beam-beam force modifies the one turn map as

\[ K \cdot M = \begin{pmatrix} 1 & 0 \\ -1/f & 0 \end{pmatrix} \cdot \begin{pmatrix} \cos(2\pi \nu) & \beta^* \sin(2\pi \nu) \\ -\sin(2\pi \nu)/\beta^* & \cos(2\pi \nu) \end{pmatrix} \]

\[ M_t = \begin{pmatrix} \cos(2\pi \nu) & \beta^* \sin(2\pi \nu) \\ -\sin(2\pi \nu)/\beta^* - \cos(2\pi \nu)/f & \cos(2\pi \nu) - \beta^* \sin(2\pi \nu)/f \end{pmatrix} \]

Tune shift:

\[ 2 \cos(2\pi \nu + 2\pi \Delta \nu) = 2 \cos(2\pi \nu) - \beta^* \sin(2\pi \nu)/f \]

Beta beat

\[ \Delta \beta = \beta^* \sin(2\pi \nu) / \sin(2\pi (\nu + \Delta \nu)) \]
Linear model, cont’d

• Assuming the tune change is small compare with the one turn tune,

\[ 2 \cos(2\pi \nu + 2\pi \Delta \nu) \sim 2 \cos(2\pi \nu) - 4\pi \Delta \nu \sin(2\pi \nu) \]

\[ \Delta \nu = \frac{\beta}{4\pi f} = \xi \]

• Stability condition

\[ \left| 2 \cos (2\pi \nu) - \beta^* \sin (2\pi \nu) / f \right| < 2 \]
Linear stability condition

• The beam-beam parameter has to satisfy:

\[
\xi = \frac{1}{4\pi} \frac{\beta^*}{f} < \begin{cases} 
\frac{\cot(\pi \nu)}{2\pi} & n < \nu < n + 0.5 \quad n \in \mathbb{Z} \\
-\frac{\tan(\pi \nu)}{2\pi} & n + 0.5 < \nu < n + 1 \quad n \in \mathbb{Z}
\end{cases}
\]

• This wouldn’t explain the observed beam-beam parameter limitation of the existing machines.
Nonlinear kick

![Graph showing nonlinear kick and head-on collision vs long-range beam-beam collision]
Multi-pole expansion
Nonlinear tune shift

(0.685, 0.695) w/o beam-beam effect

Every particle has its own tune shift as a function of its transverse amplitude.

Incoherent effect.

Nonlinear resonance induces the beam-beam limit.

Need sophisticated simulation efforts to understand and predict.
Beam-beam simulation

• **Soft Gaussian Model**
  – Bunch always preserve Gaussian shape in transverse direction.
  – Analytical formula exists even with non-round beam (Bassetti & Erskine)

\[
E_x - iE_y = -\frac{ine}{2\varepsilon_0\sqrt{2\pi(\sigma_x^2 - \sigma_y^2)}} \left[ w \left( \frac{x + iy}{\sqrt{2(\sigma_x^2 - \sigma_y^2)}} \right) - \exp \left( -\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} \right) w \left( \frac{x\sigma_y}{\sigma_x} + i\frac{y\sigma_x}{\sigma_y} \right) \right]
\]

• Self consistent model, field is calculate from transverse distribution via Poisson Solver
Symplectic Map

• To model the collision of two long bunches, the only known symplectic map, synchro-beam mapping can be used to keep symplecticity during long-term tracking.

• The effect is calculated for the slice of the opposing beam at position:

\[ s = \frac{(z_1 - z_2)}{2} \]
Synchro-Beam Map

• With the redefined coordinate:

\[
\begin{align*}
X &= x + p_x S(z, z_*), & P_X &= p_x, \\
Y &= y + p_y S(z, z_*), & P_Y &= p_y, \\
Z &= z, & P_Z &= \epsilon - \frac{p_x^2 + p_y^2}{4}
\end{align*}
\]

• The map:

\[
\begin{align*}
x^{new} &= x + S(z, z_*) f_X(X, Y; Z), \\
p_x^{new} &= p_x - f_X(X, Y; Z), \\
y^{new} &= y + S(z, z_*) f_Y(X, Y; Z), \\
p_y^{new} &= p_y - f_Y(X, Y; Z), \\
z^{new} &= z, \\
\epsilon^{new} &= \epsilon - \frac{1}{2} f_X(X, Y; Z)[p_x - \frac{1}{2} f_X(X, Y; Z)] \\
&\quad - \frac{1}{2} f_Y(X, Y; Z)[p_y - \frac{1}{2} f_Y(X, Y; Z)] \\
&\quad - g(X, Y; Z).
\end{align*}
\]
Beam-Beam compensation (wire)

- For long-range beam-beam force is proportional to $1/r$

$$\Delta r' = \frac{2N r_0}{\gamma r} = \frac{2N ce^2 \mu_0}{4\pi mc \gamma r}$$

- Current-caring wire for long-range beam-beam compensation. The current produces $B_\varphi$ to generate same kick amplitude with opposite sign.
Beam-Beam compensation (e-lens)

- To compensate head-on collision, no magnet can do the job. Only candidate is another charged bunch with same transverse distribution.

e-lens experiment @ RHIC for head-on beam-beam compensation
e-lens, cont’d

<- No e-lens @ 2e11 p per bunch

With e-lens @2e11 p per bunch ->

<- With e-lens @ 2.5e11 p per bunch

With e-lens @3e11 p per bunch ->
e-lens, cont’d

- Electron beam must have same Gaussian transverse distribution. When collide with ion beam, the beam-beam force can compensate the one from the other ion beams.

[Simulation S. White]
A BRIEF INTRODUCTION TO BEAM-BEAM EFFECT IN LINACs
Disruption parameter

• The beam-beam limit in ring does not exist if the beam is used only once.

• Therefore beam-beam parameter can be much larger.

• Usually another parameter is important: Disruption parameter

\[ d = \frac{\sigma_{z,opp}}{f_{bb}} \]
Disruption effect with large disruption parameter

- The strong nonlinear field will distort the transverse distribution.

The beam will rotate in phase space while traveling through the opposing beam.

\[ n \sim \sqrt{d} \]

Emittance growth due to nonlinearity and mismatch.
Beamstrahlung

• In lepton colliders or lepton-ion colliders, the electron beam will radiate in the opposing beam. This is name Beamstrahlung, Beam+Bremsstrahlung.

• Beamstrahlung parameter:

\[ \Upsilon \sim \frac{5 \ r_e^2 \gamma N_i}{6 \ 2\alpha \sigma_z \sigma_r} \]

• Much less then unity: classical SR
• Much larger than unity: Quantum effect
Reference

• Lecture of Beam-beam effects, W. Herr
• Beam-beam effect study in ERL based eRHIC, Ph.D Thesis, Y. Hao, Indiana University
• Y. Luo, et.al. PRSTAB, 15, 051004, 2012