THE COOLER INJECTOR SYNCHROTRON AT IUCF

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Submitted to the faculty of the University Graduate School
in partial fulfillment of the requirement
for the degree
Doctor of Philosophy
in the Department of Physics,
Indiana University

July 1998
Accepted by the Graduate Faculty, Indiana University, in partial fulfillment of the requirement for the degree of Doctor of Philosophy.

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July, 1998

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To

my parents,

my wife Xiuying

and

daughter Weisi
Acknowledgments

This dissertation could not have been completed without the constant encouragement, guidance, suggestions, criticisms, support and help from numerous people. I would first of all give my special thanks to Professor Shyh-Yuan Lee of Indiana University, Chair of my Dissertation Committee. Throughout the years of my Ph.D. study, he was always there to mentor, encourage, and support me both academically and morally. His expertise in various areas of physics initiated and reinforced my interest in accelerator physics, and his kindness, patience, and understanding walked me through many difficult times. The present research would not have been possible without his total support in all aspects.

I would like to thank the other members of my Committee, Prof. Bennet B. Brabson, Prof. Malcolm Macfarlane, Prof. Peter Schwandt of Physics Department, and Prof. Andrew J. Hanson of Computer Science Department, for their constructive advice and expertise suggestions to my thesis.

I am also grateful to the staff of the Indiana University Cyclotron Facility. In particular, I would like to thank all the members of the CIS project group for their work to make the experiments possible. Special thanks goes to Dr. Dennis Friesel, the CIS project manager, for his support and help on the CIS design and experiments. I would also like to thank Dr. George P. Berg for his patient discussion and direction of the dipole design and mapping, rf cavity field calculation and helping me solve many other problems. Thanks to Dr. Alex Pei for his helpful discussion and help on the rf cavity design and operation. Thanks also to Brett Hamilton, Mark Ball, Tom Rinkel, Terry Sloan, John Calahan, Walt Fox, David Jenner, Mark Luxnat for their help and discussion of various problems. Thanks are also due to former members of the accelerator physics group at IUCF, Dr. Derun Li, Dr, Lian Wang, and Dr.
Jiayang Liu. They gave a lot of help during my first two years of study. Thanks also for the help from the group members of the accelerator physics at IUCF, Dr. Scott Berg, Dr. Paul Chu, Dr. Dong-o Jeon.

This work was funded in part by Department of Energy, National Science Foundation and the Indiana University and their support is gratefully acknowledged.

Finally, I thank my parents for giving me their constant love and support over the years. And most of all, I thank my wife, Xiuying Zhang, for her constant support, and my daughter Weisi Kang, for being such a wonderful and loving child, during the years of my Ph.D. study.
Abstract

Xiaojian Kang

THE COOLER INJECTOR SYNCHROTRON AT IUCF

A fast ramping synchrotron at IUCF, Cooler Injector Synchrotron (CIS), which will be used as the injector for the existing Cooler, is designed to accelerate high-intensity polarized protons (deuterons) from 7 MeV (5 MeV) to 200 MeV (107 MeV). The circumference of CIS is 1/5 of that of the Cooler ring. The CIS lattice is composed of four superperiods, each of which is composed of a drift space and a dipole magnet with 90° bending angle and 12° edge angle at both ends. The effects of the eddy current on the dynamic aperture and on the vacuum chamber temperature are studied. Passive eddy current compensating coils were installed to minimize the eddy current sextupole effect. The emittance growth due to the stripping foil scattering is compared with experimental measurement. The adiabatic capture of high intensity proton beam is studied. Experimental results of the adiabatic capture confirmed the prediction of the ramping simulation. Single-turn extraction and transport beam line system is designed and installed to achieve the bucket-to-bucket beam transfer from CIS to Cooler. With the proper positions and strengths of the quadrupoles and dipoles along the beam line, the beam parameters can be matched properly both transversally and longitudinally at the Cooler injection point. Experimental measurements of beam properties of the CIS will be presented.
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Chapter 1

Introduction

The Indiana University Cyclotron Facility (IUCF) is a National Facility funded by the National Science Foundation (NSF). Research on a wide range of disciplines is carried out using particle beams. The major facilities include a 15 MeV injector cyclotron, 200 MeV main cyclotron, and Cooler ring with electron cooling. Figure 1.1 shows the layout of the IUCF and beam transport lines to experiment areas. The research is carried out with internal targets in the Cooler ring as well as with external beams from the cyclotron when not in use to fill the Cooler. In the cooling section of the Cooler ring, an electron beam that travels at the same velocity as that of the proton beam can efficiently reduce the proton beam emittance. This dramatically improve the beam’s quality by increasing beam lifetime and lowering the longitudinal spread in the proton beam energy to 2 keV at 200 MeV. The small emittance then allows new levels of precision and flexibility for nuclear physics research. However, experiences with the Cooler also revealed some limitations such as the limited luminosity, small duty factor, and long accumulation time.

To improve the duty factor for nuclear physics experiments, Indiana University and NSF jointly funded the construction of a dedicated, low energy, rapid cycling
Figure 1.1: Indiana University Cyclotron Facility layout. The main components of the accelerator system are (1) the polarized ion source HIPIOS, (2) the unpolarized duoplasmatron ion source, (3) the 15 MeV injector cyclotron, (4) the 200 MeV main cyclotron, (5) the Cooler, and (6) the Cooler Injector Synchrotron with its RFQ and linac. Major experimental areas include two target stations (A) and (G), (S) a Siberian snake for polarization experiments, (T) a magnetic channel, (E) a beam swinger for (p, n) charge exchange reactions, (K) the K600 high-resolution spectrometer, and (N) the polarized neutron line. Applied research is conducted on (X) radiation therapy and radiobiology, and (Y) on radiation effects in materials science.
Introduction

A synchrotron, called the Cooler Injector Synchrotron (CIS), in August of 1994 [1]. CIS is aimed to enhance the capabilities of the existing synchrotron and Cooler ring by removing many of the present difficulties regarding the injection of beam from cyclotron into the Cooler, and giving a significant increase in luminosity.

As shown in Figure 1.2, the whole CIS project consists of a pulsed negative ion source, a 7 MeV $H^-$ pre-injector, a 9 m long injection beam line, a rapid cycling 200 MeV proton synchrotron, and the extraction beam line from CIS to Cooler. CIS has the ability to accelerate 7 MeV high-intensity beams of polarized protons (or 5 MeV deuterons) from an RFQ/DTL to 200 MeV (105 MeV) for injection into Cooler ring (RFQ stands for Radio Frequency Quadrupole, DTL for Drift Tube Linac).

The design of CIS lattice is to provide an ideal model of a compact synchrotron with excellent performance. Based on economics and performance requirement, a lattice with the circumference of 17.364 m, one fifth of the Cooler ring, is chosen. The lattice has four superperiods, each composed of a drift space and a dipole magnet with 90° bending angle. Chapter 2 describes the CIS lattice design and parameters in detail. A constraint of the CIS lattice is that the betatron tunes should stay away from any resonances.

To ensure the performance of the CIS ring, the dipole quality is a very critical factor. The CIS ring has four identical 90° bend, normal conducting dipole magnets with a magnetic lengths of 2 m, bending radius of $\rho = 1.273$ m, and edge angles of 12° at both ends. The laminated magnets are designed for the initial ramping rate of 1 Hz and future operation at rates up to 5 Hz. The dipole is designed to provide a "good field" region of the elliptical vacuum chamber size of 5 cm × 10 cm. Over this region, the maximum relative field deviation is $\Delta B/B < 1 \times 10^{-3}$ up to a maximum dipole field of 1.7 T. In addition to the requirement of the dipole field strength and its deviation, multipole field moments are important field quality factors. Due to the field symmetry, the quadrupole field and multipole moments higher than sextupole
Figure 1.2: Schematic layout of CIS project

will be small. The sextupole field will reduce the dynamic aperture and thus limit the performance of the machine. Field maps are calculated in 2D and 3D codes to determine the dipole field strength and its sextupole components. The sextupole field due to the eddy current induced in the dipole vacuum chamber, which is proportional
to the ramping rate of the dipole field, is also calculated and corrected. Chapter 3

discuss the CIS dynamic aperture vs dipole field quality.

Chapter 4 describes the optimization procedure of the CIS rf cavity based on an
rf cavity in use at IUCF. The requirement for the rf cavity operating from 1.3 to 10.1
MHz, a nearly 8:1 frequency swing, has been achieved by optimizing the external
ferrite biasing. With heavy ferrite loading, the cavity is made short enough to be
accommodated in the space available in the CIS ring. Tuning is accomplished by
imposing an external dc magnetic field on the ferrite that is predominantly parallel
to the cavity rf field. The effective permeability of the ring $\partial B/\partial H$ steadily decreases
with an increase of the bias field due to ferrite saturation. An external quadrupole
magnet provides a dc magnetic field strength inside the ferrite from 0 to 0.27 T, near
the saturation value. Thus the effective rf permeability can change from maximum
(around 800) to nearly zero, which insures the wide frequency range. Using the 2D
finite element code MagNet [14], we optimized the cross section. Subsequently, 3D
calculations were done to determine the fringe field effects of the quadrupole field and
the total length of the cavity.

The $H^-$ beam from the RFQ/DTL, with an energy of 7 MeV, pulse length of 25
$\mu$sec and emittance of 10 $\pi\mu$m, is strip-injected into CIS. The $H^-$ beam is converted to
protons by passage through a 4.5 $\mu$g/cm$^2$ thick, 6 mm $\times$ 22 mm Carbon foil fabricated
at the IUCF target laboratory. The emittance growth during strip injection and its
dependence on vacuum was analyzed in Chapter 5. Since the CIS commissioning in
March 1997, many experimental runs [3] have been carried out on CIS to improve its
performance. Experiment results show that CIS can accelerate a proton beam to 250
MeV in one second with up to $1.2\times10^{10}$ protons per bunch, using a duoplasmatron
source of 40 $\mu$A output intensity and 400 $\mu$s pulse length. Further efforts such as the
installation of debuncher, orbit bump timing, etc. are still being made to enhance its
capability to the design goal of $2\times10^{10}$ per bunch.
In order to inject the CIS beam into the existing Cooler, an extraction line was designed to perform the bucket-by-bucket transfer of the CIS beam to the Cooler. The dipoles and quadrupoles along the transfer beam line are tuned to match the phase space ellipse at the Cooler ring. Spin rotators in the beam lines are used for polarization matching. Cooler injection is realized using the existing septum and fast kicker in the Cooler ring. All these will be discussed in detail in Chapter 5.

The conclusion is given in Chapter 6.
Chapter 2

CIS Lattice Design

The Cooler Injector Synchrotron (CIS) is used to replace the existing cyclotrons to fill the Cooler at IUCF. The new Cooler injection system will increase the beam intensity for nuclear physics experiments at the Cooler ring. The CIS is designed to accelerate high-intensity polarized proton (deuteron) beams from 7 MeV (5 MeV) to 200 MeV (105 MeV). The beam is injected from a pre-accelerator composed of a 3 MeV RFQ and a 4 MeV DTL. The beam bunch is then injected to the Cooler storage ring for further acceleration. CIS is a zero gradient synchrotron with a superperiod of four. Each period consists of a drift space and a dipole magnet with vertical edge focusing in lieu of a separate quadrupole. In order to find the best lattice for the CIS, namely, to locate a good working point in tune diagram, i.e. betatron tunes should avoid all systematic linear and nonlinear betatron resonances and spin resonances to enhance the injection efficiency. To limit the construction costs, we studied many lattice configurations. The chosen lattice has a circumference of 17.364 m, one fifth of the Cooler ring. The general layout of the CIS is shown in Figure 2.1.

The lattice parameters are listed in Table 2.1. In this chapter, we discuss the study of the CIS lattice by varying parameters, such as the dipole length and the
edge angle, and by changing the lattice properties by putting symmetrical trim quads in the ring.

### 2.1 CIS Lattice Parameters

To minimize the cost, we choose the CIS lattice to contain only dipoles without quadrupoles as shown in Figure 2.1. Thus the only free parameters are the edge angle $\theta_e$, the length of the dipoles $L$ and the circumference of the machine. The
circumference of CIS is chosen to be 1/5 of the Cooler ring. Scanning over $\theta_e$ and $L$ of the dipoles, a number of possible lattice designs were obtained using the accelerator design program MAD [12].

**Table 2.1: IUCF CIS lattice parameters**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbol</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circumference (1/5 Cooler)</td>
<td>$C$</td>
<td>17.364 m</td>
</tr>
<tr>
<td>Horizontal tune</td>
<td>$Q_x$</td>
<td>1.4633</td>
</tr>
<tr>
<td>Vertical tune</td>
<td>$Q_z$</td>
<td>0.7788</td>
</tr>
<tr>
<td>Dipoles:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length</td>
<td>$L$</td>
<td>2.0 m</td>
</tr>
<tr>
<td>Edge angle</td>
<td>$\theta_e$</td>
<td>12°</td>
</tr>
<tr>
<td>Bending radius</td>
<td>$\rho$</td>
<td>1.2732 m</td>
</tr>
<tr>
<td>Transition energy</td>
<td>$\gamma_T$</td>
<td>1.271</td>
</tr>
<tr>
<td>Max. betatron functions</td>
<td>$\beta_{x,\text{max}}$</td>
<td>4.373 m</td>
</tr>
<tr>
<td></td>
<td>$\beta_{z,\text{max}}$</td>
<td>3.786 m</td>
</tr>
<tr>
<td>Min. betatron functions</td>
<td>$\beta_{x,\text{min}}$</td>
<td>0.996 m</td>
</tr>
<tr>
<td></td>
<td>$\beta_{z,\text{min}}$</td>
<td>3.380 m</td>
</tr>
<tr>
<td>Dispersion functions</td>
<td>$D_{x,\text{max}}$</td>
<td>1.759 m</td>
</tr>
<tr>
<td></td>
<td>$D_{x,\text{min}}$</td>
<td>1.617 m</td>
</tr>
<tr>
<td>Horizontal chromaticity</td>
<td>$C_x$</td>
<td>-0.529</td>
</tr>
<tr>
<td>Vertical chromaticity</td>
<td>$C_z$</td>
<td>-0.156</td>
</tr>
</tbody>
</table>
2.1.1 Tune Diagram

Tune changes due to the variations of edge angle $\theta_e$ and dipole length $L$ are plotted in tune diagram as shown in Figure 2.2, where betatron resonances and spin tune at

![Tune Diagram](image-url)

**Figure 2.2:** Tune diagram of the Cooler Injector Synchrotron at IUCF.
- □ stands for $\theta_e$ changing from 12° to 26° for $L = 1.7$ m, △ for $\theta_e$ from 6° to 24° for $L = 1.8$ m, * for $\theta_e$ from 6° to 22° for $L = 1.9$ m and ○ for $\theta_e$ from 12° to 26° for $L = 2.0$ m.
- The step between two points is 2°. Dashed lines are the spin resonance range within the CIS energy extent.
injection energy and the extraction energy are also shown. The working point in tune
diagram should be away from the systematic betatron and spin resonances shown in
Figure 2.2.

We note that the betatron tunes are near systematic third order resonances driven
by sextupole fields. This is extremely important since both the dipole field and eddy
current field have sextupole components. Thus the third order resonances should be
corrected to avoid these 3rd order systematic resonances with a lattice composed only
of dipoles.

Higher order resonances occur at

\[ kQ_x + lQ_z = nP \] (2.1)

where \(|k| + |l|\) is the order of resonances, \(n\) is an integer and \(P\) is the superperiod.
However, higher order resonances will have a weaker strength due to the weak field
strength. The tune diagram, Figure 2.2, shows all the resonance lines up to fourth
order.

As CIS will be used to accelerate polarized beams, it is also required to avoid spin
resonances \([5]\). The strongest spin resonance is \(Q_z = G\gamma + k\), where the \(G\) factor
for protons is 1.7928 and \(k\) is any integer. In the energy range from 7 MeV to 220
MeV, \(G\gamma\) varies from 1.8062 to 2.2132. The dashed lines in Figure 2.2 show the spin
resonance range in the energy range (for \(k = -1\)).

Considering all the above factors, the working point is chosen at the point de-
noted by the symbol \(\bullet\), where the horizontal and vertical tunes are 1.463 and 0.779,
respectively. The corresponding dipole length is 2 meters and the dipole edge angle
is 12°.
2.1.2 Betatron Functions

When scanning over the edge angle and the length of dipoles, the lattice functions are also changed due to the variations of the effective focusing forces in both planes. Figure 2.3 shows the lattice function distributions of $\beta_x$, $\beta_z$ and $D_x$ (horizontal dispersion function) at the working point. We note that $\beta_z \approx 3.5 \text{ m}$ and $\beta_x \approx 1.0 \text{ m}$ at the injection stripper location. The maximum horizontal betatron function $\beta_x$ is 4.373 m, which happens at the center of each dipole. Considering the injection emittance of $34 \pi \mu m$, the maximum beam size is 12.2 mm, about half the vacuum chamber size of 25 mm.

![Betatron Functions Diagram](image)

**Figure 2.3:** The CIS lattice functions $\beta_x$, $\beta_z$ and $D_x$ for $L = 2.0 \text{ m}$, $\theta_c = 12^\circ$
2.1.3 Dispersion Function and Chromaticity

Momentum spread could result in closed orbit changes through the dispersion so as to cause emittance growth during injection. At a $D_x \approx 1.5$ m, the emittance growth due to energy loss at the stripper thickness of $4.5 \mu g/cm^2$ is tolerable [See Section 5.2.1]. The final momentum spread of the injected beam is about $2 \times 10^{-3}$. Thus the chromaticity of the machine dose not cause large tune spread. The effects of the dispersion and the chromaticities on the variations of $\theta_e$ and $L$ are also studied. It becomes clear that the chromaticities of the CIS lattice are small.

2.1.4 Momentum Compaction and Transition Energy

![Graph showing variation of momentum compaction factor $\alpha$ with respect to edge angle for different dipole lengths.](image)

Figure 2.4: Momentum compaction factor $\alpha$ variation with respect to $L$ and $\theta_e$
To avoid the problems associated with passing the transition energy during acceleration, we also need to keep tracking the changes of the momentum compaction factor $\alpha (\alpha = 1/\gamma^2)$, where $(\gamma - 1)mc^2$ is the transition energy) when other parameters are adjusted. Figure 2.4 shows the change of $\alpha$ with $\theta_c$ and $L$. At the working point, $\gamma = 1.271$, and the corresponding transition energy of 254.3 MeV is above the CIS energy range of 7 to 220 MeV.

2.1.5 Field and Alignment Error Analysis

Error analysis has been carried out by putting random errors into the dipole fields, dipole parameters and the dipole alignment. We have given a random $\pm 3 \times 10^{-4}$ relative field strength error distribution, $\pm 0.05^\circ$ random rotation alignment errors around the beam axis to all dipoles and $\pm 10\%$ random errors to edge angles $\theta_c$ of all the dipoles. Horizontal tune changes of $\pm 1.5\%$ and vertical tune changes of $\pm 4.5\%$ were assumed. The maximum closed orbit changes are 0.911 mm and 3.302 mm in the horizontal and vertical planes, respectively.

2.2 The CIS Lattice With Trim Quadrupoles

In order to have some flexibility of adjusting betatron tunes, four trim quadrupoles will installed in each straight section. However, we do have the options of using two or four trim quads with their positions adjustable.

2.2.1 Tune Diagram

Figure 2.5 shows the tuning range of the CIS with these trim quads. In the case of two trim quads, the machine is stable for the vertical tune below 1.0, which is shown in Figure 2.5 by the symbols $\Delta$ below the line of $Q_z = 1$. 
Figure 2.5: Tune diagram for the CIS with trim quads, where ○ stands for the working point without trim quads, △ for the lattice with two trim quads and all others having four trim quads with their locations adjusted with respect to the dipoles.

Figure 2.6 shows the comparison of the MAD calculation and the experimental measurement of the tune shifts by the four symmetrical trim quads. The trim quads can shift the horizontal tune by $\Delta Q_x = 0.114$ and vertical tune by $\Delta Q_z = 0.250$.

In the case of four asymmetrical quads with superperiod of two, there is a vertical tune gap due to the integer resonance. Within this gap, the vertical motion is unstable as shown in Figure 2.7. The corresponding lattice function distributions are shown...
Figure 2.6: Tune shift by four symmetrical trim quads. Where ○ stand
for CIS working point, △ for the calculation curve, and ●
for the experimental measurement

in Figure 2.8.

2.2.2 Momentum Compaction and Transition Energy

Figure 2.9 displays the variation of momentum compaction factor α. It turns out that
α does not change very much for different strengths of the quadrupoles. It, however,
offers the possibility of studying transition crossing in this lattice. In the case of
two trim quads, momentum compaction α changes sign at the quadrupole strength
between 7.68 and 7.80. This leads to a possible imaginary γ_T lattice (α < 0).
2.2 The CIS Lattice With Trim Quadrupoles

**Figure 2.7:** Maximum vertical betatron functions for the CIS with trim quads. Notations and conditions are as same as those used for Figure 2.5 on page 15.

**Figure 2.8:** Lattice functions for the CIS with four trim quads and each quadrupole 20 cm away from the dipole edge.
Figure 2.9: Momentum compaction factor $\alpha$ for the CIS with trim quads. Notations and conditions are as same as those used for Figure 2.5 on page 15

2.3 Summary of Lattice Study

In conclusion, we have studied the CIS lattice by varying parameters, such as circumference, the dipole length, the edge angle, the trim quads position and strength. We found that the CIS lattice parameters, which are listed in Table 2.1 on page 9, will meet our proposed requirements for beam performance. With the employment of trim quadrupoles, the machine possesses flexibility for betatron tune adjustments. Furthermore, $\gamma_T$ can be also varied by the trim quadrupoles. Thus, the CIS will become an interesting machine for accelerator physics studies such as transition energy crossing and imaginary $\gamma_T$ lattice in longitudinal beam dynamics studies.
Chapter 3

CIS Dipole Field and Dynamic Aperture

As shown in Figure 2.1 on page 8, the CIS ring has four identical 90° normal conducting dipole magnets with a magnetic lengths of 2 m and bending radius of \( \rho = 1.273 \) m. While the initial ramping rate for filling the Cooler storage ring is 1 Hz, the laminated magnets are designed for future operation at rates up to 5 Hz. A steel lamination thickness of 1.52 mm was found to be sufficient to overcome the eddy current problem in the steel core for a cycling rate up to 5 Hz. The laminations are stacked together into several wedges which are machined and assembled into a 90° bending magnet [13]. Figure 3.1 shows the dipole magnet wedge shape and lamination dimensions. The maximum field of 1.7 T is higher than required to accelerate protons up to the extraction energy of 200 MeV and deuterons up to 105 MeV.

The dipole magnet has a vertical gap of 5.82 cm, a radius of 1.273 m, a bending angle of 90°, an effective field length of 2 m and edge angles of 12° at entrance and exit faces. The required “good field” region within the vacuum chamber measures 5.0 cm vertically and 10 cm horizontally. Over this region, the maximum relative field
deviation is $\Delta B / B < 1 \times 10^{-3}$ up to the maximum field of 1.7 T. No field gradient is necessary. The edge angle of 12° at the entrance and exit of the dipole provides the needed vertical focusing. Table 3.1 summarizes the CIS dipole parameters.

(a) One wedge of CIS dipole  
(b) Upper half of dipole lamination and coil in cross section

**Figure 3.1:** CIS dipole wedge and lamination

Ion optics and lattice considerations determine the required “ideal” magnetic dipole field in 3 dimensions. Such ideal field cannot be achieved in reality because of material properties, machining and assembling uncertainties, etc.. Due to the symmetry of the magnet gap, the quadrupole and higher order multipoles, except sextupole, will be small. The sextupole field may affect the dynamic aperture of the ring. Two and three dimensional magnetic field codes are used to calculate dipole and sextupole components. Another source of the sextupole field is the eddy currents in the dipole vacuum chamber, which are proportional to the ramping rate of the dipole field. Calculations were performed to estimate the eddy current field and the correction method. The sextupole coefficients of the dipole field will have effect on natural chromaticities and dynamic aperture. The dynamic apertures in different
cases are calculated and discussed in this Chapter.

3.1 CIS Dipole Calculation and Measurement

With the finite element code “MagNet” [14], 2D and 3D dipole field calculations were performed to determine the optimum dimensions of the CIS dipole and its field quality [13]. 2D calculation was done to determine the cross section geometry while 3D calculation was used to determine the endpack shape in order to minimize saturation and eddy current. The field strength and sextupole components were determined by both 2D and 3D calculations.

Before dipoles were installed in the CIS ring, field measurements were carried out in order to determine the final endpack shape experimentally. The purpose of the endpack is to ensure the good quality of the field at the two ends of the dipole and

<table>
<thead>
<tr>
<th>Table 3.1: CIS dipole properties</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dipole length</strong></td>
</tr>
<tr>
<td><strong>Bending radius</strong></td>
</tr>
<tr>
<td><strong>Bending angle</strong></td>
</tr>
<tr>
<td><strong>Edge angle</strong></td>
</tr>
<tr>
<td><strong>Vertical gap</strong></td>
</tr>
<tr>
<td><strong>Proton energy</strong></td>
</tr>
<tr>
<td><strong>Dipole rigidity</strong></td>
</tr>
<tr>
<td><strong>Dipole strength</strong></td>
</tr>
<tr>
<td><strong>Field deviation</strong></td>
</tr>
<tr>
<td><strong>“Good field” region</strong></td>
</tr>
</tbody>
</table>
other design goals as discussed below. Both the theoretical and the experimental sextupole coefficients are presented in the tables and figures.

3.1.1 Dipole Field Calculation

<table>
<thead>
<tr>
<th>I (kA-turns)</th>
<th>(B_0) (T)</th>
<th>(E_p) (MeV)</th>
<th>(B_2) (T/m²)</th>
<th>(K_2 = B_2/B_0\rho) (1/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.0</td>
<td>0.3434</td>
<td>9.12</td>
<td>-0.0537</td>
<td>-0.1228</td>
</tr>
<tr>
<td>13.0</td>
<td>0.5578</td>
<td>23.89</td>
<td>-0.0910</td>
<td>-0.1281</td>
</tr>
<tr>
<td>18.0</td>
<td>0.7714</td>
<td>45.18</td>
<td>-0.1245</td>
<td>-0.1268</td>
</tr>
<tr>
<td>23.0</td>
<td>0.9835</td>
<td>72.41</td>
<td>-0.1609</td>
<td>-0.1285</td>
</tr>
<tr>
<td>28.0</td>
<td>1.1919</td>
<td>104.62</td>
<td>-0.2082</td>
<td>-0.1372</td>
</tr>
<tr>
<td>33.0</td>
<td>1.3932</td>
<td>140.40</td>
<td>-0.2678</td>
<td>-0.1510</td>
</tr>
<tr>
<td>38.0</td>
<td>1.5693</td>
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<td>-0.1881</td>
</tr>
<tr>
<td>43.0</td>
<td>1.6896</td>
<td>200.52</td>
<td>-0.5809</td>
<td>-0.2700</td>
</tr>
<tr>
<td>50.0</td>
<td>1.7833</td>
<td>221.18</td>
<td>-1.2228</td>
<td>-0.5386</td>
</tr>
</tbody>
</table>

I 2D Calculation

As shown in Figure 3.1, the C-shape of the magnet was chosen with the gap opening to the center of the synchrotron. With the C-magnet we avoided some mechanical problems to build and assemble such a magnet. As will be shown later, the quadrupole and other odd order field components can be made small enough not to cause problems. The gap height of 5.82 cm is determined by the design acceptance of the lattice.
Figure 3.2: Field calculation curve when coil current is 50 kA-turns. Solid line is calculation field strength. Curve with the solid squares is the polynomial fit in the fitted region.

The large ratio of gap width to gap height of 5.6 is needed to limit the sextupole component to an acceptable level at the high field of 1.68 T where saturation has started to become significant.

Based on the above configuration, systematic calculations of the field for coil currents (ampere-turns $NI$) in the range from 0 to 50 kA-turns were made. The field along the center line of the magnet gap was fitted by a third order polynomial in order to evaluate the higher order components.

Table 3.2 shows results of the 2D calculation for the available coil current range. Figure 3.2 shows the dipole field in the mid-plane around the center of the pole pieces at maximum current. The magnetic multipole components are obtained by fitting
Figure 3.3: Calculation results of the dipole field component $B_0$

the field by a Taylor series from $R = 1.2282 \text{ m}$ to $1.3182 \text{ m}$:

$$B(x) = B_0 + B_1(x - x_0) + \frac{1}{2}B_2(x - x_0)^2 + \frac{1}{6}B_3(x - x_0)^3$$  \hspace{1cm} (3.1)

where $B_0$ is dipole field, $B_1 = \frac{\partial B}{\partial x}$ is quadrupole component and $B_2 = \frac{\partial^2 B}{\partial x^2}$ is sextupole component, $x_0 = 1.2732 \text{ m}$ is the central path of particles (defined by the magnetic center of the gap). Two different fits were done to the calculation for $NI = 38 \text{ kA-turns}$, one with, the other without the odd power terms $B_1$ and $B_3$. Since the fit was of similar quality with $B_1 = B_3 = 0$ we decided to omit these terms except for the lowest currents in which case there was a small $B_1$-term. The inclusion of 4th order and higher terms was not considered necessary in the fitted range of $9 \text{ cm}$. Table 3.2 shows the fitting results of all calculations. In the table, the relation between proton
energy in [MeV] and the dipole rigidity $B\rho$ in [Tm] is:

$$T = \sqrt{(299.7925B\rho)^2 + E_0^2 - E_0}$$

(3.2)

where $\rho = 1.2732$ m is the bending radius and $E_0 = 938.2723$ MeV is proton rest mass energy.

Figure 3.3 shows the dipole component $B_0$ as function of the total current $NI$ per coil together with a linear approximation which represents the calculation well in the current range up to about $NI = 30$ kA-turns. Saturation effects become increasingly important at higher currents. At 43 kA-turns, the calculated field of 1.5693T is 8.1% below the linear approximation.

II Hole in the Upper Return Yoke of Dipole 3

Extraction of 200 MeV protons out of the CIS ring is accomplished with a fast traveling wave kicker at straight section D1-D2 (See Fig. 2.1 on page 8) and a Lambertson magnet at straight section D2-D3. With the length of the Lambertson magnet limited by the available space, the extracted beam cannot clear the return yoke of the CIS main dipole magnet D3 located down stream. Thus, the extraction beam pipe will pass through a hole in the upper return yoke of D3.

The presence of the hole in the return yoke should not significantly disturb the main dipole field in the gap and the magnetic field within the hole had to be small enough not to disturb the extracted beam. At high coil current of $NI = 50$ kA-turns, the field in the gap is not disturbed by more than about 0.3 mT which is considered negligible. Without shielding the field in the hole is of the order of 2.7 mT - 4.3 mT. Therefore a cylindrical soft iron shield of $1/4$ inch thickness and $1/4$ inch distance from the wall of the hole was introduced. This reduced the field in the hole to about 0.3 mT which was considered negligible.
III Endpack Optimization

The endpack is the part at two ends which is shaped to provide the required field integral of the magnet. The endpack will be designed to provide the following dipole field properties:

1. Effective Field Length $L_{eff} = 2.0$ m at radius of $R = 1.273$ m, independent of magnetic field in the range of $B = 0.35$ T to 1.7 T.

2. Effective edge angle of $12^\circ$ as required by the lattice calculation.

3. Sextupole component $B_2 < 1.0$ T/m².

4. Good field region of 8 cm at 1.7 T and 10 cm at 1.0 T.

Field calculations to determine the shape of the endpack were performed in two steps. First, the 2D profile is determined to fulfill the first two requirements and then subsequent 3D calculations are used to shape the endpack design.

For the 2D profile, we investigated two types of pole end profiles. A simple $45^\circ$ diagonal cut with the depth of the cut as free parameter, and a Rogowsky type profile [13] approximated by 2 straight cuts in which both angles were varied to find the optimum. The results for both profiles are independent of the field in the required region. However, the effective field boundary (EFB) of the Rogowsky approximation coincides very closely with the physical end of the magnet. We decided to adopt the Rogowsky approximation.

As a last step to determine the endpack, 3D calculations using the MagNet code were performed. The $\int Bdl$ is the quantity which defines the path of the circulating particles. Since the field inside the magnet has a negative natural sextupole field in the radial direction, we tried to compensate by giving the field a positive sextupole by increasing the field for radii smaller and larger than the central ray. This is done
by adding materials (what we call noses) to the pole pieces in the beam direction. We then calculated the field in the mid-plane of the dipole and the region in the fringe field and analyzed $\int Bdl/L_{eff}$ as function of the radius $R$, where “$L_{eff}$” is “Effective Field Length”. The average multipoles of the integrated dipole field are obtained by using a similar fitting method shown in Figure 3.2. Table 3.3 shows the results of the dipole component and average sextupole component including the approximate Rogowsky endpack profile. The Dipole component of the 3D calculation is also shown in Figure 3.3.

### 3.1.2 Dipole Field Mapping

#### I Mapping Setup

In the region of 13° inside the dipole from the edge and 30cm away from the edge, the dipole field was measured by using the magnet mapping system(16 Hall probe

<table>
<thead>
<tr>
<th>I(kA-turns)</th>
<th>$B_0$(T)</th>
<th>$E_p$(MeV)</th>
<th>$B_2 = B''(T/m^2)$</th>
<th>$K_2 = B_2/B_0p(1/m^3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.0</td>
<td>0.3006</td>
<td>7.00</td>
<td>-0.006</td>
<td>-0.015676</td>
</tr>
<tr>
<td>8.0</td>
<td>0.3435</td>
<td>9.13</td>
<td>-0.007</td>
<td>-0.016003</td>
</tr>
<tr>
<td>18.0</td>
<td>0.7722</td>
<td>45.26</td>
<td>-0.015</td>
<td>-0.015258</td>
</tr>
<tr>
<td>28.0</td>
<td>1.1897</td>
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<td>-0.096</td>
<td>-0.063378</td>
</tr>
<tr>
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<td>138.88</td>
<td>-0.192</td>
<td>-0.108871</td>
</tr>
<tr>
<td>38.0</td>
<td>1.5469</td>
<td>170.53</td>
<td>-0.390</td>
<td>-0.198022</td>
</tr>
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<td>-0.318799</td>
</tr>
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<td>1.7114</td>
<td>205.25</td>
<td>-0.961</td>
<td>-0.441045</td>
</tr>
</tbody>
</table>

**Table 3.3:** 3D calculation results with endpack optimization
array) at IUCF [15] to optimize the shape and dimension of the endpack profile experimentally.

Precise magnet field mapping provides reliable parameters for beam calculations, highlights magnet field problems, verifies magnet field calculations and establishes a database when problems arise in the future. The mapper system consists of a X-Y table with 112 (X) by 64 (Y) inch travel. The mapper arm is driven by stepping motors whose position is read out by absolute encoders (1 step = 0.00025 inch). A variety of Hall probe assemblies (in-line array, finger, cube) exist to accommodate different size and shape of magnets and field orientations. The Hall probe readout system consists of a HP scanner, HP digital voltmeter and Hall current supply. The readout system also reads the magnet current, the Hall current and the room temperature with the probe readings. For the calibration of the Hall probes, a series of NMR probes with typical precision of $\Delta B/B \sim 10^{-5}$ are available in the range of 0.044 T to 2.1 T.

II Mapping Data Analysis Method

To analyze the field mapping data, we calculated the curve of the average integrated dipole field $B_{av} = \int Bdl(R)/(L_{eff})$ as function of the radius $R$. Adjusting the edge angle can adjust the $L_{eff}$, thus change the integrated dipole field $B_{av}$ curve. From the $B_{av}$ curve, we know the field at the radius of 1.2732 m is $B_0$. The following function is used to fit the $B_{av}$ curve so as to get the sextupole component:

$$B(x) = B_0(x) + \frac{1}{2}B_2(x - x_0)^2$$

(3.3)

The fitting procedure is to select the parameter edge angle, $B(x)$ and $B_2$ to get the minimize difference between $B(x)$ and $B_0$. After we got the $B_{av}$ and an optimum edge angle which may not be 12°, we can find an effective radius $R_{eff}$ at which the $L_{eff}$ is equal to the theoretical dipole length 2.0 m. And at the theoretical radius $R = 1.2732$ m, the $L_{eff}$ is $L_{eff}$ rather than the theoretical design value 2.0 m.
III Mapping Results of Four Dipoles

Using the above data analysis procedure, dipole and sextupole field components are obtained and compared with the MagNet calculation results as shown in Figures 3.4 and 3.5. The four dipoles have essentially identical filed strengths. In Figure 3.5, the measured sextupole coefficient $B_2$, which is close to the 3D calculation results, increases quickly at higher field. The sextupole component can be corrected by the correction coils outside the vacuum chamber in the dipole region by providing a correction coil current which is a function of the main dipole current. The shape and positions of the correction coil will be discussed in Section 3.2.3.

As mentioned previously, the effective dipole length, radius and edge angle may differ slightly from the design values. To estimate the effects of these variations on

![Graph showing the relationship between coil current (kA.turns) and magnetic field strength (B in T) for different dipoles and calculation methods. The graph includes data points for 1st, 2nd, 3rd, and 4th dipoles as well as 2D and 3D calculations.]

**Figure 3.4**: Calculation and measurement results (solid lines) of CIS dipole field strengths
Figure 3.5: Sextupole coefficients of four CIS dipoles. Solid lines: mapping results

Table 3.4: Variation of tunes and lattice functions during the ramping

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Designed</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Variation(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_x$</td>
<td>1.4633</td>
<td>1.4449</td>
<td>1.4682</td>
<td>1.59</td>
</tr>
<tr>
<td>$\nu_y$</td>
<td>0.7788</td>
<td>0.7788</td>
<td>0.8083</td>
<td>3.79</td>
</tr>
<tr>
<td>Max. $\beta_x$ (m)</td>
<td>4.3727</td>
<td>4.3727</td>
<td>4.5723</td>
<td>4.57</td>
</tr>
<tr>
<td>Max. $\beta_x$ (m)</td>
<td>3.7857</td>
<td>3.7022</td>
<td>3.8035</td>
<td>2.68</td>
</tr>
<tr>
<td>Max. $D_x$ (m)</td>
<td>1.7593</td>
<td>1.7593</td>
<td>1.8210</td>
<td>3.51</td>
</tr>
<tr>
<td>$\gamma_T$</td>
<td>1.2710</td>
<td>1.2557</td>
<td>1.2718</td>
<td>1.26</td>
</tr>
</tbody>
</table>
the tunes and lattice functions during the ramping procedure, calculations have been done and the results are shown in Table 3.4. The variations are small enough to be neglected. The variation of chromaticities will be discussed in the Section 3.3 on page 45 together with the effect of eddy current.

3.2 Eddy Current in the Vacuum Chamber Wall

When the metallic vacuum chamber in a magnet gap is exposed to a rapidly varying magnetic field, the unavoidable eddy currents generated in the vacuum chamber wall become important because eddy currents can generate multipole errors [22] in the gap field.

![Diagram of Dipole Magnets and Correction Coils]

Figure 3.6: Geometry of two kinds of vacuum chambers: (a). Ellipse (b). Polygon

When a chamber is between iron poles, which can be approximated by infinite permeability upper and lower plane boundaries, the magnetic field can be obtained
3. CIS Dipole Field and Dynamic Aperture

easily by the analytical formulas [17]. Various calculation results for two cross sections of vacuum chambers shown in Figure 3.6 will be discussed in the following. For the CIS ring, we use an elliptical vacuum chamber.

3.2.1 Theoretical Analysis of the Eddy Current

I Multipole Expansion of Eddy Current Field

When the vacuum chamber is between the two parallel dipole iron plates with infinite permeability, the image current method is used to achieve the boundary conditions. Using Beth’s complex representation for the magnetic field \( H = H_y + i H_x \), the field at position \( z \) due to a current filament \( I \) at \( z_c = x_c + iy_c \) in two infinite permeable parallel iron plates is given by [17]:

\[
H = \frac{I}{4g} \left( \coth \left( \frac{\pi (z - z_c)}{2g} \right) + \tanh \left( \frac{\pi (z - z_c^*)}{2g} \right) \right). \tag{3.4}
\]

For a vacuum chamber wall, the magnetic field for a current has the integral form:

\[
B(z) = \frac{\mu_0}{4g} \int_{v.c.} I(z_c) \left[ \coth \left( \frac{\pi (z - z_c)}{2g} \right) + \tanh \left( \frac{\pi (z - z_c^*)}{2g} \right) \right] ds_c. \tag{3.5}
\]

If the main dipole field \( B_0 \) is uniform, the induced current density in the vacuum chamber is \( j = \sigma \dot{B} x \), where \( \sigma \) is the electrical conductivity and \( x \) is the distance from the chamber center. So the total eddy current field is:

\[
B(z) = \frac{\mu_0 \sigma h \dot{B}}{4g} \int_{v.c.} x \left[ \coth \left( \frac{\pi (z - z_c)}{2g} \right) + \tanh \left( \frac{\pi (z - z_c^*)}{2g} \right) \right] ds_c, \tag{3.6}
\]

where \( v.c. \) stands for integration along the vacuum chamber wall.

From Eq. (3.6), the eddy current is proportional to the vacuum chamber wall thickness \( h \), the dipole field rate of change \( \dot{B} \) and the horizontal position \( x \). The integral predicts the total induced field.
3.2 Eddy Current in the Vacuum Chamber Wall

Expanding $B(z)$ in a Taylor series about the origin to get:

$$B(z) = \frac{\mu_0 \sigma h \dot{B}}{2\pi} \sum_{n=0}^{\infty} \frac{1}{n!} \left( \frac{\pi}{2g} \right)^{n+1} z^n \int_{\text{v.e.}} x(\alpha_n + \beta_n) ds_c$$  \hspace{1cm} (3.7)

where

$$\alpha_n = \left. \frac{\partial^n \tanh(z - \frac{\pi z}{2g})}{\partial z^n} \right|_{z=0}, \hspace{1cm} \beta_n = \left. \frac{\partial^n \cosh(z - \frac{\pi z}{2g})}{\partial z^n} \right|_{z=0}.$$  \hspace{1cm} (3.8)

The eddy current field also can be written in a more concise multipole expansion form:

$$B(z) = B_y + iB_x = B_0 \sum_{n=0}^{\infty} (b_n + ia_n) z^n$$  \hspace{1cm} (3.9)

The multiple coefficients can be expressed as:

$$b_n = \frac{1}{B_0 n!} \frac{\partial^n B_y}{\partial x^n} \bigg|_{x=y=0}, \hspace{1cm} a_n = \frac{1}{B_0 n!} \frac{\partial^n B_x}{\partial x^n} \bigg|_{x=y=0}$$  \hspace{1cm} (3.10)

Thus from Eq.(3.7), the magnetic multipole coefficients $a_n$ and $b_n$ are:

$$b_n + ia_n = \frac{\mu_0 \sigma h \dot{B}}{2\pi n! B_0} \left( \frac{\pi}{2g} \right)^{n+1} \int_{\text{v.e.}} x(\alpha_n + \beta_n) ds_c$$  \hspace{1cm} (3.11)

When the vacuum chamber is symmetric with respect to the horizontal mid-plane, the integral in Eq.(3.11) becomes real, i.e. the skew multipole coefficients $\alpha_n$ vanish. Similarly, if the vacuum chamber is symmetric with respect to the vertical mid-plane, multipoles with $n = odd$ are also zero. The CIS vacuum chamber nearly satisfies the horizontal and vertical mid-plane symmetries, therefore only $b_n$ with $n = even$ survives.

II Power Loss in the Vacuum Chamber Wall

The power loss per unit length in the vacuum chamber wall is

$$\frac{dp}{dl} = \int_{\text{v.e.}} (j \hbar ds_c)^2 \cdot \frac{1}{\sigma \hbar ds_c} = \sigma h \dot{B}^2 \int_{\text{v.e.}} x^2 ds = \sigma h \dot{B}^2 a^3 G_p$$  \hspace{1cm} (3.12)
where $G_p$ is called the geometric factor.

$$G_p = \begin{cases} 4 \int_0^{\pi/2} \sin \phi (\cos^2 \phi + \sin^2 \phi)^{1/2} d\phi, & \text{ellipse} \\ 4 \left(1 + m^2\right)^{1/2} / 3 + \epsilon - m, & \text{polygon} \end{cases}$$  \hspace{1cm} (3.13)$$

and $m = (b - c)/a$, $\epsilon = b/a$.

### III Temperature Rise of the Vacuum Chamber Wall

The temperature of the vacuum chamber wall will rise due to the eddy current power loss. It is reasonable to consider that there will be a layer of air with the thickness of $l_a$ conducting the heat from the wall. Thus the heat conduction equation is [20]:

$$d^2\Delta T \over dx^2 - m^2 \Delta T = -q \over k_w$$  \hspace{1cm} (3.14)$$

where $q$ is the power per unit volume, $q = (dp/dl)/V$, $m = [k_a/(l_a h k_w)]^{1/2}$, $k_w, k_a$ are the thermal conductivities of the wall and air.

In a steady-state, the heat produced by the eddy current will totally be conducted by the air layer. Then the temperature rise will be simply [21]:

$$k_a \Delta T \over l_a = \frac{1}{S} \left( \frac{dp}{dl} \right)$$  \hspace{1cm} (3.15)$$

and $S$ is the area of the wall per unit length.

### IV Effect of Eddy Current on Chromaticity

The change of the horizontal and vertical chromaticities due to the sextupoles are:

$$\begin{cases} \Delta C_x = \frac{1}{4\pi} \int \Beta_x S(s) D(s) ds \\ \Delta C_z = -\frac{1}{4\pi} \int \Beta_z S(s) D(s) ds \end{cases}$$  \hspace{1cm} (3.16)$$

where $\Beta$ is the betatron function, $D(s)$ the dispersion function. $S(s) = B''/(B_0 \rho)$ is the sextupole strength function. Eddy current produces $b_2 = \frac{1}{2B_0} \partial^2 B_x \partial s^2$, thus $S(s) = \cdots$
2b_2/\rho$, where \( \rho \) is the radius of the dipole. Using the limit \( a/g >> 1 \), \( b_2 \) becomes:

\[
b_2 \approx \frac{\mu_0 \sigma h B_0}{B_0 g}
\]  

(3.17)

**Figure 3.7:** CIS dipole field and ramping rate for 1 Hz operation of CIS

### 3.2.2 Numerical Results

#### I Geometry and Field Parameters

The geometry parameters for the vacuum chambers are

1. Ellipse:
   
   \( a = 50 \text{ mm} \quad b = 25 \text{ mm} \quad g = 58.2 \text{ mm} \)

2. Polygon:
   
   \( a = 50 \text{ mm} \quad b = 25 \text{ mm} \quad c = 15 \text{ mm} \quad g = 58.2 \text{ mm} \)
We shall assume typical stainless-steel resistivity of $\sigma^{-1} = 0.8 \ \mu \Omega \text{m}$. In both cases the vacuum chamber wall thickness is $h = 1 \ \text{mm}$.

The magnetic field $B_0$ and the field ramping rate $\dot{B}$ changes with increasing particle energy. The sextupole is proportional to the quantity $\frac{\dot{B}}{B_0}$ which is shown in Figure 3.7 along with the change of $B_0$, $\dot{B}$ for 1 Hz operation of CIS.

**Table 3.5:** Multipole coefficients $b_n (\text{m}^{-n})$ of eddy current field

<table>
<thead>
<tr>
<th>n</th>
<th>Ellipse</th>
<th>Polygon</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-0.52175$\times 10^{-3}$</td>
<td>-0.56910$\times 10^{-3}$</td>
</tr>
<tr>
<td>2</td>
<td>0.12283$\times 10^{0}$</td>
<td>0.11209$\times 10^{0}$</td>
</tr>
<tr>
<td>4</td>
<td>-0.12601$\times 10^{2}$</td>
<td>0.95063$\times 10^{1}$</td>
</tr>
<tr>
<td>6</td>
<td>0.13050$\times 10^{4}$</td>
<td>-0.89176$\times 10^{4}$</td>
</tr>
<tr>
<td>8</td>
<td>0.29670$\times 10^{6}$</td>
<td>0.76808$\times 10^{7}$</td>
</tr>
<tr>
<td>10</td>
<td>0.23100$\times 10^{8}$</td>
<td>-0.89018$\times 10^{10}$</td>
</tr>
<tr>
<td>12</td>
<td>0.98885$\times 10^{10}$</td>
<td>0.48433$\times 10^{13}$</td>
</tr>
</tbody>
</table>

II Multipole Components $b_n$

For the two kinds of vacuum chambers, Table 3.5 shows the eddy current multipole components $b_n$ in the case of maximum $\frac{\dot{B}}{B_0}$ during the ramping procedure. They have similar low-order components but very different higher moments($n \geq 4$). However, for the higher multipoles, their effects will be very small provided that the beam size is less than 50% of vacuum chamber width. The induced fundamental dipole $b_0$ and
sextupole \( b_2 \) are of the major concerns.

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
\frac{\dot{B}}{B_0} (1/\text{sec}) & b_0 (\times 10^{-3}) & b_2 (\text{m}^{-2}) & S_2 (\text{m}^{-3}) & b_0 (\times 10^{-3}) & b_2 (\text{m}^{-2}) & S_2 (\text{m}^{-3}) \\
\hline
0.0000 & 0.0000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 \\
5.2384 & -0.4472 & 0.10983 & 0.17252 & -0.4878 & 0.10018 & 0.15736 \\
6.1114 & -0.5217 & 0.12813 & 0.20128 & -0.5691 & 0.11687 & 0.18359 \\
4.6218 & -0.3946 & 0.09690 & 0.15222 & -0.4304 & 0.0839 & 0.13884 \\
3.8188 & -0.3260 & 0.08007 & 0.12577 & -0.3556 & 0.07303 & 0.11472 \\
2.8959 & -0.2472 & 0.06072 & 0.09538 & -0.2697 & 0.05538 & 0.08700 \\
1.8335 & -0.1565 & 0.03844 & 0.06039 & -0.1707 & 0.03506 & 0.05508 \\
1.5178 & -0.1296 & 0.03182 & 0.04999 & -0.1413 & 0.02903 & 0.04560 \\
0.0000 & 0.0000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 \\
\hline
\end{array}
\]

**Table 3.6:** Dipole and sextupole coefficients \( b_0, b_2 \)

III  **Dipole Component \( b_0 \) and Sextupole Coefficient \( b_2 \)**

For the two different chambers, the dipole and sextupole components \( b_0, b_2 (1/\text{m}^2) \) are shown in Table 3.6. There is not much difference between these coefficients for the two kinds of vacuum chambers.

Figure 3.8 on page 38 shows the eddy-current produced field on the horizontal mid-plane for the two kinds of vacuum chambers at maximum \( \frac{\dot{B}}{B_0} \). Although they have a different field distribution at the edge, the two chambers have similar \( b_0 \) and \( b_2 \) as discussed above, and thus have the similar effect on beam performance. In the following discussion, we will focus on the ellipse vacuum chamber chosen for CIS.
Figure 3.8: Eddy current field in the mid plane

Figure 3.9: Sextupole component $b_2$ of eddy current
Figures 3.9 shows the change of $b_2(1/m^2)$.

IV Change of Chromaticities Due to $b_2$

<table>
<thead>
<tr>
<th>$\dot{B}/B_0$</th>
<th>$S_2(1/m^3)$</th>
<th>$C_x$</th>
<th>$C_y$</th>
<th>$S_2(1/m^3)$</th>
<th>$C_x$</th>
<th>$C_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0000</td>
<td>0.0000</td>
<td>-0.5290</td>
<td>-0.1560</td>
<td>0.0000</td>
<td>-0.5290</td>
<td>-0.1560</td>
</tr>
<tr>
<td>5.2384</td>
<td>0.1654</td>
<td>0.1308</td>
<td>-0.8029</td>
<td>0.1509</td>
<td>0.0731</td>
<td>-0.7463</td>
</tr>
<tr>
<td>6.1114</td>
<td>0.1930</td>
<td>0.2407</td>
<td>-0.9108</td>
<td>0.1761</td>
<td>0.1734</td>
<td>-0.8448</td>
</tr>
<tr>
<td>4.6218</td>
<td>0.1459</td>
<td>0.0532</td>
<td>-0.7268</td>
<td>0.1332</td>
<td>0.0023</td>
<td>-0.6768</td>
</tr>
<tr>
<td>3.8188</td>
<td>0.1206</td>
<td>-0.0479</td>
<td>-0.6275</td>
<td>0.1100</td>
<td>-0.0900</td>
<td>-0.5863</td>
</tr>
<tr>
<td>2.8959</td>
<td>0.0914</td>
<td>-0.1641</td>
<td>-0.5135</td>
<td>0.0834</td>
<td>-0.1960</td>
<td>-0.4822</td>
</tr>
<tr>
<td>1.8335</td>
<td>0.0579</td>
<td>-0.2978</td>
<td>-0.3823</td>
<td>0.0528</td>
<td>-0.3180</td>
<td>-0.3625</td>
</tr>
<tr>
<td>1.5178</td>
<td>0.0479</td>
<td>-0.3376</td>
<td>-0.3432</td>
<td>0.0437</td>
<td>-0.3543</td>
<td>-0.3268</td>
</tr>
<tr>
<td>0.0000</td>
<td>0.0000</td>
<td>-0.5290</td>
<td>-0.1560</td>
<td>0.0000</td>
<td>-0.5290</td>
<td>-0.1560</td>
</tr>
</tbody>
</table>

As shown in Eq. (3.16), chromaticities will be affected by the sextupole coefficients of the eddy current field. Table 3.7 shows all the chromaticity changes during the ramping. $C_x$ and $C_y$ show big changes from their original values.

V Power Loss and Temperature Rise

The power loss per unit length is less than 13 W/m. If we think that the temperature will be the room temperature 4 cm away from vacuum chamber since the air is a poor heat conductor, the maximum temperature rises of the vacuum chamber are 92°C and
76°C respectively for the ellipse and polygon structure. In fact, since we neglected the effects of the air convention and radiation, we believe the actual temperatures will be lower than the above values.

### 3.2.3 Sextupole Component Correction

From the discussions in Section 3.1 and 3.2.2, the total sextupole field in the dipole gap region mainly come from two sources: (1). Dipole body field, (2). Eddy current field. Since it can produce unexpected chromaticities and reduce the beam dynamic aperture [See Section 3.3], sextupole field correction is needed. Correcting sextupoles along the ring could be used to compensate the sextupole field. A more economical solution to the problem, however, are self-correcting coils which are capable of compensating sextupole components in the dipole region.

#### I Self-correction Method for Eddy Current Sextupole Field

The concept of self-correcting coils has been used in the AGS Booster [22]. The self-correction method has several advantages.

- It eliminates the nonlinearity at the source since the self-correction coils follow the geometry of the vacuum chamber within the dipole.

- Vacuum chamber positional tolerances are no longer important because the canceling fields automatically have the same displacement coordinates. This feature is not available when lumped correction magnets are used.

- External correction magnets can only correct in first order for the eddy current aberrations. Self-correction method can remove sextupole, suppress partially higher order multipoles, and partially correct the dipole component.
• A single array of correction coil can be used to correct sextupole components from both dipole body field and eddy current by an external power supply.

• It requires no extra space in the ring for the correction.

After self-correction, the optical properties are close to the computed lattice design. Additional lattice chromaticity sextupole magnets and other correction magnets, if any, can be used to maximize high current performance.

As shown in Figure 3.5 on page 30, the dipole body field has a defocusing sextupole components at high field. On the other hand, the eddy current field is proportional to the horizontal distance from the center of the chamber. In Figure 3.6 on page 31, three turns of correction coil per quadrant in two locations are shown: one turn at \( a/3 \) and two turns at \( 2a/3 \). The correction coils with the above structure can produce a positive sextupole field to compensate that of the dipole field, and produce the opposite field to cancel the sextupole field from eddy current.

II Correction of the Sextupole Component of the Dipole Field

In Figure 3.5 on page 30, the sextupole component of the main dipole field is seen to increase faster at high \( B \) field. If the correction coil current is a function of dipole field \( B \), i.e.

\[
I_c = f(B)
\]  

(3.18)

the correction coil can provide a positive sextupole component which will cancel the sextupole field from dipole system. In this case, \( I_c \) is properly programmed as a function of main dipole ramping current supply.
III Correction of Eddy Current Sextupole Component

The eddy current sextupole component is proportional to the ramping rate $\dot{B}_0$ of the main dipole as shown in Eq. (3.17). If the correction coil current $I_c$ is proportional to $\dot{B}$:

$$I_c = K_c \cdot \dot{B} \quad (3.19)$$

where $K_c$ is the self-correction coefficient, the self-correction field will be

$$B_c(z) = \frac{\mu_0 I_c}{4g} \sum_{k=1}^{4m} \left[ \coth \left( \frac{\pi (z - z_c)}{2g} \right) \tanh \left( \frac{\pi (z - z_c^*)}{2g} \right) \right]$$

$$= \frac{\mu_0 K_c \dot{B}}{2\pi} \sum_{n=0}^{\infty} \frac{1}{n!} \left( \frac{\pi}{2g} \right)^{n+1} z^n \sum_{k=1}^{4m} (\alpha_{cn} + \beta_{cn}) \quad (3.20)$$

where the $m$ is the number of correction coil turns per quadrant.

Expanding it in the Taylor series and use the multipole form for the eddy current,

$$B_c(z) = \frac{\mu_0 K_c \dot{B}}{2\pi} \sum_{n=0}^{\infty} \frac{1}{n!} \left( \frac{\pi}{2g} \right)^{n+1} z^n \sum_{k=1}^{4m} (\alpha_{cn} + \beta_{cn}) \quad (3.21)$$

$$= B_0 \sum_{n=0}^{\infty} (b_{cn} + ia_{cn}) z^n \quad (3.22)$$

The multipole coefficients of the eddy current are:

$$b_{cn} + ia_{cn} = \frac{\mu_0 K_c \dot{B}}{2\pi n! B_0} \sum_{k=1}^{4m} (\alpha_{cn} + \beta_{cn}) \quad (3.23)$$

where $a_{cn} = 0$ and odd $b_{cn} = 0$ due to the correction coil symmetry.

To cancel the eddy current field completely, $B_c(z)$ needs to be equal to $B(z)$ in Eq. (3.6) for any value of $z$, which is impossible for a fixed value $K_c$. However, from Eq. (3.23) and Eq. (3.11), the self-correction coefficient $K_c$ can be chosen to remove one of the multipole components when $b_{cn} = -b_n$:

$$K_c(n) = -\frac{\sigma h}{\sum_{k=1}^{4m} (\alpha_{n} + \beta_{n})} \int_{v.c.} x(\alpha_{n} + \beta_{n}) ds_c$$

$K_c$ depends on the vacuum chamber geometry and the positions of the correction coils for a multipole moment $b_n$. While $b_n$ can be canceled completely, other moments are
partially removed. Since the major component affecting the beam performance is the sextupole $b_2$ and higher moments are small, we choose $K_c = K_c(2)$ to cancel the sextupole component completely.

**Table 3.8:** Multipole coefficients $b_n (m^{-n})$ after correction

<table>
<thead>
<tr>
<th>n</th>
<th>Eddy current $b_n$</th>
<th>Correction coil $b_{nc}$</th>
<th>Final $b_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-0.52175E-03</td>
<td>0.23567E-03</td>
<td>-0.28608E-03</td>
</tr>
<tr>
<td>2</td>
<td>0.12283E+00</td>
<td>-0.12283E+00</td>
<td>0.00000E-00</td>
</tr>
<tr>
<td>4</td>
<td>-0.12601E+02</td>
<td>-0.14214E+02</td>
<td>-0.26815E+02</td>
</tr>
<tr>
<td>6</td>
<td>0.13050E+04</td>
<td>0.67208E+05</td>
<td>0.68513E+05</td>
</tr>
<tr>
<td>8</td>
<td>0.29670E+06</td>
<td>-0.87884E+08</td>
<td>-0.87588E+08</td>
</tr>
<tr>
<td>10</td>
<td>0.23100E+08</td>
<td>-0.49850E+11</td>
<td>-0.49827E+11</td>
</tr>
<tr>
<td>12</td>
<td>0.98885E+10</td>
<td>0.86039E+14</td>
<td>0.86049E+14</td>
</tr>
</tbody>
</table>

**A Self-correction coefficient $K_c$** Since the self-correction coefficient $K_c$ does nor depend on the dipole field variation, it can be calculated first for the fixed geometry. In order to cancel sextupole completely,

$$K_c = -0.3308 \quad [A \cdot \text{sec/T}] \quad (3.24)$$

This means the current in the correction coil is from 0 to 1.8A with the change of the dipole field. In practice, pick-up coils are put in between the main dipole coil arrays to obtain a correction current proportional to $\dot{B}$. 
B Multipole coefficients $b_n$ From the calculation results, the sextupole coefficients vanish after correction. Table 3.8 shows other components after correction. From the table, $b_0$ was reduced but higher moments are increased since the correction coil field has bigger higher moments. Fortunately, they have less effects on the beam performance as long as beam size is less than half size of the chamber.

C Eddy current field correction In CIS, since vertical betatron motion is small, it is important to correct the field distribution near the mid plane between the dipole gap. Figure 3.10 shows the correction results of the field on the horizontal mid-plane. The final field is fairly flat and can be considered as a dipole field at least in the region $x < a/2$. The magnitude of the final dipole component is only 0.04% of the main dipole field. The dipole field component $b_0$ can be trivially corrected by minute readjustment of the main dipole field $B_0$.

![Graph](image_url)

**Figure 3.10:** Eddy current field and correction coil field on the mid plane
3.3 CIS Dynamic Aperture

3.3.1 The Effect of Sextupoles

The vector potential for sextupole field is

\[
A_x = A_z = 0, \quad A_s = \frac{B_2}{6}(x^3 - 3xz^2)
\] (3.25)

where

\[
B_2 = \frac{\partial^2 B_z}{\partial x^2} \bigg|_{x=z=0}
\] (3.26)

and the sextupole field is

\[
B_z + iB_x = \frac{B_2}{2}(x^2 - z^2 + 2ixz)
\] (3.27)

The Hamiltonian for particle motion is

\[
H = \frac{1}{2} \left[ \dot{x}'^2 + K_x x^2 + \dot{z}'^2 + K_z z^2 \right] + \frac{S(s)}{6}(x^3 - 3xz^2)
\] (3.28)

where \( S(s) = -B_2/B\rho \) is the sextupole coefficient. Equations of motion are

\[
\begin{aligned}
\dot{x}'' + K_x x + \frac{1}{2} S(x^2 - z^2) &= 0 \\
\dot{z}'' + K_z z - Sxz &= 0
\end{aligned}
\] (3.29)

If there are off-center closed orbit in both directions,

\[
x = x_{co} + x_{\beta} \quad z = z_{co} + z_{\beta}
\] (3.30)

Equations of motion become

\[
\begin{aligned}
x_{\beta}'' + (K_x + S_{x_{co}})x_{\beta} - S_{z_{co}z_{\beta}} + \frac{1}{2} S(x_{\beta}'^2 - z_{\beta}'^2) &= -K_x x_{co} - \frac{1}{2} S(x_{co}'^2 - z_{co}'^2) \\
z_{\beta}'' + (K_z - S_{x_{co}})z_{\beta} - S_{z_{co}x_{co}} - S_{x_{co}z_{\beta}} &= -K_x x_{co} + S_{x_{co}z_{co}}
\end{aligned}
\] (3.31)

From the equations it is concluded that the sextupole has the following effects:
• Dipole field errors:

\[
\begin{align*}
\frac{\Delta B_z}{B_0} &= -K_x x_{co} - \frac{1}{2} S(s)(x_{co}^2 - z_{co}^2) \\
\frac{\Delta B_x}{B_0} &= K_x x_{co} - S x_{co} z_{co}
\end{align*}
\]  
(3.32)

• Quadrupole gradient errors:

\[
\Delta K_x = S(s) x_{co} \quad \Delta K_z = -S(s) x_{co}
\]  
(3.33)

and chromaticity errors:

\[
\begin{align*}
\Delta C_x &= \frac{1}{4\pi} \int \beta_x S(s) D(s) ds \\
\Delta C_z &= -\frac{1}{4\pi} \int \beta_x S(s) D(s) ds
\end{align*}
\]  
(3.34)

• Skew quadrupole error:

\[
\Delta q = -S(s) z_{co}
\]  
(3.35)

• The resonances driven by the sextupoles are:

\[
\begin{align*}
\nu_x &= lP \\
3\nu_x &= lP \\
\nu_x \pm 2\nu_x &= lP
\end{align*}
\]  
(3.36 - 3.38)

where \( l \) is an integer and \( P \) is the lattice superperiod. The dynamic aperture will become smaller due to these resonances.

### 3.3.2 Total Sextupole Components in Dipole Region

Total sextupole coefficient in the dipole region consist of two parts: one is from the dipole field and increases with \( B \) as shown in Figure 3.5 on page 30, and the other
3.3 CIS Dynamic Aperture

from the eddy current field (proportional to $\dot{B}$) as shown in Table 3.6 on page 37. Figure 3.11 shows the total sextupole coefficients. As discussed in Section 3.2, sextupole coefficients due to eddy current can be reduced to zero by compensation coils. Figure 3.12 shows the variation of the horizontal and vertical chromaticities due to the sextupole coefficients.

3.3.3 Dynamic Aperture of the Calculation

The dynamic aperture can be calculated by doing particle tracking by MAD program, of which the transfer matrices in the TRANSPORT method are up to the order of two by default.

Figure 3.13 shows the dynamic apertures using the sextupole coefficients from the

![Figure 3.11: Total sextupole coefficients due to dipole field and eddy current field](image-url)
2D and 3D dipole field calculation, while Figure 3.14 shows the dynamic apertures using the total and measured sextupole coefficients. Only the dynamic apertures in the cases with the worst sextupole coefficients were calculated.

In Figure 3.14, the total sextupole field causes the dynamic aperture smaller than the vacuum chamber at low energy because of the bigger values of $S(s)$. However, if the sextupole component from the eddy current is canceled by the correction coil, the dynamic aperture is twice as big as the vacuum chamber within the operation energy range.

### 3.3.4 Limitation on Dipole Field Ramping Rate

To reduce the eddy current effect, we can either compensate the eddy current completely or reduce the value of $\dot{B}/B_0$. Figure 3.14 shows that the dynamic aperture
Figure 3.13: Dynamic aperture due to the calculated sextupole coefficients
Figure 3.14: Dynamic aperture due to the measured and total sextupole coefficients
is smaller than the vacuum chamber at the low energy stage due to systematic sextupoles in dipole and eddy current on vacuum chamber. This is also reflected in the variation of chromaticities in Figure 3.12. So it is necessary to reduce the value of $\dot{B}/B_0$ in the low energy stage to reduce the total sextupole coefficients. When the energy is high, we can have bigger $\dot{B}/B_0$ without increasing the total sextupole coefficients too much. The bigger the value $\dot{B}/B_0$, the larger the sextupole coefficients of the eddy current and thereby the smaller the dynamic aperture. Calculations are done to find the maximum $\dot{B}/B_0$ allowed while keeping the dynamic aperture at least twice as big as the vacuum chamber. The results are shown in Table 3.9. Maximum $\dot{B}$ allowed is small in low energy stage. The ramping rate of the dipole field will have smaller slope in the low energy stage and increases at higher energy.

**Table 3.9:** Maximum values of $\dot{B}/B_0$ for proper dynamic aperture

<table>
<thead>
<tr>
<th>Energy (MeV)</th>
<th>$B_0$ (T)</th>
<th>Max. $S_2$ (1/m$^3$)</th>
<th>Max. $\frac{\dot{B}}{B_0}$ (1/sec)</th>
<th>Max.$\dot{B}$ (T/sec)</th>
<th>D.A. size (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.8</td>
<td>0.3372</td>
<td>0.080</td>
<td>2.5000</td>
<td>0.8431</td>
<td>92</td>
</tr>
<tr>
<td>12</td>
<td>0.3941</td>
<td>0.130</td>
<td>4.0625</td>
<td>1.6011</td>
<td>100</td>
</tr>
<tr>
<td>25</td>
<td>0.5708</td>
<td>0.150</td>
<td>4.6875</td>
<td>2.6758</td>
<td>90</td>
</tr>
<tr>
<td>45</td>
<td>0.7699</td>
<td>0.158</td>
<td>4.9375</td>
<td>3.8012</td>
<td>100</td>
</tr>
<tr>
<td>72</td>
<td>0.9806</td>
<td>0.180</td>
<td>5.6250</td>
<td>5.5160</td>
<td>90</td>
</tr>
<tr>
<td>103</td>
<td>1.1822</td>
<td>0.175</td>
<td>5.4687</td>
<td>6.4650</td>
<td>96</td>
</tr>
<tr>
<td>147</td>
<td>1.4279</td>
<td>0.260</td>
<td>8.1250</td>
<td>11.6017</td>
<td>96</td>
</tr>
<tr>
<td>179</td>
<td>1.5881</td>
<td>0.345</td>
<td>10.7812</td>
<td>17.1215</td>
<td>90</td>
</tr>
<tr>
<td>208</td>
<td>1.7239</td>
<td>0.515</td>
<td>16.0938</td>
<td>27.7447</td>
<td>100</td>
</tr>
</tbody>
</table>
From the dynamic aperture calculation, our conclusions are listed below.

- The size of the dynamic aperture depends largely on the absolute value of $K_2 = \frac{1}{B_\rho} \frac{\partial B}{\partial x}$. The bigger the $|K_2|$, the smaller the dynamic aperture. This is because our betatron tune is unfortunately very close near the systematic sextupole resonances.

- The systematic sextupole field may reduce the dynamic aperture smaller to a size than the vacuum chamber. Thus the sextupole field arising from eddy current should be corrected. The resulting dynamic aperture is twice bigger than the vacuum chamber within the operation energy range.

- The tolerable sextupole field happens when the dynamic aperture is equal to the vacuum chamber. The results in Table 3.9 show that dipole field has to ramp slower in the lower energy stage.
Chapter 4

Design of CIS RF Systems

The RFQ/DTL will provide 7 MeV (5 MeV) polarized proton(deuteron) beam for the CIS ring. In the ring, the rf system is designed to capture and accelerate the beam from 7 MeV (5 MeV) to a maximum of 200 MeV (105 MeV). The corresponding rf frequency is from 1.3 to 10.1 MHz when the rf system operates at $h = 1$. The rf system can also support acceleration cycling frequencies up to 5 Hz. A wide band tunable rf cavity was designed using non-uniform ferrite biasing to achieve 10:1 frequency coverage with good performance and lower power requirement. With heavy ferrite loading, the cavity is made short enough to accommodate the space available in the CIS ring. Tuning is accomplished by imposing an external dc magnetic field on the ferrite that is predominantly parallel to the cavity rf field. The effective permeability of the ring, which is $\partial B / \partial H$, steadily decreases with an increase of the bias field due to ferrite saturation. In this chapter, the determination of the cavity structure by the calculation of the external biasing field and the operating principle of the cavity will be discussed.
4.1 Fundamentals of Cylindrical Rf Cavities

4.1.1 Resonant Modes of a Cylindrical Cavity

The cylindrical cavity is the most common resonant structure used in accelerator applications. The transverse magnetic field (TM modes) inside the cavity in cylindrical coordinates \((\rho, \phi, z)\) has the forms [4]:

\[
\begin{align*}
E_s &= C k^2 \rho J_m(k_\rho \rho) \cos m\phi \cos k_z s \\
E_\rho &= -C k_s k_\rho J'_m(k_\rho \rho) \cos m\phi \sin k_z s \\
E_\phi &= C m k_\rho \frac{1}{\rho} J_m(k_\rho \rho) \sin m\phi \sin k_z s \\
B_s &= 0 \\
B_\rho &= -i C \frac{m k}{c_\rho} J_m(k_\rho \rho) \sin m\phi \cos k_z s \\
B_\phi &= -i C k k_\rho \frac{1}{c} J'_m(k_\rho \rho) \cos m\phi \cos k_z s
\end{align*}
\]

(4.1)

where \(k_\rho, k_s\) are wave numbers in the radial and longitudinal modes, \(k = \sqrt{k^2_s + k^2_\rho}\).

The boundary conditions require that \(E_\rho = 0, E_\phi = 0\) at \(z = 0, l\) and \(E_s = 0, E_\phi = 0\) at \(r = R_0\). Thus the wave number \(k\) is written as:

\[
k_{mnp} = \sqrt{\frac{p^2 \pi^2}{l^2} + \frac{j^2_{mn}}{a^2}} = \frac{\omega_{mnp}}{v} = \frac{2\pi}{\lambda_{mnp}}
\]

(4.3)

where \(j_{mn}\) is the zero points of \(J_m(k_\rho R_0)\), i.e. \(J_m(j_{mn}) = 0\), and \(m, n, p\) are mode numbers. For azimuthally symmetric modes, \(m = 0, n\) is the radial mode number, \((n - 1)\) is the number of nodes in the radial variation of \(E_s\), \(p\) is the longitudinal mode number. It is zero when \(E_s\) is constant along the \(s\) direction.
4.1.2 Properties of the Cylindrical Resonant Cavity

Due to the symmetry of the cavity structure, we assume that all fields have azimuthal symmetry and the electric field has no longitudinal variation. Thus the only modes possible in the cavity are $\text{TM}_{0n0}$. $\text{TM}_{0n0}$ modes are optimal for particle acceleration and have the following advantages:

1. $k_{0n0} = \frac{j_{0n}}{R_0} = \frac{\omega_{0n0}}{v}$. Wave number, frequency of $\text{TM}_{0n0}$ only depend on cavity radius $R_0$.

2. The longitudinal electric field is uniform along the propagation direction of the beam and its magnitude is maximum on axis.

3. The transverse magnetic field is zero on axis. This is very important since otherwise the transverse field could deflect the beam.

A cylindrical cavity can support a variety of resonant modes. Higher-order modes are generally undesirable since the energy shunted in higher-order modes is wasted. For the fundamental accelerating mode $\text{TM}_{010}$, 

$$E_s = E_0 J_0 \left( \frac{2.405}{R_0} \rho \right) \quad B_\phi = i \cdot \frac{\sqrt{\mu_r}}{c} E_0 J_1 \left( \frac{2.405}{R_0} \rho \right)$$

(4.4)

Figure 4.1 shows the normalized longitudinal electric field and azimuthal magnetic field as a function of radius.

4.1.3 Some Fundamental Parameters of the Cavity

I Transit-time Factor $F_t$

The definition of $F_t$ is:

$$F_t = \sin \frac{h g}{2 R_0} = \sin \frac{h w_0 g}{2v} = \sin \frac{h \pi g}{\lambda g}$$

(4.5)
where $g$ is the accelerating gap width, $h$ the harmonic number. The effective voltage seen by the passing particle is $V = V_0 \cdot F_t$. If $g = \lambda/2$, $F_t = 2/\pi = 0.636$ for $h = 1$. The effective gap can be reduced by adding nose cones or cavity drift tubes [9], in which case $g$ must be big enough to avoid the sparking problem.

II Shunt Impedance $R_{sh}$

The shunt impedance is related to the power dissipation:

$$R_{sh} = \frac{V_{rf}^2}{2P_d} \quad (4.6)$$

where $P_d$ is the total power dissipated in the cavity wall.
III Quality Factor $Q$

$Q$ is defined as:

$$Q = \frac{\text{power stored in the cavity}}{\text{power loss}} = \frac{P_s}{P_d} \quad (4.7)$$

On the other hand, $Q$ is related to the high frequency as follows. At high frequency, a cavity can be represented by a parallel $RLC$ circuit. The voltage $V(\omega)$ is

$$V(\omega) = z(\omega)I = \frac{V_0}{\sqrt{1 + Q^2 (\eta - \frac{1}{\eta})^2}} \quad (4.8)$$

where $V_0 = IR$, $Q = \omega L/R$ and $\eta = \omega/\omega_r$ with $\omega_r$ the resonance frequency. Figure 4.2 shows different resonance curves $V(\omega)/V_0$ at different $Q$ values. Resonance curve with bigger $Q$ value has better frequency selection ability.

![Figure 4.2: Resonance curves at different $Q$-values](image-url)
Eq. (4.8) can be re-written as:

\[
\frac{V(\omega)}{V_0} = \frac{1}{\sqrt{1 + Q^2(\frac{\omega^2 - \omega_r^2}{\omega_r^2})}} \approx \frac{1}{\sqrt{1 + Q^2 \cdot \frac{4(\Delta \omega)^2}{\omega_r^2}}} \tag{4.9}
\]

where \( \Delta \omega = \omega - \omega_r \).

The \( Q \) value is simply defined as

\[
Q = \frac{f_r}{\Delta f_{1/2}} \tag{4.10}
\]

where \( \Delta f_{1/2} \) is the Full Width at Half Maximum (FWHM). Within \( \Delta f_{1/2} \), it is considered that the cavity is excited. For a very big \( Q \), \( \Delta f_{1/2} \) is small, and the driving frequency should be precisely timed.

### 4.1.4 Power Exchange With Resonant Cavity

Power must be coupled to the resonant cavity to maintain the electromagnetic field. There are two ways of power coupling [23]:

1. Electrically coupling. Power feed is on the axis. In this case power is electrically coupled to the cavity since the current in the power feeds interacts predominantly with electric fields.

2. Magnetically coupling. The power is coupled to the cavity by a driving loop in the cavity; therefore, interaction is predominantly through magnetic fields. The loop is attached to a transmission line that is terminated by a matched resistance \( R \). The optimum size of the coupling loop corresponds to maximum power transfer with minimum perturbation on the \( TM \) field inside the cavity.
4.2 Design of the RF Cavity with External Ferrite Biassing

A wide tuning range rf cavity with external quadrupole ferrite biasing was proposed [24] for the CIS ring. Figure 4.3 and 4.4 show the cross section and azimuthal plane of this kind of cavity. In the ferrite materials, the biasing magnetic field parallel to the cavity rf field produces the effective rf permeability proportional to $\partial B/\partial H$, which steadily decreases with an increase of the biasing field due to magnetic polarization of the ferrite. The ferrite material is used both to achieve resonance by a imposed biasing magnetic field (and different permeability) and to reduce the length of the cavity. RF power is coupled into the cavity by a driving port which is directly tapped to the center conductor of the cavity. The loop biasing magnet is used for the input impedance matching, which will be discussed in Section 4.3. This kind of scheme was proposed by S. Papureanu and first adopted at the Max Planck Institute. A cavity of this type has been in use in the IUCF Cooler for the past few years [25]. However, the geometry of the external quadrupole and cavity can be further optimized by calculating the biasing field in the ferrite. The biasing field may leak into the beam pipe. Some shielding measures need to be taken.

Using the finite element code MagNet [14], 2D calculations were performed to optimize the cross section, and subsequent 3D calculations were done to determine the fringe field effects of the quadrupole field and decide on the total length of the cavity.

4.2.1 Material Parameters

Table 4.1 lists parameters of the materials used for the cavity. Figure 4.5 shows the B-H curve for the ferrite material Phillips 8C12. Since the rf field in the ferrite material
Figure 4.3: Cross section of RF cavity

Figure 4.4: Azimuthal plan of RF cavity. Ferrite disks are divided into two groups: loop biasing magnet for the input impedance matching and main biasing magnet for biasing field.
Table 4.1: RF cavity parts and materials

<table>
<thead>
<tr>
<th>Part Name</th>
<th>Material/Magnetic properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnet return/pole piece</td>
<td>Inland Steel 1006 or better B(H) curve: IN06</td>
</tr>
<tr>
<td>Ferrite disk</td>
<td>Phillips 8C12, B(H) curve: FC12 (φ498 × φ270 × 25mm)</td>
</tr>
<tr>
<td>Current coil</td>
<td>Cu</td>
</tr>
<tr>
<td>Inner and outer rf conductor</td>
<td>Cu</td>
</tr>
<tr>
<td>Shielding pipe</td>
<td>1008 steel, B(H) curve: SA08</td>
</tr>
</tbody>
</table>

is in the azimuthal direction, the effective rf permeability [27] will be \( \mu_\parallel = \partial B/\partial H \), which is the slope of the B-H curve and steadily decreases with an increase of the biasing field. The magnetic field in Inland Steel 1006, used as magnet iron, can be up to 1.5 T without saturation problem. Coil current of 1000 Ampere-turns was used for all the calculations. The current is large enough to produce a magnetic field near to the saturation value in ferrite, thus very small effective permeability.

4.2.2 Optimization of RF Cavity Cross Section Geometry

The RF cavity cross section geometry can be optimized by the 2D cylindrical calculation of the quadrupole field in the return and ferrite. Due to the symmetry, the calculation models only one-eighth of the cross section shown in Figure 4.3. The typical results of the field distribution is shown in Figure 4.6.
Figure 4.5: B(H) curve of ferrite Phillips 8C12

Figure 4.6: 2D calculation results of the quadrupole biasing field
I The Thickness of the Magnet Flux Return Yoke

When the magnet flux return yoke has the thickness of 30 mm, the field in the magnet return is 0.92T, well below the saturation value in the return material. The field inside the ferrite is around 0.24T, which is close to the saturation and gives small permeability. Since the saturation value of the flux return material is about five times as that of the ferrite material, the return thickness can be as thin as one-fifth of the half-pole width as long as the return has enough mechanical strength. Increasing the return width does not affect the field inside the ferrite disk significantly. In the final magnet design the return has the thickness of 60 mm.

II Optimization of Pole Width

The purpose of the external quadrupole structure is to provide maximum effective biasing field in the ferrite for a fix current. In the ferrite, only the azimuthal component of biasing field \( B \) contributes to the rf field. Along the circle inside the ferrite, the integral

\[
I_T = \int_{circle} B_T d\theta
\]  

(4.11)

should be as large as possible, where \( B_T \) is the magnitude of the tangential component of \( B \) along the circle. As \( B_{rf} \) is proportional to \( 1/r \) between inner and outer conductors, for the whole ferrite disk, the surface integral

\[
B_{int} = \int_{r_i}^{r_o} \frac{I_T}{r} dr
\]  

(4.12)

can serve as figure of merit and should be maximized.

When the pole width is small, a large portion of the field inside the ferrite disk is azimuthal but the value is small. Increasing the pole width will increase the \( B \) value but less field will be azimuthal. So we can expect an optimum pole width for which \( B_{int} \) is maximum.
Calculation of $B_{int}$ were done by scanning the pole width from 140 mm to 280 mm. The optimum pole width with the maximum $B_{int}$ is around 235 mm and within about ±5 mm $B_{int}$ is for practical purpose not sensitive to the change of the pole width.

III Shielding Against Leaking Quadrupole Field Inside the Bore of Ferrite Disks

In the case of no shielding pipes, the radial gradient of the leaking field with a predominant quadrupole component inside the bore of ferrite disk where the beam will pass is around 150 G/m. The calculation results show that the leaking field gradient increases with the increase of the pole width and decrease of the ferrite radial thickness.

The leaking field inside the bore of the ferrite disks will have focusing effect on the beam. By inserting a quadrupole (assumed to be 1 m long) with the strength of $K_1 \ [1/m^2] = 150 \ [G/m] / B \rho \ [Gm] \ (B \rho = 3800 \ Gm \ for \ 7 \ MeV \ proton)$ in the CIS ring, we can calculate the change of the tune and lattice functions due to the leaking field. The results are shown in the column with $K_1 = \pm 0.04 \ (1/m^2)$ in Table 4.2. Although the working point changes only less than 2%, the change of lattice functions cannot be neglected.

Decreasing the pole width or increasing the ferrite disk thickness can reduce the gradient of the leaking quadrupole field. However, the pole width has been optimized and the width of the ferrite disks are fixed by availability. Therefore we decided to install a 2 mm in thickness shielding pipe inside the ferrite disks as shown in Figure 4.3 on page 60 to reduce the quadrupole field to have negligible effect on the beam.

Calculation results show that after the shielding, the field gradient between the shielding pipe and the inner conductor is around 500 G/m, while the field inside the
4.2 Design of the RF Cavity with External Ferrite Biasing

<table>
<thead>
<tr>
<th>$K_1 = \frac{1}{B_0} B' \left( \frac{1}{m^2} \right)$</th>
<th>0</th>
<th>0.04</th>
<th>-0.04</th>
<th>0.005</th>
<th>-0.005</th>
</tr>
</thead>
<tbody>
<tr>
<td>no field</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hz. tune $\nu_x$</td>
<td>1.4633</td>
<td>1.4680</td>
<td>1.4590</td>
<td>1.4639</td>
<td>1.4628</td>
</tr>
<tr>
<td>Vt. tune $\nu_z$</td>
<td>0.7788</td>
<td>0.7676</td>
<td>0.7899</td>
<td>0.7774</td>
<td>0.7802</td>
</tr>
<tr>
<td>Trans. energy $\gamma_T$</td>
<td>1.271</td>
<td>1.271</td>
<td>1.270</td>
<td>1.271</td>
<td>1.270</td>
</tr>
<tr>
<td>Max. $\beta_x$ (m)</td>
<td>4.373</td>
<td>4.948</td>
<td>4.788</td>
<td>4.433</td>
<td>4.427</td>
</tr>
<tr>
<td>Max. $\beta_z$ (m)</td>
<td>3.786</td>
<td>4.047</td>
<td>4.049</td>
<td>3.817</td>
<td>3.817</td>
</tr>
<tr>
<td>Min. $\beta_x$ (m)</td>
<td>0.996</td>
<td>0.898</td>
<td>0.906</td>
<td>0.984</td>
<td>0.983</td>
</tr>
<tr>
<td>Min. $\beta_z$ (m)</td>
<td>3.380</td>
<td>3.171</td>
<td>3.165</td>
<td>3.366</td>
<td>3.353</td>
</tr>
<tr>
<td>Max. $D_x$ (m)</td>
<td>1.759</td>
<td>1.822</td>
<td>1.837</td>
<td>1.767</td>
<td>1.769</td>
</tr>
<tr>
<td>Min. $D_x$ (m)</td>
<td>1.617</td>
<td>1.560</td>
<td>1.562</td>
<td>1.610</td>
<td>1.610</td>
</tr>
<tr>
<td>Chromaticity $C_x$</td>
<td>-0.529</td>
<td>-0.545</td>
<td>-0.519</td>
<td>-0.530</td>
<td>-0.527</td>
</tr>
<tr>
<td>Chromaticity $C_z$</td>
<td>-0.156</td>
<td>-0.174</td>
<td>-0.140</td>
<td>-0.158</td>
<td>-0.154</td>
</tr>
</tbody>
</table>

pipe is reduced to 0.2 G and the gradient is 2 G/m, which is negligible.

The inside diameter of the shielding pipe is 140mm, which is large enough to accommodate the beam vacuum pipe. The gap between shielding pipe and ferrite disk ($\sim$ 60 mm) is also large enough so that the field inside the ferrite disk will not be reduced by the existence of the pipe.

### IV Gyromagnetic Resonance Inside the Ferrite

The condition for gyromagnetic resonance in a ferrite material is [See Appendix A]

$$ H = \frac{f}{2.8} $$

(4.13)
where $H$ is the internal dc magnetic field in units of Oersted and $f$ is the rf frequency in units of MHz. In the ferrite disk of the rf cavity, $H$ is about 1000 A/m or 12.57 Oersted for a coil excitation of 1000 A-turns. Therefore $f = 35.2$ MHz. The ramping of the rf frequency (2.0 to 10 MHz) is below the above resonance line.

### 4.2.3 3D Calculation of the Quadrupole Biasing Field

In order to design the longitudinal structure more efficiently, 3D calculations of the biasing field by MagNet [14] were done based on the model shown in Figure 4.7. Ten ferrite disks are used in the cavity and divided into two groups. Between the disks, there are 19 mm gaps for forced air cooling. The purpose of the loop biasing magnet is to allow the impedance matching of the driving loop [Section 4.3]. 3D calculations will decide the optimized cavity length, estimate the leaking field in the shielding pipe, and check the uniformity of the field in the ferrite disks and magnet irons.

![Figure 4.7: Rf cavity model 3D calculations](image-url)
I Qualitative Analysis of the Field Distribution

In order to produce a uniform field inside all the ferrite disks, the magnetic iron core should be longer than the ferrite stack. After several initial calculations, we found that for this purpose the iron core needs to be only 20 mm longer than the ferrite stack. This structure is shown in Figure 4.7.

The resulting magnetic field of the 3D model indicated that the maximum field value in the magnet iron is 1.04T, which is well below the saturation value of the low carbon iron, and this maximum value happens at the two round corners at two ends. The magnet iron has no location with saturation problem. The field inside the ferrite disks are uniform along the cylindrical axis direction. The field in the shielding pipe is slightly higher at both ends. The quantitative results are given below.

II Total Length of Cavity

On the bottom plane of the 3D calculation model, the tangential components of the field in the ferrite is maximum. On this plane, fields along the inner edge, middle and outer edge of the ferrite disks are checked. Figure 4.8 shows the field along the middle line of the ferrite disks. The field in the ferrite has the following properties:

1. The field inside the 10 ferrite disks is fairly uniformly distributed.

2. The field in disks at two ends is slightly higher than in the those in the middle.

3. The fringe field stops quickly outside the ferrite. The ferrite helps to reduce the spread of the magnetic field at two ends of the cavity.

These results show that the model shown in Figure 4.7 on page 66 is feasible with the length of the magnet iron 548 mm. Considering the width of the coil, the total length of the cavity will be 608 mm with a 70 mm gap between the loop magnet and main magnet.
III The Length of the Shielding Pipe

In the 3D model the length of the shielding pipe is 548mm, the same length as that of the magnet iron. To check the screening effect of the shielding pipe, Figure 4.9 shows the magnetic field along the different contour lines parallel to the cylindrical axis on the bottom plane. The shielding pipe reduces the field in the cavity region to a negligible value and the fringe field outside the cavity is below 3 G. To check the effect of the fringe field on the tune change and lattice functions, we calculated the focal strength \( K_f \) of the leaking field.

\[
K_f = \int_{\text{field region}} \frac{1}{B\rho} \cdot \frac{\partial B(r, l)}{\partial r} dl
\]  

(4.14)

and has the values shown in Table 4.3 on page 70 for 7 MeV protons \((B\rho = 0.38 \text{ Tm})\).
Figure 4.9: Leaking quadrupole field strength in the shield pipe with

\[ R = 70 \text{ mm}. \] From bottom to top the field along the contour

lines of \( r = 0, 10, 35, 69 \) mm is shown

From the fact that the field along the axis is roughly zero and the \( K_f \) values at different radius is the same within about \( \pm 15\% \), we verified that the fringe field is predominantly a quadrupole field. For \( K_f = 5 \times 10^{-3}/\text{m} \), which is bigger than the values in Table 4.3 but 8 times less than the strength without shielding, the change of lattice parameters is shown in Table 4.2 on page 65. From these results we conclude that the fringe field within the shielding pipe region will have negligible effect on the beam performance. The length of the shielding pipe can be of the same length as the cavity.

In addition, the maximum field in the wall of the shielding pipe, which happens at the ends of the pipe, is 0.65T, is much less than the saturation value of the pipe
Table 4.3: Focal strength of fringe field

<table>
<thead>
<tr>
<th>Contour line radius (mm)</th>
<th>10</th>
<th>35</th>
<th>69</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_f$ (1/m)</td>
<td>$3.2 \times 10^{-3}$</td>
<td>$2.5 \times 10^{-3}$</td>
<td>$2.4 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

material.

IV The Uniformity of the Field Inside the Ferrite Disks

The field inside each ferrite disk should be as close to each other as possible. Qualitatively, the uniformity can be viewed from the Figure 4.8. For more precise results, the integral of $B_T$ inside each ferrite disk has been calculated using Eq. (4.12) on page 63. The variation of $B_T$ is less than 2%, which indicates that the field in each disk is fairly uniform.

4.2.4 Summary of the Biasing Field Calculation

In a summary of the cavity structure design, the cross section of the cavity, as shown in Figure 4.3 on page 60, is optimized by the 2D calculations of the external biasing field. From the 3D calculation results, there is no saturation problem in the magnet and the field inside the ferrite disks has a good uniformity. The effect of the cavity field on the beam performance can be reduced by shielding pipe within the cavity region and the effects on beam performance by the fringe field is negligible.

The calculation error of the biasing field may come from the following sources. First, the B-H curve from the manufacturer’s data sheet for ferrite material has a big hysteresis area. The B-H curve used for the calculation is the average value of the upper and lower branches of the hysteresis loop. Secondly, the calculation accuracy
depends on the size of the finite element. But the number of elements is limited by the program. Larger element size may cause oscillations in the calculation results in some points.

4.3 Determination of RF Parameters of the Cavity

A variety of rf parameters have to be chosen for the cavity. Some relate directly to beam stability criteria and are therefore easy to determine. Others are often determined by non-physical criteria like availability and economics.

Some known parameters related to the cavity are:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ring circumference $L$ (m)</td>
<td>17.364</td>
</tr>
<tr>
<td>Initial energy (MeV)</td>
<td>7(p), 5(d)</td>
</tr>
<tr>
<td>Final energy (MeV)</td>
<td>200(p), 105(d)</td>
</tr>
<tr>
<td>Transition energy $\gamma_T$</td>
<td>1.271</td>
</tr>
<tr>
<td>Repetition rate</td>
<td>1 – 5 Hz</td>
</tr>
<tr>
<td>Acceleration time</td>
<td>600 – 120 msec</td>
</tr>
<tr>
<td>Harmonic number $h$</td>
<td>1</td>
</tr>
<tr>
<td>Initial stationary bucket area</td>
<td>0.033 eV·sec</td>
</tr>
</tbody>
</table>

4.3.1 Cavity Frequency Range

The revolution frequency range of proton (deuteron) from 7 MeV (5 MeV) to 200 MeV (105 MeV) is 2.099 MHz (1.258 MHz) to 9.779 MHz (5.551 MHz). The cavity frequency range is chosen as 1.25 MHz to 10 MHz.
4.3.2 Rf Cavity Voltage

Rf cavity voltage must be high enough to provide a stationary bucket to capture the initial dc beam in the CIS ring. Some of the synchrotron motion parameters in $(\delta, \phi)$ space are [4]:

- Synchrotron tune

$$\nu_s = \sqrt{\frac{heV |\eta \cos \phi_0|}{2\pi\beta^2 E}}$$  \hspace{1cm} (4.15)

- Bucket height

$$\delta_B = 2\sqrt{\frac{eV}{2\pi\beta^2 Eh |\eta|}} Y(\phi_0)$$  \hspace{1cm} (4.16)

- Bucket area:

$$A = 16\sqrt{\frac{eV}{2\pi\beta^2 Eh |\eta|}} \alpha(\phi_0)$$  \hspace{1cm} (4.17)

where $Y(\phi_0)$ and $\alpha(\phi_0)$ are given by:

$$Y(\phi_0) = \sqrt{|\cos \phi_0 - \frac{\pi - 2\phi_0}{2} \sin \phi_0|}$$  \hspace{1cm} (4.18)

$$\alpha(\phi_0) = \frac{1}{4\sqrt{2}} \int_{\phi_u}^{\pi-\phi_0} \left\{ -\frac{|\eta|}{\eta} (\cos \phi + \cos \phi_0 - (\pi - \phi - \phi_0) \sin \phi_0) \right\}^{\frac{1}{2}} d\phi$$  \hspace{1cm} (4.19)

and the width of the separatrix is $\pi - \phi_0 - \phi_u$, where $\phi_u$ is the left turning point of the separatrix:

$$\cos \phi_u + \phi_u \sin \phi_0 = -\cos \phi_0 + (\pi - \phi_0) \sin \phi_0$$  \hspace{1cm} (4.20)

The initial beam from the RFQ/DTL has an energy spread of $\pm 70$ KeV. The strip injection of the $H^-$ ions adds an extra energy spread of 70 KeV [Section 5.2].
4.3 Determination of RF Parameters of the Cavity

Considering that the rf debuncher [Section 4.5] can eliminate the initial energy spread before injection, the stationary bucket area is 0.033 eV·sec; this requires that rf cavity voltage be 760V.

On the other hand, from the rf cavity structure, the rf voltage is decided by the magnetic flux density in the ferrite:

\[
V_{rf} = \omega_r l \int_{r_1}^{r_2} B(r) dr = \omega_r l B_1 r_1 \ln \frac{r_2}{r_1} \approx \omega_r \tilde{B} A
\]  

(4.21)

where \( A = l(r_2 - r_1) \) is the effective area of the ferrite core and \( \tilde{B} \) is the peak magnetic flux. For a given \( r_2 \), \( V_{rf} \) is maximum when \( \frac{r_2}{r_1} = e \approx 2.7 \). For our case, \( \frac{r_2}{r_1} = 1.8 \), which is limited by availability.

If the average energy gain per turn is \( V_g \), the acceleration rate is

\[
\frac{dE}{dt} = V_g \cdot \frac{\beta_c}{L}
\]

(4.22)

where \( L \) is CIS ring circumference. The total acceleration cycle time is

\[
T_c = \frac{L}{c} \cdot \frac{E_0}{V_g} \int_{r_1}^{r_f} \frac{\gamma d\gamma}{\sqrt{\gamma^2 - 1}} \\
= \frac{L}{c} \cdot \frac{E_0}{V_g} \left( \sqrt{\gamma_f^2 - 1} - \sqrt{\gamma_i^2 - 1} \right) \\
= \frac{L}{c} \cdot \frac{E_0}{V_g} (\beta_f \gamma_f - \beta_i \gamma_i)
\]

(4.23)

In the case of repetition rate of 1 Hz, \( T_c = 0.6 \) s and the average energy gain per turn is \( V_g = 51 \) V; for 5 Hz operation, \( V_g = 255 \) V.

4.3.3 Cavity Q Value

The quality factor of the rf cavity is dominated by the \( Q \) value of ferrite itself. In our case \( Q \) is only around 40 [26]. The \( Q \) is so low that within \( \Delta f_{\perp} \) it is not adequate to excite the cavity. This can be accomplished by adjusting the the external biasing
field current so that the permeability is changed to tune the system always to some required frequency near resonance.

Other frequency and ferrite permeability dependent parameters, such as skin depth, surface resistance, inductance and capacitance, will change during beam acceleration. The characteristic impedance

\[ R_c = \sqrt{\frac{L}{C}} \approx 60\sqrt{\mu_r \ln \frac{r_2}{r_1}} \quad (\Omega) \]  \hspace{1cm} (4.24)

is about 1000 Ω for a typical value of \( \mu_r = 800 \) in the ferrite.

### 4.4 Operational Principle of the Cavity

The cavity designed for CIS has the structure of external quadrupole magnet biasing. It has the following advantages:

1. The basing scheme is simple and effective. The design is straightforward.

2. The biasing field is predominantly parallel with the rf field. The parallel bias has higher efficiency.

3. The symmetry of the quadrupole field provides cancellation of biasing field at the cavity axis, minimizing magnetic disturbance to the beam.

4. Because the biasing structure is totally outside the rf cavity, and is not seen by the rf field, the design is clean and does not suffer from the disturbances (especially at high order modes) brought by the biasing structure.

#### 4.4.1 Biasing Principle

The use of ferrite loading has two purposes: tuning and reducing the cavity length. If the rf field \( \vec{H}_{rf} \) is small compared to the bias field \( \vec{H}_{dc} \), the effective rf permeabilities
**Figure 4.10:** A typical ferrite B-H curve and definitions of effective rf permeabilities. The effective permeability for parallel biasing is a function of the rate change of the B-H curve as shown by the slope of the tangent lines at points A and C, while for perpendicular biasing it is a function of the ratio of B/H at any point on the B-H curve.

are [27]:

\[
\mu_\parallel = \frac{\partial B}{\partial H} \quad (\vec{H}_{rf} / \vec{H}_{dc}) \quad (4.25)
\]

\[
\mu_\perp = \frac{B}{H} \quad (\vec{H}_{rf} \perp \vec{H}_{dc}) \quad (4.26)
\]

Figure 4.10 shows the definitions of \(\mu_\parallel\) and \(\mu_\perp\) and the biasing field needed for the same frequency range. Assuming that the operating rf frequency is sufficiently far below the frequency for gyromagnetic resonance and that the rf field is small compared to the bias field, the effective rf permeability is the tangential slope of the B-H curve for the parallel bias (for example A and C), or the ratio of B and H for the perpendicular bias (such as lines B and D). To provide the same tune change,
that is, the same change of permeability, the perpendicular bias requires significantly larger bias field $\Delta H_\perp$ than the parallel bias field $\Delta H_\parallel$.

In the rf cavity, the rf field has only the azimuthal component. The external quadrupole magnet provides dc magnetic field strength inside the ferrite from 0 to 0.27 T, near the saturation value. Thus effective rf permeability can change from maximum (around 800) to nearly zero, which insures the wide frequency range required.

### 4.4.2 Power Transfer to the Cavity

As shown in Figure 4.4 on page 60, the cavity is divided into two sections: the loop biasing magnet, which is used for the impedance matching for the power coupling, and the main biasing magnet, which provides the biasing field in the ferrite material. Power is coupled into the cavity by a driving port which is directly tapped to the center conductor of the cavity. To transfer power efficiently from the amplifier to the cavity, it is important that the driving port of the cavity presents a fixed impedance matching that of the transmission line at all frequencies.

Since the power delivered to the driving port equals to the power dissipated on the gap resistance at resonance, we have the following relation:

$$R = R_0 \cdot \left(\frac{V}{V_0}\right)^2$$  \hspace{1cm} (4.27)

where $R_0$ and $V_0$ are gap resistance and voltage, $R$ and $V$ are resistance and voltage at the driving port. $R$ should match the impedance (usually 50 $\Omega$) of the transmission line. Since $R_0$ changes due to frequency dependent ferrite losses, the ratio of $\frac{V}{V_0}$ should be changed accordingly in order to keep a constant $R$. The driving port voltage is

$$V = \oint_s \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} \propto \mu \cdot s$$  \hspace{1cm} (4.28)

Thus

$$R = R_0 \cdot \left(\frac{\mu_d}{\mu}\right)^2 \left(\frac{s}{s_0}\right)^2$$  \hspace{1cm} (4.29)
where \( s \) and \( s_0 \) are the magnetic areas enclosed by the driving loop and entire cavity, and \( \langle \mu \rangle \) is the average permeability of the ferrite rings of the entire cavity and \( \mu_d \) the average permeability of the ferrite rings enclosed by the driving loop. By adding two trim coils to the quadrupole magnet adjusting the bias strength of the ferrite rings enclosed by the driving loop, the \( \frac{V}{V_0} \) ratio can be changed. The trim coil current have a large effect on the driving port impedance but a small effect on the overall cavity tuning.

A 300 Watt solid state amplifier will be used to drive the cavity. Cooling of the cavity is via forced air. The ferrite rings have cooling spaces between them. The center conductor of the cavity is perforated and the outer conductor is made of copper strips rather than solid copper.

### 4.5 RF Debuncher

![Diagram of the Debuncher and Related Components](image)

**Figure 4.11:** RFQ/DTL linac and the debuncher

The beam from the RFQ/DTL 425 MHz linac have very tight bunch structures with about 2.35 ns bunch spacing. At the energy of 7 MeV, the beam bunches from the linac have an energy spread of \( \pm 70 \) keV. The energy spread will increase
the longitudinal beam emittance after in the CIS ring, thus reduce the rf cavity capture efficiency. A debuncher, which consists of one rf cavity operating at the same frequency as the the RFQ/DTL, can be used to rotate the longitudinal phase space shape so that the injection beam has a smaller momentum spread. Figure 4.11 shows the position of debuncher, which is 2.792 m from the RFQ/DTL exit.

4.5.1 Longitudinal Dynamics of Debuncher

The equation for the relative phase is

$$\frac{d\Delta \psi}{ds} = \omega \left( \frac{dt}{ds} - \frac{dt_s}{ds} \right) = \omega \left( \frac{1}{v} - \frac{1}{v_s} \right) = -\frac{\omega}{mc^2 \beta_s^3 \gamma_s^8} \Delta E,$$

(4.30)

where

$$\Delta E = E - E_s \quad \Delta \psi = \psi - \psi_s = \omega(t - t_s)$$

(4.31)

and $\Delta E$ is the energy deviation, $v$ and $v_s$ are velocities of a particle and the synchronous particle, and $\beta_s, \gamma_s$ are relativistic factors of the synchronous particle.

![Diagram of RFQ/DTL and Debuncher](image)

**Figure 4.12:** Positions of debuncher after RFQ/DTL. $\Delta s = 2.792$ m

In the drift space from DTL to the debuncher as shown in Figure 4.12, the change of the phase difference is

$$\Delta \psi = -\frac{\omega}{mc^2 \beta_s^3 \gamma_s^8} \cdot \Delta E \cdot \Delta s$$

(4.32)
4.5 RF Debuncher

Figure 4.13: Rf voltage on the debuncher

At the debuncher location, the debuncher voltage should have the form as shown in Figure 4.13 to cancel the energy spread. The time average of the rf voltage with the phase difference $\Delta \psi$ is

$$\Delta V = \frac{1}{\Delta t} \int_{-\Delta t/2}^{\Delta t/2} V(t) dt = \frac{V_m}{\omega_0 \Delta t} \int_{-\Delta t/2}^{\Delta t/2} \sin(\omega_0 t + \Delta \psi) d(\Delta \omega_0 t)$$

$$= \frac{eV_g}{\omega_0 \Delta t} \cdot 2 \sin \Delta \psi \sin \left( \frac{\omega_0 \Delta t}{2} \right) = V_m T_{tr} \sin \Delta \psi \quad (4.33)$$

where the transition factor $T_{tr}$ is defined as

$$T_{tr} = \sin \left( \frac{\omega_0 \Delta t}{2} \right) = \sin \left( \frac{\omega_0 \beta}{2\beta_s c} \right) \quad (4.34)$$

Thus the requirement for the rf peak voltage is

$$V_m = \frac{|\Delta E|}{T_{tr} \sin |\Delta \psi|} \quad (4.35)$$

The rf electric field in the gap of the debuncher is

$$\mathcal{E}_z = \frac{1}{g} \frac{|\Delta E|}{T_{tr} \sin |\Delta \psi|} \quad (4.36)$$
Particle beams from the RFQ/DTL have an energy spread of ±70 keV at 7 MeV. After drifting a distance of $\Delta s = 2.792$ m, the maximum phase difference is $57.96^\circ$ and the minimum peak voltage required for a debuncher operating at 425 MHz is 82.58 kV.

### 4.5.2 Determination of Debuncher Structure

![Debuncher structure and calculation model](image)

**Figure 4.14**: Debuncher structure and calculation model

As shown in Figure 4.14(a), the debuncher is a pillbox cavity. The available code SUPERFISH [30] is used to calculate its resonant frequency, shunt impedance and quality factor, etc.. One quarter of the whole structure was chosen as the calculation model due to its symmetry. The design goal is to optimize a cavity structure having a resonant frequency of 425 MHz and a sufficient high voltage between the gap. Starting from the initial model, the resonant frequency is increased by reducing the
Table 4.4: Geometry parameters of debuncher models (Length: mm)

<table>
<thead>
<tr>
<th>Models</th>
<th>$L$</th>
<th>$L_i$</th>
<th>$R_i$</th>
<th>$\theta_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>207.6</td>
<td>156.8</td>
<td>203.95</td>
<td>108.6°</td>
</tr>
<tr>
<td>$\theta_h$</td>
<td>4°</td>
<td>$R_h$</td>
<td>15.21</td>
<td>$r_3$=9.652</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Models</th>
<th>$G$</th>
<th>$R_n$</th>
<th>$R_{nb}$</th>
<th>$r_1$</th>
<th>$r_2$</th>
<th>$f$ (MHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>15.07</td>
<td>24.93</td>
<td>48.78</td>
<td>3.175</td>
<td>11.176</td>
<td>373.108</td>
</tr>
<tr>
<td>Model 2</td>
<td>19.07</td>
<td>20.93</td>
<td>44.11</td>
<td>3.175</td>
<td>11.176</td>
<td>410.419</td>
</tr>
<tr>
<td>Model 3</td>
<td>21.11</td>
<td>18.89</td>
<td>41.72</td>
<td>2</td>
<td>8</td>
<td>424.501</td>
</tr>
</tbody>
</table>

Table 4.5: Calculation and measurement results of debuncher models

<table>
<thead>
<tr>
<th>Models</th>
<th>$f$ (MHz) (Calculated)</th>
<th>$f$ (MHz) (Measured)</th>
<th>$\Delta f$ (MHz)</th>
<th>$Q$</th>
<th>TTF</th>
<th>Shunt Impedance (MΩ/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>373.108</td>
<td>370.756</td>
<td>2.352</td>
<td>23587</td>
<td>0.9748</td>
<td>28.183</td>
</tr>
<tr>
<td>Model 2</td>
<td>410.419</td>
<td>408.874</td>
<td>1.545</td>
<td>25361</td>
<td>0.9664</td>
<td>34.387</td>
</tr>
<tr>
<td>Model 3</td>
<td>424.501</td>
<td>422.985</td>
<td>1.516</td>
<td>26004</td>
<td>0.9599</td>
<td>36.869</td>
</tr>
</tbody>
</table>

The size of the nose inside the cavity. Table 4.5 shows the calculation and measurement of the different debuncher models with the structures in Table 4.4. The resonant frequency of the final model is chosen as 424.5 MHz on the consideration that the tuning capacitor of the debuncher has a tuning range of 500 KHz.
Chapter 5

CIS Injection and Extraction

The $H^-$ beam from the RFQ/DTL, with an energy of 7 MeV, pulse length of 25 $\mu$s and emittance of 9.8 $\pi\mu$m, is strip-injected into the CIS via DC magnet chicanes and two bumper magnets located in adjacent straight sections of the injection point. The $H^-$ beam is converted to protons by passage through a 4.5 $\mu$g/cm$^2$ thick, carbon foil fabricated in the IUCF target laboratory. Theoretical analysis of the foil scattering process has been carried out to find the emittance growth rate due to the foil and vacuum rest gas scattering. The beam emittance after the accumulation must be less than 34 $\pi\mu$m in order to get a desired emittance of less than 10 $\pi\mu$m at the final energy of 200 MeV (10 $\pi\mu$m is the acceptance of the Cooler). After the proton beam is accumulated in the CIS ring, the adiabatic capture process is adopted using the rf cavity described in Chapter 4 to capture the stored beam into an rf bucket with high efficiency. Then the beam is accelerated to 200 MeV in 600 ms when the repetition rate is 1 Hz, or in 120 ms at 5 Hz. In the CIS ring, beam position monitors (BPM), wall gap monitors (WGM) and beam profile monitors with stops are used for beam diagnostics. Experimental results show that $1.2\times10^{10}$ protons per bunch have been ramped to 200 MeV with emittance less than 10 $\pi\mu$m.
The proton beam bunches are extracted by a fast traveling kicker and vertical septum at high energy, and are transferred to the Cooler. The dipoles and quadrupoles along the transfer beam line can be adjusted properly for the best beam phase space matching at the Cooler injection point. Partial snakes (spin rotators) in the beam lines are used for the polarized proton beam spin manipulation. Cooler injection is realized by the existing septum and fast kicker in the Cooler ring.

5.1 CIS Beam Diagnostics System

![Diagram of CIS beam diagnostics system]

**Figure 5.1**: CIS diagnostics system, injection and extraction elements

For the linac pre-injector, CIS ring and extraction beam line, there are several
kinds of beam diagnostics methods: beam position monitor, beam profile monitor, wall gap monitor, and ping tune system.

5.1.1 Beam Position Monitors (BPMs)

Beam position monitors [37] usually consist of two or four conductor plates or buttons. The left/right, up/down(L/R, U/D) signals produced by the image current on the conductor plates will be summed (Σ) or subtracted (Δ) to give the beam intensity or normalized beam position information.

While the beam intensity is proportional to the sum signal of all the electrodes, the beam position can be determined by the sum and difference signals. For the parallel cylindrical electrodes, the beam position is determined by [35]

\[
\frac{\Delta}{\Sigma} = \frac{R - L}{R + L} = \frac{4 \sin \phi}{\phi \cdot b} \cdot \frac{x}{b} + \text{higher order terms of } (x, z)
\]

or in the log-amp representation

\[
20 \log_{10} \frac{R}{L} = \frac{160 \sin \phi}{\phi \ln 10} \cdot \frac{x}{b} + \text{higher order terms of } (x, z)
\]

where \( \phi \) is the electrode angular width, \( b \) is the electrode radius, and \( x \) is the normalized beam position. In the log-amp representation, the beam signal has higher sensitivity and less nonlinear effect.

For the split can case, the beam position is

\[
\frac{x}{b} = \frac{\Delta}{\Sigma} = \frac{R - L}{R + L} = \frac{2R - \Sigma}{\Sigma};
\]

and the response to the beam position is linear with half of the sensitivity of that of the strip line electrodes.

For the 7 MeV \( H^- \) transport line, where the injection pulse is 200 to 300 \( \mu s \) long, with repetition rates of 1 to 5 Hz, the pickups are 4-quadrant electrostatic...
type with high input impedance, front-end amplifiers. The four signals $L + R$, $L - R$, $U - D$ and $U + D$ are normalized in the computer to provide the position and intensity information. The minimum detectable signal is approximately $10 \, \mu A$ and the predicted range in which the injection beam line will operate is from 10 to 400 $\mu A$.

The CIS ring uses elliptically shaped 4 quadrant electrostatic pickups. For the elliptical cross section, the 4 quadrant electrodes are divided in such a way that each segment gives the same response. So, the up/down segments are much smaller than the left/right segments. Calculations were done with the MagNet[14] to make sure that all electrodes have the same surface integral of the electric field, which is proportional to the charge density. 8 BPMs are positioned at the entrance and exit of each of 4 bending magnets, as shown in Figure 5.1 on page 84. Logarithmic amplifiers are used for the detectors. The difference of the log amps, $L - R$ and $U - D$ gives the normalized positions. The sum of the four signals provides the intensity.

5.1.2 Beam Profile Monitors

A harp is used to measure the beam profile in both the injection beam line and the CIS ring. The harp uses a 5 printed circuit board stack, each with 24 $\phi 50 \, \mu m$ wires on each of the horizontal and vertical detector boards, spaced 1 mm apart. The first, third and fifth boards have 6 wires, spaced 4 mm apart, and are biased with a voltage (usually 90 volts) to collect the scattered electrons. The second and fourth boards are aligned in the horizontal and vertical planes as detector grids. The signals from the 24 wires give the horizontal and vertical beam profiles.

The harp has been the primary diagnostic during the commissioning of the CIS ring. Two harps have been placed in the injection beam line, one at the exit of the linac and the other between the $31^\circ$ and $21^\circ$ injection dipoles. One harp is installed
after the first dipole D1 in the ring, as shown in Figure 5.1 on page 84. The first harp in the injection line and the harp in the ring are mounted in front of a stop, allowing one to monitor intensity while adjusting the the profile. The harps can be moved in and out of the beam path using air actuated insertion devices. The flexibility of this setup allows an operator to observe a harp display of the first turn beam along with multi-turns, and also to accumulate maximum intensity with the harp out of the beam path.

5.1.3 Wall Gap Monitors (WGMs)

Longitudinal beam information can be efficiently picked up by wall gap monitors (WGM). The pickup works by intercepting the induced charge by breaking the conducting wall with a load resistor $R$, shielded by an outside conductor. The monitor’s output voltage is equal to $-I_b R$ between the low frequency limit of $L/R$ and the high frequency limit of $1/RC$, where $L$ is the inductance of the shielding conductor and $C$ is the wall gap capacitance. The response of a WGM is roughly independent of the transverse beam distribution inside the beam pipe.

One WGM is placed after dipole D3 of the CIS ring. The beam intensity can be estimated by

$$I = \frac{V_{pp}}{R}$$  \hspace{1cm} (5.4)

where $V_{pp}$ is the peak-peak voltage of the WGM output.

5.1.4 Ping Tune System (PTS)

The ping tune system [36] (PTS) in the CIS ring measures the horizontal and vertical beam position on a turn-by-turn basis for kicked beam with currents in the range from 1 $\mu$A to 1 mA. A fast Fourier transformation of this position data is performed by a
PC-based DSP (Digital Signal Processor) module at a rate of 10 measurements per second; from this FFT analysis the betatron fractional tune is obtained. Log amps are used in the system to improve the performance by increasing the sensitivity and decreasing the nonlinear effect. The CIS ring revolution frequency can range from 2.0987 MHz to 10.13 MHz corresponding a proton beam energy ranging from 7 MeV to 220 MeV. A 10 kV fast horizontal and vertical tune kicker is used to kick the beam when measuring the tunes. The PTS uses the signal from BPM2 to get the tune and monitor the beam positions.

5.2 Beam Injection to CIS Ring

The stripping injection method is chosen for CIS injection to achieve higher beam intensity. The H\(^-\) beam produced by a duoplasmatron source is accelerated by an RFQ to 3 MeV and by a DTL to 7 MeV. The CIS injection elements include a 4.5 \(\mu\)g/cm\(^2\) thick, 7 mm \(\times\) 22 mm carbon strip foil at the center of the injection section, three DC chicane dipoles, and two bumper magnets located in the two sections adjacent to the injection section (refer to Figure 5.1 on page 84).

During injection, 7 MeV H\(^-\) beam is converted to protons by the strip foil. Three DC chicane dipoles produce a bump in the closed orbit near the strip foil. Then two bumpers are switched on to bump the closed orbit passing through the strip foil so that the injection beam and circulating beams overlap. Once the injection process is completed, the bumpers are turned off to restore the CIS ring equilibrium orbit. The bumpers allow the closed orbit to be moved off the strip foil in \(\sim\) \(\mu\)s compared to the chicane decay time of ms.
5.2 Beam Injection to CIS Ring

5.2.1 Emittance Growth in the Ring

During the strip injection, H− and proton beam particles will be randomly Coulomb scattered by the foil atoms when passing through the foil. This scattering produces random angular kicks and energy loss of the beam causing beam emittance growth. Even after the bumpers are switched off, there is a time delay of 20 to 50 μs before the closed orbit comes off the strip foil completely. So, the emittance growth due to the scattering cannot be neglected. The emittance growth can also come from the multiple scattering of the particle beams by the rest gas in the vacuum chamber.

I Multiple Scattering Angle and Energy Loss

The rms multiple Coulomb scattering angle for a charged particle traversing a medium is [8]

\[ \theta \approx \frac{13.6 z_p}{\beta c [\text{MeV}]} \sqrt{\frac{t}{X_0}} \quad [\text{mrad}] \] (5.5)

where \( p, \beta c \) and \( z_p \) are momentum, velocity and charge number of the incident beam particle, \( X_0 \) is the radiation length and \( t \) is the target thickness in [μg/cm²]. \( X_0 \) is given by

\[ X_0 = \frac{716.4 A}{Z(Z+1) \ln(287/\sqrt{Z})} \quad [\text{g/cm}^2] \] (5.6)

where \( Z, A \) are the atomic charge and the mass number of the medium. Here \( Z = 6 \), \( A = 12 \) for the carbon strip foil.

Charged particles passing through matter lose energy primarily by ionization. The mean rate of energy loss (or stopping power) is given by the Bethe-Bloch equation [8]:

\[ -\Delta E = K \frac{Z z_p^2}{A \beta^2} \left[ \ln \frac{2m_e c^2 \beta^2 \gamma^2}{I} - \beta^2 \right] t [\mu g/cm^2] \quad [\text{MeV}] \] (5.7)
where

\[ K = 0.3071 \times 10^{-6} \text{[MeV/(\(\mu g/cm^2\))] is a constant;} \]

\[ I = 16Z^{0.9} \text{eV is the effective ionization potential of the medium atom;} \]

\[ m_e c^2 = 0.511 \times 10^6 \text{eV is the rest energy of electron.} \]

II Emittance Growth Due to the Strip Foil Scattering

The random beam angular kicks and energy loss of the incident H\(^-\) beam when passing through the strip foil will result in beam emittance growth.

First we estimate the effect of the angular kicks. According to the Courant-Snyder invariant, an angular kick \(\delta y' = \theta\) results in a change of the betatron action

\[ \Delta I_\theta = I(y, y' + \theta) - I(y, y') = 2\theta(\alpha y + \beta y') + \beta \theta^2 \]  \hspace{1cm} (5.8)

For random scattering, the emittance growth can be averaged over betatron oscillations and kick angles

\[ \Delta \epsilon_\theta \equiv \langle \Delta I_\theta \rangle = \langle \beta \theta^2 \rangle \approx \langle \beta_\perp \rangle \langle \theta^2 \rangle \]  \hspace{1cm} (5.9)

The emittance growth of H\(^-\) beam per passage is then

\[ \Delta \epsilon_\theta = 58.8746 \frac{\beta_\perp [m]}{\beta^2 (pc[MeV])^2 X_0[\mu g/cm^2]} [\pi \text{mm} - \text{mrad}] \]  \hspace{1cm} (5.10)

The energy loss will increase the momentum deviation and thus change the closed orbit of the off-momentum particle. From the energy relation

\[ p^2 c^2 = E^2 - E_0^2 \]  \hspace{1cm} (5.11)

one can get the relation between the momentum deviation \(\delta_s\) and the energy deviation \(\Delta E/E\) and the kinetic energy deviation \(\Delta T/T\) due to the scattering

\[ \delta_s = \frac{\Delta p}{p} = \frac{E}{p^2 c^2} \Delta E = \frac{1}{\beta^2} \frac{\Delta E}{E} = \frac{E}{E + E_0} \frac{\Delta T}{T} = \frac{\gamma}{\gamma + 1} \frac{\Delta T}{T} \]  \hspace{1cm} (5.12)
There is a change of \((\delta y_D, \delta y'_D) = (D\delta_s, D'\delta_s)\) of the off momentum closed orbit corresponding to the momentum deviation \(\delta_s\). Since the phase space position coordinates do not change by any finite impulse, the betatron orbit will have a change

\[ \delta y = -D\delta_s, \quad \delta y' = -D'\delta_s \]  
\(5.13\)

The resulting change of the Courant-Snyder invariant is

\[
\Delta I_E = I(y - D\delta_s, y' - D'\delta_s) - I(y, y') \\
= \frac{1}{\beta} \left[ (y - D\delta_s)^2 + [(\alpha y + \beta y') - (\alpha D + \beta D')\delta_s]^2 \right] - \frac{1}{\beta} \left[ y^2 + (\alpha y + \beta y')^2 \right] \\
= \frac{1}{\beta} \left[ D^2 + (\alpha D + \beta D')^2 \right] \delta_s^2 - \frac{2}{\beta} (\alpha y + \beta y')(\alpha D + \beta D')Dy |\delta_s| \]  
\(5.14\)

When averaging over the betatron coordinates \(y, y'\), the second term in the above equation vanishes. The emittance growth can be obtained as

\[
\Delta \epsilon_E \equiv \langle \Delta I_E \rangle = \frac{\mathcal{H}}{\pi} \delta_s^2 \times 10^6 \quad [\text{mm mrad}] \]  
\(5.15\)

with

\[
\mathcal{H} = \frac{1}{\beta} \left[ D^2 + (\alpha D + \beta D')^2 \right] \quad [\text{m rad}] \]  
\(5.16\)

where \(\alpha[\text{rad}], \beta[\text{m}], D[\text{m}]\) and \(D'[\text{rad}]\) are \(\alpha\)-parameter, betatron function, dispersion function and its slope at the strip foil.

Combining Eqs. 5.10 and 5.15, the total emittance growth due to strip foil scattering is

\[
\Delta \epsilon = \Delta \epsilon_\theta + \Delta \epsilon_E \quad [\text{mm mrad}] \]  
\(5.17\)

III Emittance Growth Due to Beam-Gas Scattering

Multiple scattering of the particle beams by the gas molecules in the vacuum chamber can also cause the beam emittance growth. The equivalent target thickness per second
at the room temperature is [4]:

\[
x = 1.641\beta P_g [\text{nTorr}] A_g \quad [\mu \text{gm/cm}^2]
\]

(5.18)

where \( P_g \) is the partial pressure of a gas and \( A_g \) is the gram molecular weight of the gas.

From Eq. (5.10), the emittance growth rate due to the gas molecule scattering is

\[
\frac{d\epsilon}{dt} = 96.613 \langle \beta_\perp \rangle [\text{m}] \frac{\beta}{\beta} \left( \frac{z_p}{pc[\text{MeV}]} \right)^2 \frac{P_g [\text{nTorr}] A_g}{X_0g [\mu \text{gm/cm}^2]} \quad [\pi \text{mm - mrad/sec}]
\]

(5.19)

where \( X_0g \) is the radiation length of the gas and \( \langle \beta_\perp \rangle \) is the average transverse betatron function in the accelerator.

### IV Calculation Results of the Emittance Growth

<table>
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Table 5.1 shows the calculation results of the effect due to the strip foil scattering and vacuum gas scattering in the CIS ring. The following is the summary of the calculation of the emittance growth during the strip injection:

- The normalized horizontal emittance of the injection beam from RFQ/DTL is $1.2 \pi \text{ mm-mrad}$, which is equal to $9.81\pi \text{ mm-mrad}$ un-normalized emittance at 7 MeV.

- The beam emittance will shrink when accelerating from 7 MeV to 200 MeV. The limit for the CIS extraction horizontal emittance is the Cooler acceptance, which is $10 \pi \text{ mm-mrad}$. The corresponding emittance limitation at 7 MeV is $56 \pi \text{ mm-mrad}$. The relation between the emittance for two energy cases is

$$\frac{\varepsilon_2}{\varepsilon_1} = \frac{\beta_1 \gamma_1}{\beta_2 \gamma_2}$$  \hspace{1cm} (5.20)

- The growth of the momentum deviation due to the scattering is very large. The initial momentum deviation is $\pm0.5\%$. The final momentum deviation is almost double: $\pm0.99\%$. This is too large for the rf cavity adiabatic capture. To reduce it, a debuncher is needed to reduce the initial value, and the strip injection time needs to be reduced to reduce the final value of the momentum deviation.

- During the first 300 $\mu$s injection time, the final horizontal beam emittance is less than the limitation. The increment of the horizontal emittance is small because of the small betatron function (1.005 m).

- The vertical beam emittance has a bigger increment step because of a larger betatron function (3.383 m) at the strip foil.

- The emittance growth mainly comes from the multiple Coulomb scattering. The energy staggering has a much smaller contribution to the total emittance growth.
• Beam positions at the strip foil: in the injection process, the closed orbit is first shifted 33 mm towards the strip foil by the dc chicanes, then bumped another 24 mm by the bumpers. Adding the beam size of 11 mm, the maximum beam position displacement from the CIS ring equilibrium orbit is 69 mm. The vacuum chamber at the strip foil is much bigger than 69 mm.

• The emittance growth due to the gas molecule scattering in the vacuum chamber is not big enough to cause beam loss. During the 500 ms time at 7 MeV, the final maximum beam size (33 × 33 mm) is still less than the vacuum chamber size (100 × 50 mm).

5.2.2 Closed Orbit Bump for CIS Injection

In order to inject the beam properly, the closed orbit need to be bumped to the position of the strip foil during the beam injection. Two magnetic bumpers B2 and B4 can be used to bump the closed orbit onto the strip foil. After the injection, the two bumpers are switched off to move the closed orbit off the strip foil completely. The two injection bumpers are located at asymmetric positions relative to the strip foil. To make things worse, the two bumpers have different vacuum chambers as there is a wall gap monitor within B4, thus the two bumpers have different magnetic field decay curves after switch-off because of the eddy current effect.

The closed orbit bump for a magnetic kicker is

\[
\Delta x(s) = x(s) - x_c(s) = \frac{\beta(s)\beta(s_0)}{2 \sin \pi \nu} \theta_0 \cos(\pi \nu - |\phi_s - \phi_0|) \tag{5.21}
\]

where \(\beta, \phi, \nu\) are betatron function, phase and tune, \(\theta_0\) is the kicker angle at the location “0” and \(\Delta x(s)\) is the closed orbit bump at the location “s”.

The following are the calculations of the closed orbit bump by B2 and B4.
Figure 5.2: Closed orbit bump by two asymmetric injection bumpers.

Solid line: asymmetric vacuum chambers; dashed line: symmetric vacuum chambers.

- The closed orbit can be shifted by 24 mm towards the strip foil when the bumpers are working at their full strength, 380 G. The position of the foil is adjustable both horizontally and vertically.
- The bump of the closed orbit is asymmetric and the closed orbit has a distortion of less than 5 mm outside the two bumpers, as shown in Figure 5.2.
- 100 µs after the bumpers are switched off, the circulating beam leave the strip foil completely.

The closed orbit outside B2 and B4 will be nonzero due to the asymmetric bumper
positions and decays. If the two bumpers are installed symmetrically, the closed orbit bump is also symmetric but still has small distortion outside the two bumpers before and after the switch-off. After the switch-off, the closed orbit bumps are asymmetric again due to the different decay curves of the two bumpers. The main reason for the nonzero closed orbit outside the bumpers is that the phase advance between two bumpers is larger than 180°. We can use different strengths for the two bumpers to overcome the asymmetric effect. But it doesn’t help to reduce the closed orbit bumps outside the two bumpers.

![Graph](image)

**Figure 5.3:** Injection profiles on BPM4. Injection beam: 220 μs, 250 μA. Maximum of 18 mA (8×10^{10} protons) were accumulated in 100 μs at 7 MeV and 5 mA at 3 MeV. The top curve shows the BPM4 signal of 7 MeV multi-turn accumulation, while the middle curve shows the BPM4 signal of 7 MeV single-turn injection. The curve at the bottom is the BPM4 signal of 3 MeV single-turn injection.
Figure 5.4: 7 MeV beam injection profile from BPM and WGM with RF cavity non-adiabatic capture. Upper: BPM4 multi-turn signal; Lower: WGM signal: 8.5 mA. Injection beam is the same as in Figure 5.3.

Figure 5.5: Circulating beam in the CIS ring. Horizontal: 0.2 μs/div, Vertical: 10 mV/div
5.2.3 Beam Injection and Accumulation

Proton beam injection and accumulation were done at 3 MeV (RFQ only) and 7 MeV (RFQ + DTL). Beam profiles can be observed on BPMs and HARP/STOP. Figure 5.3 shows the first turn and multi-turn (accumulation) beam injection profiles on BPM4. In the experiment, stored beam intensity scales with the source intensity (< 1000 μA). In Figure 5.4, 7 MeV injection beam profiles were also observed independently on a WGM monitor. From the beam signal on the WGM, only 8.5 mA is captured in the bucket with 200 V non-adiabatic RF cavity capture. Figure 5.5 shows the circulating beam in the CIS ring at period of 0.476 μs, or revolution frequency of \( f_0 = 2.09683 \) MHz.

5.3 Simulation and Experiment of CIS Ramping

After the 200 μs, a 1 Hz beam pulse from the RFQ/DTL is injected in the CIS ring by strip injection, the rf cavity will capture the dc beam and accelerate it from 7 MeV to 200 MeV. To achieve a high capture efficiency, adiabatic capture is needed, that is, the rf cavity voltage is turned on adiabatically.

5.3.1 Synchrotron Motion Equations

The Hamiltonian for the synchrotron motion in \((\frac{\Delta E}{\omega_0}, \phi)\) phase space is [4]:

\[
H = \frac{1}{2} \beta^2 E \left( \frac{\Delta E}{\omega_0} \right)^2 + \frac{1}{2\pi} \left[ \cos \phi - \cos \phi_s + (\phi - \phi_s) \sin \phi_s \right]
\]  

(5.22)

where \( \hbar \) is the harmonic number, \( \omega_0 = 2\pi f_0 \) is the revolution frequency, \( V \) is the rf voltage, \( \phi_s, \beta \) are the phase and velocity of the synchronous particle and \( \eta = \frac{1}{\pi} - \frac{1}{2\pi} \).
the phase slip factor. Equations of motion derived from the above Hamiltonian are:

\[
\begin{align*}
\frac{d\phi}{dt} &= \frac{\hbar \eta \omega_0^2}{\beta^2 E} \left( \frac{\Delta E}{\omega_0} \right) \\
\frac{d}{dt} \left( \frac{\Delta E}{\omega_0} \right) &= \frac{1}{2\pi} eV (\sin \phi - \sin \phi_s) 
\end{align*}
\]  

(5.23)

Equation of motion in \((\delta, \phi)\) space can be obtained from Eq. (5.23) by using the relation \(\delta = \frac{\Delta \mu}{\rho_0} = \omega_0 \eta \left( \frac{\Delta E}{\omega_0} \right)\):

\[
\begin{align*}
\frac{d\phi}{dt} &= i \omega_0 \eta \delta \\
\frac{d\delta}{dt} &= \frac{\omega_0 e V}{2\pi \beta^2 E} (\sin \phi - \sin \phi_s) 
\end{align*}
\]  

(5.24)

For the synchrotron motion in the adiabatic region, The Hamiltonian is time independent or the time variation of the Hamiltonian is small in a synchrotron period. The adiabatic condition can be described as: [4]:

\[
\left| \frac{1}{\omega_s^2} \frac{d\omega_s}{dt} \right| \ll 1 
\]  

(5.25)

where \(\omega_s\) is the synchrotron frequency. Then the Hamiltonian has a stable fixed point at \((\phi_s, 0)\) and an unstable fixed point at \((\pi - \phi_s, 0)\). The phase space trajectory of the separatrix in \((\frac{\Delta E}{\omega_0}, \phi)\) space is

\[
\left( \frac{\Delta E}{\omega_0} \right)^2_{sx} + \frac{e V \beta^2 E}{\pi \omega_0^2 \hbar \eta} [\cos \phi + \cos \phi_s - (\pi - \phi - \phi_s) \sin \phi_s] = 0 
\]  

(5.26)

The two turning points of the separatrix are \((\phi_u, 0)\) and \((\pi - \phi_s, 0)\), where \(\phi_u\) is given by

\[
\cos \phi_u + \phi_u \sin \phi_s = - \cos \phi_s + (\pi - \phi_s) \sin \phi_s 
\]  

(5.27)
The bucket area $A$, defined as the area enclosed by the separatrix, and the bucket height $\delta_B$ are given by

\[
A = 16 \sqrt{\frac{eV \beta^2 E}{2\pi\hbar \omega_0^2 |\eta|}} \alpha(\phi_s), \quad \delta_B = 2 \sqrt{\frac{eV \beta^2 E}{2\pi\hbar \omega_0^2 |\eta|}} Y(\phi_s)
\]

where $\alpha(\phi_s)$ is the ratio of the running bucket area and the stationary bucket area, and $Y(\phi_s)$ is the ratio of the running bucket height and the stationary bucket height. They are determined by the following equations:

\[
\alpha(\phi_s) = \frac{1}{4\sqrt{2}} \int_{\phi_s}^{\pi-\phi_s} \left[ -\frac{1}{\eta} (\cos \phi + \cos \phi_s - (\pi - \phi - \phi_s) \sin \phi_s) \right]^\frac{1}{2} d\phi
\]

(5.29)

\[
Y(\phi_s) = \left| \frac{\cos \phi_s - \frac{\pi - 2\phi_s}{2} \sin \phi_s}{\sin \phi_s} \right|^\frac{1}{2}
\]

(5.30)

### 5.3.2 Synchrotron Motion Mapping Equations

When simulating the synchrotron motion, turn-by-turn mapping equations are preferred. The mapping relations from the $n$th turn to $(n+1)$th turn are obtained by using the replacement $\frac{d}{dt} = \frac{\omega_0}{2\pi} \frac{d}{dn}$ in Eq. (5.23):

\[
\begin{cases}
\left( \frac{\Delta E}{\omega_0} \right)_{n+1} = \left( \frac{\Delta E}{\omega_0} \right)_n + \left( \frac{eV}{\omega_0} \right)_n \left( \sin \phi - \sin \phi_s \right) \\
\phi_{n+1} = \phi_n + \left( \frac{2\pi h \eta \omega_0}{\beta^2 E} \right)_{n+1} \left( \frac{\Delta E}{\omega_0} \right)_{n+1}
\end{cases}
\]

(5.31)

The mapping equations are advantageous over the numerical simulation methods in that they satisfy the symplectic condition (Jacobian = 1) while the numerical solvers may blow up the phase space area artificially. The Jacobian of the mapping shown in Eq. (5.31) is

\[
J = \frac{\partial \left[ \left( \frac{\Delta E}{\omega_0} \right)_{n+1}, \phi_{n+1} \right]}{\partial \left[ \left( \frac{\Delta E}{\omega_0} \right)_n, \phi_n \right]} = 1
\]

(5.32)
The phase space area in \((\frac{\Delta E}{\omega_0}, \phi)\) space is constant, which is also the result suggested by the Liouville’s Theorem.

In reality, we often do the synchrotron motion tracking in the \((\delta, \phi)\) phase space. Use the relation \(\delta_n = \left(\frac{\omega_0}{\beta^2 E}\right)_n \left(\frac{\Delta E}{\omega_0}\right)_n\) in Eq. (5.31), we get the mapping equations in \((\delta, \phi)\) phase space as follows:

\[
\begin{align*}
\delta_{n+1} &= \left(\frac{\omega_0}{\beta^2 E}\right)_{n+1} \left(\frac{\Delta E}{\omega_0}\right)_{n+1} \\
&= \left(\frac{\omega_0}{\beta^2 E}\right)_{n+1} \left[\left(\frac{\Delta E}{\omega_0}\right)_n + \left(\frac{eV}{\omega_0}\right)_n (\sin \phi - \sin \phi_s)\right] \\
&= \delta_n \left(\frac{\omega_0}{\beta^2 E}\right)_{n+1} \left(\frac{\beta^2 E}{\omega_0}\right)_n + \left(\frac{\omega_0}{\beta^2 E}\right)_{n+1} \left(\frac{eV}{\omega_0}\right)_n (\sin \phi - \sin \phi_s) \\
\phi_{n+1} &= \phi_n + \left(\frac{2\pi \hbar \eta \omega_0}{\beta^2 E}\right)_{n+1} \left(\frac{\beta^2 E}{\omega_0}\right)_n \delta_{n+1}
\end{align*}
\]

The Jacobian of the above mapping equation is

\[
J = \frac{\partial(\delta_{n+1}, \phi_{n+1})}{\partial(\delta_n, \phi_n)} = \left(\frac{\omega_0}{\beta^2 E}\right)_{n+1} \left(\frac{\beta^2 E}{\omega_0}\right)_n = \frac{(\beta \gamma)_n}{(\beta \gamma)_{n+1}} < 1
\]

which means the space area in \((\delta, \phi)\) shrinks during the acceleration.

### 5.3.3 Optimization of the Dipole Ramping Curve

According to the CIS design, the ring magnet allows acceleration of protons up to 200 MeV at 1Hz and 5Hz. In the 1Hz mode, the ramping time to accelerate the 7 MeV protons to 200 MeV is 600 ms. The dipole magnetic rigidity \(B\rho\) was programmed to ramp following the curve:

\[
B\rho(t) = \left[\sin \left(\frac{\pi}{2} \cdot \frac{t}{T_r}\right)\right]^j [B\rho_f - B\rho_i] + B\rho_i
\]

where \(j\) is the fitting parameter, \(T_r\) is the ramping time, and \(B\rho_i\) and \(B\rho_f\) are the initial and final values determined by the particle energy \(T\):

\[
B\rho[T\cdot \text{m}] = \frac{1}{299.7925} P = \frac{1}{299.7925} \sqrt{(T[\text{MeV}] + E_0)^2 - E_0^2}
\]
The fitting parameter $j$ should be chosen in such a way that the ramping of $B\rho$ will produce the minimum vacuum chamber eddy current and its induced sextupole components. We already know that the eddy current is proportional to the quantity $\frac{(B\rho)}{B_0B}$. From the ramping curves shown in Figures 5.6 we noticed that $\frac{(B\rho)}{B_0B}$ is minimum when $j = 3$.

A proper dipole coil current must be provided in order to get the required $B\rho$ curve. All four CIS dipole magnets are connected in series and ramped by one power supply. Measurements of $I(B\rho)$ were conducted in steps of up-ramps, i.e., in steps of 100 A where each measurement using a Hall probe took about 4 seconds [16]. The measured data of main dipole current is fitted by the analytical function

$$ I(B\rho) = a_0 + a_1 B\rho + a_2 (B\rho - B_{2\gamma})^2 + a_7 (B\rho - B_{2\gamma})^7 + a_{12} (B\rho - B_{1\gamma})^{12} + a_{s1} \sin(f_{s1}B\rho) + a_{s2} B\rho \sin(f_{s2}B\rho) $$

(5.37)
where the constants are listed below

\[
\begin{align*}
    a_0 &= -6.775 \quad a_1 = 1534.786 \quad a_2 = 21.474 \quad B_{27} = 0.942 \\
    a_7 &= 59.645 \quad a_{12} = -5.556 \quad B_{12} = 1.053 \quad a_{s1} = -1.311 \\
    f_{s1} &= 6.622 \quad a_{s2} = 0.605 \quad f_{s2} = 12.172
\end{align*}
\]

The above fitted function reproduces the measured data well within 0.1% in the field range from 0.303T to 1.780T.

With the \( I(B\rho) \) curve, the power supply voltage can be obtained immediately as

\[
V_s(t) = R \cdot I(B\rho) + L \cdot \frac{dI(B\rho)}{dt}
\]

\[
= R \cdot I(B\rho) + L \cdot \frac{dI(B\rho)}{d(B\rho)} \cdot \frac{d(B\rho)}{dt} \tag{5.38}
\]

where \( R = 0.023 \Omega \) and \( L = 0.0345 H \) are the dipole load resistance and inductance respectively.

### 5.3.4 Adiabatic Capture

The injected dc beam in the CIS ring has a uniform initial distribution in the \((\delta, \phi)\) space. The synchrotron tune \( \nu_s \) depends on the synchrotron amplitude \([4]\) and is smaller for the large amplitude particles. Due to this synchrotron tune spread effect, the injection bunch actually fills up all the stationary bucket area after capture. However, if the rf voltage is turned on adiabatically, the beam bunch can be captured in the central area of the bucket. While the strict adiabatic capture condition is difficult to meet, computer simulation shows that an rf turn-on time of a few synchrotron periods is sufficient for the adiabatic capture efficiency to reach its maximum. The rf voltage usually has the following form in order to change smoothly during the adiabatic capture process:

\[
V_{pp}(t) = \left[ 3 \left( \frac{t}{T_v} \right)^2 - 2 \left( \frac{t}{T_v} \right)^3 \right] (V_1 - V_0) + V_0 \tag{5.39}
\]
where $V_0$ and $V_1$ are the initial and final voltages and $T_r$ is the adiabatic capture time. The first 5 ms in curve (d) of Figure 5.13 on page 111 shows the summary of Rf voltage during the adiabatic capture and other ramping curves.

To show this effect, simulations were done for the cases with and without adiabatic capture. In Figure 5.7, the adiabatic capture in (e) is much better than the normal capture in (b), especially when the bucket area shrinks by $\alpha(\phi_a)$ during the acceleration in (c) and (f).

5.3.5 Beam Capture Experiment and Tune Measurement

Figure 5.8 shows the beam profile and rf cavity voltage change during the adiabatic capture. Figure 5.9 shows the beam profiles from the HARP and WGM along the beam line. The transmission efficiency is 73% from source to Linac and 83% from Linac to the position before injection. In totally 24.2 mA of proton beam, or $7 \times 10^{10}$ protons, were captured 5 ms after injection.

As mentioned in Section 5.1, the Ping Tune System can measure the CIS fractional tune at the rate of 10 measurements per second. Figure 5.10 shows the display of the horizontal tune measurement and closed orbit drift. The measured tunes, $Q_x = 1.480 \pm 0.02$ and $Q_z = 0.769 \pm 0.02$, are very close to the design values of $Q_x = 1.4794$ and $Q_z = 0.7877$.

Figure 5.11 on page 108 shows the CIS closed orbit distortion during the injection and capture. The horizontal closed orbit drifts within $\pm 5$ mm while the vertical drift is $\pm 1$ mm.

After the beam injection and capture, we want the beam to last for a long time at the peak current at 7 MeV. The scattering of the vacuum gas molecules, space charge effect, etc., can cause emittance growth and beam loss. The beam lifetime is defined as the time for the beam intensity to decay to $e^{-1}$ of its original value, or to decay
Figure 5.7: RF cavity capture of the injected beam bunch: Normal capture: (a) Initial distribution with rf voltage $V=240\text{V}$; (b) Stationary bucket capture; (c) Running bucket capture: $\phi_s = 16^\circ$; Adiabatic capture: (d) Initial distribution with no rf voltage; (e) Stationary bucket capture: $V=240\text{V}$; (f) Running bucket capture: $\phi_s = 16^\circ$;
by 8.7 dB. Figure 5.12 shows the beam intensity decay on the Spectrum Analyzer. Measurement results show that beam life time is 420 ms when vacuum is $3.8 \times 10^{-7}$ torr and 580 ms when $2.9 \times 10^{-7}$ torr. So the life time at low energy is dominated by the vacuum pressure, rather than the space charge effect.

### 5.3.6 Simulation of the CIS Ramping Procedure

The CIS ramping procedure starts with the ramping of the power supply according to Eq. (5.37), and then the ramping of $B\rho$ shown in Eq. (5.35) with $j = 3$. The rf voltage is turned on smoothly according to Eq. (5.39). All other parameters will change accordingly as follows.
Figure 5.9: Beam profiles from source to CIS ring. Beam intensity at the source: 570 $\mu$A. In the figures (a) to (d), left is the horizontal beam profile and right the vertical, and 5 mm/div for the horizontal axes, 5 ms/div for the horizontal axis in (e).
Figure 5.10: Horizontal tune measurement for 7 MeV circulating beam by Ping Tune System. Upper: tune spectrum, $Q_x=0.48$; Lower: closed orbit drift ($<\pm 5$ mm)

Figure 5.11: CIS closed orbit measurement. Horizontal: $<\pm 5$ mm. Vertical: $<\pm 1$ mm.
Figure 5.12: Beam life time measurement at 7 MeV. The horizontal axis is time and vertical axis is logarithm of beam intensity. Captured peak current: 13 mA. Vacuum: $3.8 \times 10^{-7}$ torr. Beam life time: 420 ms

Particle momentum and energy:

\[
P[\text{MeV}] = 299.7925B \rho [\text{Tm}] \quad (5.40)
\]

\[
E[\text{MeV}] = \sqrt{(299.7925B \rho [\text{Tm}])^2 + E_0^2} \quad (5.41)
\]

where $E_0 = 938.27$ MeV is the proton rest energy.
Rf frequency is then
\[
\omega_{r,f} = 2\pi h \frac{\beta c}{L} = \frac{\hbar c}{R} \left[ 1 + \left( \frac{E_0}{299.7925 B \rho [\text{Tm}]} \right)^2 \right]^{-\frac{1}{2}}
\] (5.42)
where \( R = L/(2\pi) \) is the CIS mean radius, \( h \) is the harmonic number, and \( c \) the speed of light.

The rf phase of the synchronous particle \( \phi_s \) depends on the rf voltage and the energy gain per turn. At time \( t \), when the particle period is in its \( n \)th turn, its period is \( T_0 = 2\pi h/\omega_{r,f} \). From Eq. (5.41), the energy gain in \( n \)th turn is
\[
\Delta E_n [\text{MeV}] = \frac{299.7925P}{E} \cdot \Delta(B\rho) = \frac{299.7925}{\sqrt{1 + \left( \frac{E_0}{299.7925 B \rho [\text{Tm}]} \right)^2}} \cdot \frac{d(B\rho)}{dt} T_0
\] (5.43)
The \( \phi_s \) is then expressed as
\[
\phi_s = \sin^{-1} \left[ \frac{\Delta E_n [\text{MeV}] \times 10^6}{V_{pp}[\text{V}]} \right] = \sin^{-1} \left[ \frac{10^6}{V_{pp}[\text{V}]} \cdot \frac{299.7925P}{E} \cdot \frac{d(B\rho)}{dt} T_0 \right]
\] (5.44)
where \( P, E \) and \( d(B\rho)/dt \) can be obtained easily from the previous equations.

The revolution frequency for the initial 7 MeV proton in CIS ring is 2.09725 MHz and its synchrotron period is 0.67 ms. So the adiabatic capture time \( T_v = 5 \) ms is a reasonable choice for the adiabatic capture. The maximum rf voltage available is 500 V. Figure 5.13 shows various CIS ramping curves.

The initial dc beam in the CIS ring has a uniform distribution in \( (\delta, \phi) \) space. The maximum initial momentum deviation \( \delta_m \) should be as small as possible. The beam bunches from the RFQ/DTL have \( \delta = \pm 0.3\% \) after the debuncher; \( \delta \) will increase by \( \pm 0.2 \) during the 200 \( \mu \)s injection period due to the strip foil scattering. So the total initial \( \delta_m = \pm 0.5\% \), as shown in (a) of Figure 5.14.

Since the maximum rf voltage is only 500V, the particles are not fully captured by rf cavity adiabatically. With the increment of the particle energy, there is a beam loss because the bucket shrinks faster than the particle phase space. The final acceleration
efficiency is only 63.3%. It is difficult to reduce the initial beam momentum deviation $\delta_m$. Increasing the rf voltage is expected to yield higher acceleration efficiency. Simulation results show that when $V_{pp} = 760$ V, all the particles can be accelerated to the desired energy.
5.3.7 CIS Ramping Experiment

Figure 5.15 show the ramping procedure from 7 MeV to 200 MeV, including the adiabatic capture. Figure 5.16 shows the WGM beam signal during the 225 MeV ramp. Table 5.2 gives the summary of the ramping experiments. Figure 5.17 shows a very small tune shift during the 200 MeV and 225 MeV ramps.
(a) Monitoring of Hz. c.o., \( \dot{B} \) and Vt. c.o. during the ramp

(b) WGM beam signal and RF voltage. 100ms/div.

Figure 5.15: Beam ramping from 7 MeV to 200 MeV in 1000 ms

5.3.8 Matching of the RF Parameters Between CIS and Cooler

The CIS rf parameters must be chosen properly to produce qualified beam pulses for the injection to the Cooler. In order to avoid the bunch dilution during the bucket to bucket transfer from CIS to Cooler, the ratio of the bucket height and width of the
Figure 5.16: 225 MeV ramping in 1 second. Upper: WGM beam signal. Lower: log $I$ signal from BPM5

Figure 5.17: Tune shift during the 200 MeV and 225 MeV ramping
Table 5.2: Summary of CIS ramping experiments

<table>
<thead>
<tr>
<th></th>
<th>7</th>
<th>7</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Energy (MeV)</td>
<td>0.476</td>
<td>0.476</td>
<td>0.476</td>
</tr>
<tr>
<td>Period (μs)</td>
<td>8.69</td>
<td>8.69</td>
<td>5.4</td>
</tr>
<tr>
<td>(particle number)</td>
<td>(2.6 × 10¹⁰)</td>
<td>(2.6 × 10¹⁰)</td>
<td>(1.6 × 10¹⁰)</td>
</tr>
<tr>
<td>Adiabatic capture time (ms)</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Ramping time (ms)</td>
<td>600</td>
<td>1000</td>
<td>600</td>
</tr>
<tr>
<td>RF voltage (V)</td>
<td>120~300</td>
<td>120~300</td>
<td>250~500</td>
</tr>
<tr>
<td>Final Energy (MeV)</td>
<td>100</td>
<td>200</td>
<td>225</td>
</tr>
<tr>
<td>Period (μs)</td>
<td>0.135</td>
<td>0.102</td>
<td>0.0979</td>
</tr>
<tr>
<td>Final beam (mA)</td>
<td>17</td>
<td>25</td>
<td>19.2</td>
</tr>
<tr>
<td>(particle number)</td>
<td>(2 × 10¹⁰)</td>
<td>(1.6 × 10¹⁰)</td>
<td>(1.18 × 10¹⁰)</td>
</tr>
<tr>
<td>Transmission efficiency</td>
<td>70%</td>
<td>60%</td>
<td>73%</td>
</tr>
<tr>
<td>Hz. c.o. drift (mm)</td>
<td>±8</td>
<td>±15</td>
<td>±3</td>
</tr>
<tr>
<td>Vt. c.o. drift (mm)</td>
<td>±1</td>
<td>±2</td>
<td>±2</td>
</tr>
</tbody>
</table>

two rings must be maintained,

\[
\left| \frac{\hat{\delta}}{R \theta_{CIS}} \right| = \left| \frac{\hat{\delta}}{R \theta_{Cooler}} \right|
\]

(5.45)

where \( R \) is the average radius of the ring, \( \hat{\theta} \) is the azimuthal orbital angle of the beam pulse ( \( R \hat{\theta} \) is the beam pulse length). This condition means that the rf system in CIS should reflect a proper longitudinal matching condition.

The momentum acceptance of the Cooler ring is \( \hat{\delta} = \pm 0.23\% \). Thus the final bucket height of the CIS beam pulse should be less than the above limitation, with the corresponding rf voltage less than 200 V. In the case of the above simulation with
rf voltage of 500 V, the final bucket height is 0.46%.

5.4 Beam Extraction from CIS

The CIS is designed to be used as the injector for the IUCF Cooler and is able to accelerate polarized protons (deuterons) from 7 MeV (5 MeV) to 200 MeV (107 MeV). Figure 5.18 is the schematic layout of the CIS and the Cooler.

At the final energy, particle beams are extracted vertically by a traveling wave fast kicker and a 12° Lambertson septum. Then the beams are transported to the Cooler ring by the beam line BL9A, which consists of a suitable set of quadrupoles and dipoles for beam matching at the injection point and two solenoids for spin manipulation. At the Cooler injection point, another set of septum plus kicker devices is used for the Cooler injection. The transverse and longitudinal beam parameters are matched at the injection point by adjusting the positions and the strengths of the quadrupoles and by setting the proper conditions of both CIS and Cooler rf systems, respectively.

5.4.1 Extraction Method

At the energy of 200 MeV, the particle period in the CIS is 102 ns and the beam bunch length is about 50 ns. A traveling wave fast kicker [18] with flat top of 320 ns is used to do the fast single-turn extraction from the CIS. Figure 5.19 shows the typical structure of the single-turn extraction. The septum is a specially designed dipole magnet with a thin partition (i.e. septum) which separates the deflecting field from a field free region. After being deflected by the kicker, the beam passes through the region with the horizontal deflecting field for vertical extraction while the closed orbit passes through the field-free region. The beam trajectory is kicked horizontally to enter the extraction field region of the septum. The beam parameters at the kicker
Figure 5.18: Schematic layout of CIS ring, beam transport line BL9A and Cooler injection portion
and the septum have the relations

\[
\begin{align*}
    x_s &= \delta_k \sqrt{\beta_k \beta_s} \sin \psi \\
    x'_s &= x_s \beta_s^{-1} (\cot \psi - \alpha_s)
\end{align*}
\]

(5.46)

where \( \delta_k \) is the kicker angle, \( \beta_k, \beta_s \) are the lattice \( \beta \)-functions at the kicker and the septum and \( \psi = \psi_s - \psi_k \) is the betatron phase advance.

The offset \( x_s \) must be larger than the full-width of the circulating beam with an allowance for closed orbit distortions and alignment errors plus the half-width of the extracted beam and the septum width. The kicker is often limited by technology or economics, so it is advantageous to make \( \psi \) closed to \( \pi/2 \) and \( \beta_k \) large so that \( \delta_k \) is reduced. The minimum strength of the septum field depends on how much angular deflection is needed to take the beam clear of the next machine magnet (in the case of CIS, a dipole). A large value for \( \beta_s \) reduces \( \delta_k \), but has the adverse effects of increasing the beam size and hence \( x_s \) and \( x'_s \). However, the deviation of the beam directly from the central orbit provided by the kicker is not big enough for the
structure of the septum. To minimize the cost and strength of the fast kicker, beam bumpers are used to adjust the closed orbit locally closer to the septum. Outside the four bumpers, the closed orbit is still along the CIS ring center line.

5.4.2 Traveling Wave Kicker

In order to perform the single-turn extraction, it is required that the extraction pulse rise, stay at the top for extraction, and reset to zero within the proton beam cycling time of 100 ns at the energy of 200 MeV. Ferrite loaded kickers are well suited to provide strong angle kicks, but they are not fast enough for the CIS extraction. Suitably designed transmission line kicker [19] meets the requirements for the CIS extraction.

The transmission line consists of two parallel electrodes within a vacuum tube. The impedance of 50 Ω was chosen for convenience in the design of power supply and electrode geometry. A bipolar pulse, sent in the opposite direction of the beam creates an electric and magnetic field which deflect the beam in the same direction. Therefore, this type of kicker is also referred to as "counter traveling wave kicker". The pulse propagates in the TEM mode with the speed of light $c$ along the transmission line. Under this assumption electric and magnetic field lines are perpendicular to each other and $E = B \cdot c$. A simple expression for the total angular deflection $\Theta$ for a constant electric field $E$, i.e. constant electrode separation, is

$$\Theta = \frac{E \cdot L}{c \cdot B \rho} \cdot \left( \frac{1}{\beta} + 1 \right)$$ (5.47)

where $B \rho$ is the beam rigidity (2.148 Tm for 200 MeV protons), $E$ is the electric field, $\beta c$ is the particle velocity, and $L$ is the kicker length.

As will be discussed in the following section, for CIS extraction the required kicker angle is larger than 15 mrad. When $L = 1.3$ m, $E = 30$ kV/cm, the kicker angle is 16.8 mrad. For a gap of 4.0 cm this assumes pulse voltages of ± 60 kV. For 220
MeV protons the kick angle reduces to 15.9 mrad, still sufficient for extraction of the full beam. The kicker gap is chosen to be big enough for the beam. At an energy of 7 MeV with a momentum spread of 1%, the injection beam has a maximum total horizontal and vertical spread of 3 cm and 2.2 cm. A horizontal gap of 4 cm was chosen for the design to allow for uncertainties [18].

5.4.3 Closed Orbit Bump

The fast traveling wave kicker[18] used in CIS can deflect 200 MeV protons by at least an angle of 15 mrad with a rise time of 40 ns. The resulting deviation of the beam at the septum is only 17 mm, while the beam should have a total displacement of > 40 mm in order to be extracted from the ring. So, before kicking the beam, four bumper magnets are used to bump the closed-orbit horizontally by about 25 mm from the central orbit in the vicinity of the septum. The four bumper method is often used since it doesn’t displace the closed orbit outside the bumpers.

The closed orbit bump at location $s$ from a single bumper is

$$\Delta x(s) = x_s - x_c = \frac{\sqrt{\beta_s \beta_0}}{2 \sin \pi \nu} \theta_0 \cos(\pi \nu - |\psi_s - \psi_0|) \quad (5.48)$$

where $\beta, \psi, \nu$ are betatron function, phase and tune and $\theta_0$ is the kicker angle at the location “0”.

The closed orbit bump of four bumpers along the ring is [4]:

$$x(s) = x_s - x_c = \frac{\sqrt{\beta(s)}}{2 \sin \pi \nu} \sum_{i=1}^{4} \sqrt{\beta_i \theta_i \cos(\pi \nu - |\psi_s - \psi_i|)} \quad (5.49)$$

where $\beta_i, \psi_i, \nu$ are betatron function, phase and tune, $\theta_i$ is the kicker angle.

We only want the orbit displacement within the region of the four bumpers. The condition is

$$x(s_4) = 0, \quad x'(s_4) = 0 \quad (5.50)$$
5.4 Beam Extraction from CIS

After solving the above two equations, we get the following relations [4] among the four bumper parameters.

\[
\begin{align*}
\sqrt{\beta_3} \theta_3 &= -\frac{\sqrt{\beta_1} \sin \psi_{14} + \sqrt{\beta_2} \sin \psi_{24}}{\sin \psi_{34}} \\
\sqrt{\beta_3} \theta_4 &= \frac{\sqrt{\beta_1} \sin \psi_{13} + \sqrt{\beta_2} \sin \psi_{23}}{\sin \psi_{34}}
\end{align*}
\]

(5.51)

where \( \psi_{ij} = (\psi_j - \psi_i) \) is the phase shift.

**Table 5.3:** Closed orbit bump of 25 mm for extraction

<table>
<thead>
<tr>
<th></th>
<th>Length(m)</th>
<th>( \beta_x ) (m)</th>
<th>( \psi_i(2\pi) )</th>
<th>Strength(G)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B2</td>
<td>0.2</td>
<td>1.613</td>
<td>0.260</td>
<td>989</td>
</tr>
<tr>
<td>Dipole trim 2</td>
<td>2.0</td>
<td>4.373</td>
<td>0.549</td>
<td>145</td>
</tr>
<tr>
<td>Dipole trim 4</td>
<td>2.0</td>
<td>4.373</td>
<td>0.915</td>
<td>141</td>
</tr>
<tr>
<td>B4</td>
<td>0.2</td>
<td>1.480</td>
<td>1.194</td>
<td>1020</td>
</tr>
</tbody>
</table>

Four bumpers in the CIS ring, Bumper 2, Dipole trim 2, Dipole trim 3 and Bumper 4, are used to bump the closed orbit towards the septum. Table 5.3 shows the orbit bump and required bumper strength. For example, 989 G and 1020 G are required for B2 and B4 in order to produce a closed orbit bump of 25 mm. The strength required for the dipole trim coil is less than the designed value of 200 G. Figure 5.20 also shows that the closed orbit displacement is 25 mm in the vicinities of septum and there is no closed orbit bumps outside the four bumpers.

5.4.4 Beam Extraction Trajectory

After the closed orbit is bumped 25 mm nearer the septum, the beam is kicked by the fast kicker and has a horizontal displacement of 44 mm from the CIS equilibrium.
**Figure 5.20:** Closed orbit bump and beam extraction from CIS. Note that the orbit deviations are enlarged by 10 times.

**Figure 5.21:** Beam extraction trajectory in the vertical plane
orbit at the entrance of the septum, a vertical Lambertson extraction magnet. The kicked beam from Dipole 2 has an angle of 11.57 mrad (0.66°) inwards before entering septum. So the beam displacement is 44 mm at the entrance of the septum and 38 mm at the its exit. Vertically, the beam is deflected 12.3° upwards by the septum and later returned to horizontal by another dipole TPES as shown in Figure 5.21. After it is extracted from the CIS, the beam is 0.81 m above the CIS ring orbit plane.

The beam displacement at the septum by the kicker is proportional to the fast kicker strength. For an estimation, \( \Delta x \) (mm) = \( x_\beta - x_{eo} = 1.2995 \cdot \theta_k \) (mrad).

The horizontal small angle of the extracted beam can be eliminated by the rotation of the septum. When rotated by \( \theta_r \), the septum strength \( B_s \) will have two components \( B_x \) and \( B_z \). The beam deflections in the two directions are

\[
\Delta x' = \frac{B_x l}{B \rho} = \frac{B_x l \sin \theta_r}{B \rho}, \quad \Delta z' = \frac{B_z l}{B \rho} = \frac{B_z l \cos \theta_r}{B \rho}
\]

(5.52)

where \( l = 0.508 \) m is the septum length and \( B_s = 9078 \) G is the strength needed to deflect the beam by 12.3°. When \( \theta_r = 3.1^\circ \), the horizontal small angle is canceled, as shown by the dotted line in Figure 5.20.

5.4.5 Effect of Bumper and Kicker Pulse Fluctuation

The fast kicker used for CIS extraction has a 50 ns rise time, a 320 ns flat top and a 100 ns fall time. If both the bumper and kicker power supplies have a fluctuation, the closed orbit and the extracted beam trajectory will be affected slightly. With a ±5% of the bumper pulse fluctuation, and ±5% kicker pre-pulse and top fluctuation, the change of the beam trajectory at the kicker and the septum is less than ±2 mm. However there is a ±5.9 mm beam trajectory shift at the entrance of the first dipole in the extraction beam line.
5.5 Beam Transportation from CIS to Cooler

The extracted beam from the CIS ring has an emittance of 10 πμm and momentum spread of ±0.2% and is transported from CIS to Cooler by the transport beam line 9A (BL9A) composed of a set of suitable quadrupoles, dipoles and solenoids, as shown in Figure 5.18 on page 117. Two solenoids SOL1, SOL2 and two dipoles D1, D2 are used for providing a proper spin orientation for polarized beam injection for the Cooler ring. In order to inject the beam into the Cooler properly, both the transverse and longitudinal beam parameters must be matched at the Cooler injection point. The transverse beam matching can be reached by adjusting the distance between quadrupoles and their strength.

5.5.1 Transversal Beam Matching

The beam is injected into Cooler at the septum L00 in the Cooler ring. A perfect transverse match of two lattices requires all lattice functions to be the same at the injection point. That is

\[(\beta_x, \alpha_x, \beta_y, \alpha_y, D_x, D'_x)_{BL9A} = (\beta_x, \alpha_x, \beta_y, \alpha_y, D_x, D'_x)_{Cooler}\]  \hspace{1cm} (5.53)

which means the phase ellipse at the end of BL9A is the same as the phase ellipse at the entrance of the Cooler ring. A mismatch of the phase space will result in an emittance growth or dilution of the injected beam. From Appendix B on page 145, the emittance growth factor due to the betatron parameter mismatch is

\[ f = F^2_+ = X_d - \sqrt{X^2_d - 1} \]  \hspace{1cm} (5.54)

where

\[ X_d = \frac{1}{2}(\gamma_1 \beta + \beta_1 \gamma - 2 \alpha_1 \alpha) \]  \hspace{1cm} (5.55)
A match of the dispersion function \((D_x, D'_x)\) assures that the phase ellipse for off momentum particles matches as well. Mismatch of the dispersion function will cause a bigger emittance growth factor for particles with momentum deviation \(\delta\):

\[
f = \left[ \sqrt{X_d + \sqrt{X_d^2 - 1}} + \delta \sqrt{\frac{(\Delta D)^2}{\beta} + \left( \sqrt{\frac{1}{\beta}} \Delta D' + \frac{\alpha}{\sqrt{\beta}} \Delta D \right)^2} \right]^2
\]

(5.56)

In general, there are six lattice functions to be matched requiring at least six variables or quadrupoles in the focusing structure of the upstream lattice to produce a perfect match. Matching quadrupoles must not be too close together in order to provide some independent matching power for individual quadrupoles.

**Table 5.4:** Beam parameters matched at Cooler septum

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Without solenoids</th>
<th>With solenoids</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta_x) (m)</td>
<td>1.384</td>
<td>1.4120</td>
</tr>
<tr>
<td>(\beta_y) (m)</td>
<td>1.815</td>
<td>1.8338</td>
</tr>
<tr>
<td>(\alpha_x)</td>
<td>0.218</td>
<td>0.2146</td>
</tr>
<tr>
<td>(\alpha_y)</td>
<td>-0.040</td>
<td>-0.0855</td>
</tr>
<tr>
<td>(D_x) (m)</td>
<td>-4.152</td>
<td>-3.0293</td>
</tr>
<tr>
<td>(D'_x)</td>
<td>0.083</td>
<td>1.3519</td>
</tr>
</tbody>
</table>

The fitting process of beam line 9A is done using the SYNCH [11] program. The following are major conclusions of the fitting procedure:

- Fitting variables: \((\beta_x, \alpha_x, \beta_y, \alpha_y, D_x, D'_x)\) at the Cooler septum, the injection point.
### Table 5.5: Fitting constraints for Cooler injection match

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Initial</th>
<th>Without solenoids</th>
<th>With solenoids</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_1$</td>
<td>9.9999982</td>
<td>7.3273088</td>
<td>2.3726080</td>
</tr>
<tr>
<td>$B_2$</td>
<td>-7.2650281</td>
<td>-5.7501733</td>
<td>5.8324440</td>
</tr>
<tr>
<td>$B_3$</td>
<td>6.6119067</td>
<td>7.7916690</td>
<td>8.1835909</td>
</tr>
<tr>
<td>$B_4$</td>
<td>-6.5590755</td>
<td>-6.4586484</td>
<td>6.9242199</td>
</tr>
<tr>
<td>$B_5$</td>
<td>2.2938179</td>
<td>2.5982929</td>
<td>2.7204111</td>
</tr>
<tr>
<td>$B_6$</td>
<td>-2.4567029</td>
<td>-3.9166315</td>
<td>-2.9140665</td>
</tr>
<tr>
<td>$B_{99}$</td>
<td>2.0705442</td>
<td>3.6261811</td>
<td>1.6585291</td>
</tr>
<tr>
<td>$B_{160}$</td>
<td>-1.2928168</td>
<td>-2.9227807</td>
<td>0.6225340</td>
</tr>
<tr>
<td>$S_{12}$</td>
<td>0.2098016</td>
<td>0.2000000</td>
<td>0.2000001</td>
</tr>
</tbody>
</table>

### Table 5.6: Emittance growth when changing D1, D2 positions without solenoids. The first three lines are without solenoids and the last line is with the two solenoids in BL9A

<table>
<thead>
<tr>
<th>Position Change (m)</th>
<th>Horizontal</th>
<th></th>
<th>Vertical</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mismatch Factor</td>
<td>$\epsilon_x$ Growth</td>
<td>Mismatch Factor</td>
<td>$\epsilon_y$ Growth</td>
</tr>
<tr>
<td>-0.2</td>
<td>1.00024</td>
<td>1.02355</td>
<td>1.00084</td>
<td>1.04180</td>
</tr>
<tr>
<td>-0.1</td>
<td>1.00023</td>
<td>1.02318</td>
<td>1.00106</td>
<td>1.04708</td>
</tr>
<tr>
<td>0.0</td>
<td>1.00022</td>
<td>1.02260</td>
<td>1.00117</td>
<td>1.04964</td>
</tr>
<tr>
<td>0.0</td>
<td>1.00010</td>
<td>1.02339</td>
<td>1.00335</td>
<td>1.0853</td>
</tr>
</tbody>
</table>
Figure 5.22: Betatron functions along the BL9A without solenoids after beam matching at Cooler injection point

- Constraints: the field gradients $B'$ of eight quadrupoles from QA1 to QA6, Q99 and Q109 (remember that quadrupole coefficient $K = \frac{1}{B_0} B'$), the distance SA12 between the first two quadrupoles Q1 and Q2.

- The minimum distance from the exit of dipole TPES and D1 is 3.3m and an increment of the distance will reduce the gap between D1 and D2, while the gap must be longer than the length of the solenoid.

- Moving the positions of D1 and D2 will change the emittance growth factor. Table 5.6 shows the fitting results by moving D1 and D2 from the original position. The original position gives the reasonable emittance match factor and growth in the two cases of with and without solenoids.

- The effect (0.15%) of the dispersion mismatch due to the momentum spread of
\( \sim 0.1\% \) is very small compared to that(\( \sim 2.2\% \)) of the betatron parameters. So in the SYNCH fitting procedure, it is desired to keep \( (\beta_x, \alpha_x, \beta_y, \alpha_y) \) have higher accuracy.

- The fitting results in the two cases of with and without solenoids are different. Table 5.4 and 5.5 show the fitting results of the lattice functions at the Cooler septum and the requirement for the constraints in the above two cases.

- As shown in Table 5.4, the emittance growth is 2.22\% for \( \epsilon_x \) and 4.96\% for \( \epsilon_y \) without solenoids, and 2.33\% for \( \epsilon_x \) and 8.53\% for \( \epsilon_y \) with solenoids.

- The horizontal and vertical beam envelopes are defined as

\[
\sigma_{x,z} = \sqrt{\beta_{x,z}^2 + (D_{x,y} \delta_p)^2} \tag{5.57}
\]

where \( \epsilon, \beta \) and \( D \) are beam emittance, beta-function and dispersion function, respectively. Figure 5.22 shows the beta functions of BL9A. The maximum horizontal beam envelope, which happens at the the end of dipole D49(refer to Figure 5.18 on page 117), is 9.7 mm and smaller than the “good field” region in the dipole region. The vertical beam envelope is also less than all the vertical dipole gaps along BL9A.

### 5.5.2 Beam Injection into Cooler Ring

If the fast kicker located at the center of the first dipole of the Cooler ring is used as the injection kicker, as shown in Fig. 5.23, the relation between the beam positions and angles at the septum \( (x_s, x'_s) \) and at the kicker \( (0, \delta_k) \) is

\[
\begin{align*}
  x_s &= \delta_k \sqrt{\beta_k \beta_s} \sin \psi \\
  x'_s &= \delta_k \sqrt{\frac{\beta_k}{\beta_s}} (\cos \psi - \alpha_s \sin \psi)
\end{align*} \tag{5.58}
\]
where $\beta_k, \beta_s, \alpha_s$ are betatron parameters at the kicker and septum and $\psi = |\psi_s - \psi_k|$ is the betatron phase shift between the septum and kicker. The offset $x_s$ must not be less than the half-width of the injected beam and the septum width plus the half-width of the circulating beam with an allowance for closed orbit distortions and alignment errors. The above two equations are used when the beam direction is from septum to the kicker. To reduce the kicker strength $\delta_k$, it is usually advantageous to make $\psi$ closed to $\pi/2$ and $\beta_k$ large.

From the above equations, when

$$\tan \psi = \frac{1}{\alpha_s}$$

the incoming beam will be parallel to the closed orbit.

The distance from the septum to the fast kicker is 6.459 m. Table 5.7 shows the betatron parameters at the fast kicker and the septum. The sign of $\alpha_s = -\beta'_s/2$ here is opposite to that in the output of the MAD program because MAD assumes the
Table 5.7: Betatron parameters at the Cooler septum (L00) and fast kicker (HC01)

<table>
<thead>
<tr>
<th></th>
<th>Septum(L00)</th>
<th>Fast Kicker HC01</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_x$(m)</td>
<td>4.152</td>
<td>1.233</td>
</tr>
<tr>
<td>$\beta_x$ (m)</td>
<td>1.384</td>
<td>15.117</td>
</tr>
<tr>
<td>$\alpha_x$</td>
<td>-0.218</td>
<td>0.680</td>
</tr>
<tr>
<td>$\mu_x$ (2\pi)</td>
<td>0.000</td>
<td>0.282</td>
</tr>
</tbody>
</table>

opposite beam direction to that of Eq. (5.58).

The relations among $x_s, x'_s$ and $\delta_k$ are

$$\delta_k = 0.2231x_s, \quad x'_s = 0.0459\delta_k = 0.0102x_s \quad (5.60)$$

The kicker used is the existing fast kicker HC01 between the two dipoles DIA and DIB in the Cooler Ring. It has a kick angle of 2.8 mrad for 200 MeV protons with a rise time of 90 ns and fall time of 104 ns. Table 5.8 shows the calculation results for the different closed orbit bumps. The conclusions about the Cooler injection calculation are:

- In order to reduce the strength of the kicker $\delta_k$, $x_s$ needs to be as small as possible.

- Horizontally, the beam from CIS has an emittance of 10 $\pi$mm-mrad and momentum spread of 0.1%, thus, a radius of $r_0 = 5.6$ mm at the septum(L00). If injection is parallel, the minimum $x_s$ should be larger than $2r_0$ plus the septum width. The minimum $x_s$ is 12 mm, in which case the Cooler closed orbit is bumped closer to the septum by 5 mm.
Table 5.8: Betatron parameters at the septum and the kicker

<table>
<thead>
<tr>
<th>Orbit Bump (mm)</th>
<th>$x_s$ (mm)</th>
<th>$x_s'$ (mrad)</th>
<th>$\delta_k$ (mrad)</th>
<th>$B_k$ (G)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>16.3</td>
<td>0.17</td>
<td>3.63</td>
<td>195.2</td>
</tr>
<tr>
<td>1</td>
<td>15.3</td>
<td>0.16</td>
<td>3.41</td>
<td>183.2</td>
</tr>
<tr>
<td>2</td>
<td>14.3</td>
<td>0.15</td>
<td>3.19</td>
<td>171.2</td>
</tr>
<tr>
<td>3</td>
<td>13.3</td>
<td>0.14</td>
<td>2.97</td>
<td>159.2</td>
</tr>
<tr>
<td>4</td>
<td>12.3</td>
<td>0.13</td>
<td>2.74</td>
<td>147.3</td>
</tr>
<tr>
<td>5</td>
<td>11.3</td>
<td>0.12</td>
<td>2.52</td>
<td>135.3</td>
</tr>
<tr>
<td>6</td>
<td>10.3</td>
<td>0.11</td>
<td>2.30</td>
<td>123.3</td>
</tr>
<tr>
<td>7</td>
<td>9.3</td>
<td>0.09</td>
<td>2.07</td>
<td>111.3</td>
</tr>
<tr>
<td>8</td>
<td>8.3</td>
<td>0.08</td>
<td>1.85</td>
<td>99.3</td>
</tr>
</tbody>
</table>

- Vertically, the beam from CIS has an emittance of $11 \pi \text{mm-mrad}$ as a result of the strip foil scattering. The vertical beam size therefore is 4.5 mm, as $\beta_y = 1.815$ m.

- The maximum kick angle of the fast kicker is 2.8 mrad. As long as $x_s$ is less than 12.3 mm, when closed orbit is bumped by 4 mm, the kick angle is within its ability.

5.6 Spin Manipulation of Extracted Beam

A critical requirement of the CIS ring is its ability to provide polarized proton beams for the Cooler ring. The particle spin after extraction from the CIS is in the $z$-direction (vertical). Shown in Figure 5.18 on page 117, the lattice solenoid 1, dipole
D1, solenoid 2 and dipole D2 will provide the proper spin orientation desired by Cooler experiments.

### 5.6.1 Spin Parameters

The particle spin is described by the spin 4-vector $S^i = (S_0, \vec{S})$. It has the following relation with the 2-component spinor $\Psi$:

$$S_i \equiv \langle \Psi | \sigma_i | \Psi \rangle$$  \hspace{1cm} (5.61)

where $\sigma_i$ are Pauli matrices given by

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$  \hspace{1cm} (5.62)

If $\Psi = \begin{pmatrix} u \\ d \end{pmatrix}$, the three spin vector components are

$$\begin{align*}
S_1 &= u^*d + ud^* = 2\text{Re}(u^*d) \\
S_2 &= -i(u^*d - ud^*) = 2\text{Im}(u^*d) \\
S_3 &= |u|^2 - |d|^2
\end{align*}$$

(5.63)

The spinor wave function $\Psi$ is propagated from $i$ to $f$ by a spin transfer matrix as:

$$\Psi_f(\theta) = T_s(\theta, \theta_i)\Psi_i(\theta_i)$$  \hspace{1cm} (5.64)

The spin transfer matrices for common beam line elements are given below:

**Dipole:**

$$T_d = e^{-i\frac{\gamma_2}{2} \theta \sigma_3} = \cos\left(\frac{G\gamma}{2} \theta\right) - i \cdot \sin\left(\frac{G\gamma}{2} \theta\right) \cdot \sigma_3$$

$$= \begin{pmatrix} e^{-i\frac{G\gamma}{2} \theta} & 0 \\ 0 & e^{i\frac{G\gamma}{2} \theta} \end{pmatrix}$$  \hspace{1cm} (5.65)
Solenoid:

\[ T_s = e^{-i \frac{\phi}{2} \alpha_2} = \cos \frac{\chi}{2} - i \cdot \sin \frac{\chi}{2} \cdot \sigma_2 \]

\[ = \begin{pmatrix} \cos \frac{\chi}{2} & -\sin \frac{\chi}{2} \\ \sin \frac{\chi}{2} & \cos \frac{\chi}{2} \end{pmatrix} \] (5.66)

with \( \chi = (1 + G) \theta, \quad \theta = \frac{B_1 L}{B \rho} \)

Quadrupole:

\[ T_q = e^{-i \frac{\phi}{2} \alpha_1} = \cos \frac{\chi}{2} - i \cdot \sin \frac{\chi}{2} \cdot \sigma_1 \]

\[ = \begin{pmatrix} \cos \frac{\chi}{2} & -i \sin \frac{\chi}{2} \\ -i \sin \frac{\chi}{2} & \cos \frac{\chi}{2} \end{pmatrix} \] (5.67)

with \( \chi = (1 + G \gamma) \theta, \quad \theta = \frac{\Delta B_x}{B \rho} \cdot \Delta L, \quad \Delta B_x = \left( \frac{\partial B_x}{\partial x} \right) z \)

In the above equations \( G \) is a constant factor for hadrons and \( \gamma \) is the proton energy. For protons, \( G = 1.7928474, \ \gamma = 1.21316 \) for 200 MeV protons. Also, \( \nu_s = G \gamma \) is the spin tune.

5.6.2 Spin Closed Orbit Change in the Extraction Beam Line

A partial snake is often used to overcome the effect of an imperfection resonance on the spin closed orbit. If the partial snake has the spin rotation angle of \( \phi \), the spin closed orbit change will be [5]:

\[ \begin{align*}
\cos \alpha_1 &= \frac{-1}{\sin \pi \nu_s} \sin G \gamma (\pi - \theta) \sin \frac{\phi}{2}, \\
\cos \alpha_2 &= \frac{1}{\sin \pi \nu_s} \cos G \gamma (\pi - \theta) \sin \frac{\phi}{2}, \\
\cos \alpha_3 &= \frac{1}{\sin \pi \nu_s} \sin \pi G \gamma \cos \frac{\phi}{2}, \\
\cos \pi \nu_s &= \cos \pi G \gamma \cos \frac{\phi}{2}
\end{align*} \] (5.68)
Figure 5.24: Change of spin closed orbit angles due to 5% partial snake in CIS extraction beam line

Figure 5.25: Change of spin closed orbit components due to 5% partial snake in CIS extraction beam line
where \((\cos \alpha_1, \cos \alpha_2, \cos \alpha_3)\) are direction cosines of spin closed orbit \(\hat{n}_{co}\) along \((\hat{e}_1, \hat{e}_2, \hat{e}_3)\) axes.

For a 5% partial snake in the CIS extraction beam line, the changes of the angles and components of \(\hat{n}_{co}\) are shown in Figures 5.24 and 5.25.

### 5.6.3 Spin State of Extracted Beam

The extracted beam from the CIS ring has the spin state:

\[
\Psi_i = \begin{pmatrix} 1 \\ 0 \end{pmatrix}
\]  

(5.69)

which corresponds to the three spin vector \(S = \{0, 0, 1\}\).

The solenoid 1, dipole D1, solenoid 2 and dipole D2 are used to provide the proper spin state of the polarized proton beams for the Cooler.

The total transfer matrix is therefore,

\[
T_{d2s2d1s1} = T_d(\theta_2) \cdot T_s(\chi_2) \cdot T_d(\theta_1) \cdot T_s(\chi_1)
\]

\[
= \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix}
\]

(5.70)

The expression for the final matrix is in the Appendix E on page 159.

The final spinor is

\[
\Psi_f = T_{d2s2d1s1} \cdot \Psi_i = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} T_{11} \\ T_{21} \end{pmatrix}
\]

(5.71)

The spin vector is

\[
\vec{S} = \langle \Psi_f | \vec{\sigma} | \Psi_f \rangle
\]

(5.72)
Therefore the three components are:

\[ S_1 = 2 \cdot \text{Re}(T_{11}^*T_{21}), \quad S_2 = 2 \cdot \text{Im}(T_{11}^*T_{21}), \quad S_3 = |T_{11}|^2 - |T_{21}|^2 \quad (5.73) \]

Based on the matrices obtained above, numerical calculations are done to get the change of three spin vector components with the solenoid magnetic field.

The length of the solenoids is 0.5842 m. When \( \chi = 2.0 \), for 200 MeV protons, \( B \rho = 2.14 \text{ Tm} \), \( B_\parallel = 2.6239 \text{ T} \).
Chapter 6

Conclusions

The design of and initial experiments on the Cooler Injector Synchrotron (CIS) have been described in this dissertation. CIS is the most compact synchrotron in the world to provide 200 MeV (107 MeV) high intensity polarized proton (deuteron) beams. The experimental results show that with the successful design and construction of the machine, the CIS can efficiently deliver high intensity beams to the Cooler ring at IUCF.

An extensive study has been done in order to find an optimized CIS lattice. With a total length of 17.364 m in circumference, CIS lattice consists of four superperiods, each of which is composed of a drift space and a dipole magnet with 90° bending angle and 12° edge angle at both ends. The absence of a regular FODO lattice using separate quadrupoles makes CIS a compact machine to meet the proposed requirements for the beam performance. By adding four short trim quadrupoles symmetrically, the machine possesses flexibility for betatron tune adjustments and transition energy ($\gamma_T$) variation. Thus, the CIS will become an interesting machine for accelerator physics studies such as transition energy crossing and imaginary $\gamma_T$ operation in longitudinal beam dynamics studies. Because of its lower construction and operation cost and
operational simplicity (few adjustable elements), it is also an ideal model for the accelerators applied in the areas of proton therapy and material science research.

Four normal conducting main dipole magnets were designed and built for the CIS which allow proton acceleration up to 220 MeV with a magnetic field of $B_{max} \approx 1.78$ T. Dipole field calculations in two and three dimensions were used to optimize magnet endpacks and determine the effective field length and sextupole components. Field mapping was done for each dipole to determine the field strength and sextupole components experimentally. The effects of the eddy current in the vacuum chamber wall on the beam is not negligible but can be overcome by the proposed self-correction coil method. The study of the CIS dynamic aperture shows that CIS has an aperture of 100 mm $\times$ 200 mm, which is big enough to accommodate the vacuum pipe. The good quality dipole field with its small sextupole components contributes to the large aperture.

The CIS operating energy range requires that the rf cavity has a wide tuning range from 1.3 MHz to 10 MHz. Based on an rf cavity in use at IUCF, a wide tuning range rf cavity with external quadrupole ferrite biasing was proposed for the CIS ring. By using the finite element code MagNet [14], two and three dimensional calculations of the rf cavity external biasing field were performed to optimize its structure. The external quadrupole magnet provides dc magnetic field strengths inside the ferrite from 0 to 0.27 T. The effective relative permeability can change from maximum (around 800) to nearly zero, which insures the wide frequency range. The symmetry of the quadrupole field provides cancellation of the biasing field at the cavity axis, minimizing the magnetic disturbance to the beam. Because of the ferrite materials in the cavity, the fringe field is very small and has a negligible effect on the beam performance.

Strip injection is used to inject the $H^{-}$ beam from an RFQ/DTL pre-injector into CIS. The $H^{-}$ beam is converted to protons by a 4.5 $\mu$g/cm$^2$ thick, 6 mm $\times$ 22
Conclusions

mm carbon foil fabricated at the IUCF target laboratory. Beam position monitors, beam profile monitors, wall gap monitors and a ping tune system are used in the CIS ring for beam diagnostics. After the successful injection and accumulation of the incoming beams, an rf adiabatic capture method was proposed to enhance the beam transmission efficiency. The simulation of the adiabatic capture and synchrotron motion in the CIS ring suggested a proper set of ramping curves, rf voltage, dipole power supply, etc. for the CIS ring beam acceleration. At the injection energy of 7 MeV, a maximum of $7 \times 10^{10}$ particles per bunch has been stored and captured successfully. In later beam ramping experiments, $1.2 \times 10^{10}$ particles per bunch, over 70% of the accumulated protons, have been captured and accelerated to 100 MeV, 200 MeV, and 225 MeV. Low capture efficiency due to lower than ideal rf cavity voltage, gas molecule scattering and other perturbations may contribute to the beam loss during the ramping. On the other hand, closed orbit drift and tune shift are very small during the ramping process. So far, experimental results show that CIS has the ability to provide high intensity proton beams for the Cooler ring to improve its stored beam intensity and luminosity. The good performance was ensured by quality of design and manufacture of the ion source, RFQ/DTL, dipoles, rf cavity, control system, etc.. With the higher intensity ion source, CIS is expected to have even better performance.

At high energy, particle beam bunches are extracted from the CIS ring by a fast traveling kicker and vertical septum, and are bucket-by-bucket transferred to the Cooler by the new transport beam line BL9A. Study of the kicker-septum extraction method and beam trajectory provided the kicker and bumper strength for CIS extraction. A fast traveling wave kicker with a rise time less than 50 ns is used to do the single-turn extraction. Four bumpers are used to displace the closed orbit near the extraction septum magnet to reduce the strength of the fast kicker. The extraction beam line BL9A from the CIS to the Cooler can provide a nearly perfect beam
matching at the Cooler injection point if the quadrupoles along the beam line are properly adjusted. The solenoids and dipoles in the extraction beam line can provide any desired spin state for the Cooler ring in the case of polarized beam acceleration. Beam bunches from the CIS are injected into the Cooler by a fast kicker located at the center of the first dipole of the Cooler ring. The Cooler closed orbit should be bumped close to the septum in order to inject the parallel incoming beam properly.

With the successful construction of the CIS ring and the transport beam line from CIS to Cooler, many Cooler experiments can be done more easily and efficiently. In addition, more research projects have been proposed on the CIS ring itself. One of them is the study of the space charge effect on the beam emittance at the injection energy, 7 MeV. The emittance can be measured by both the beam profile monitors at the exit of the CIS ring and a combination of a quadrupole before the three monitors with one of them. As mentioned above, accelerator physics studies such as transition energy crossing and imaginary $\gamma_T$ lattice in longitudinal beam dynamics studies can be performed on CIS ring. To find more applications of compact synchrotrons like CIS, slow extraction methods will be studied to explore the suitability of the machine for medical use.
Appendix A

Gyromagnetic Resonance in Ferrite Materials

The characteristic property of a magnetized ferrite is that while it has a scalar
dielectric constant, it has a tensor permeability at microwave frequencies. The macro-
scopic theory of microwave ferrite material is based on the equation of motion of the magnetization vector [28]. In the presence of a constant magnetic field $\mathbf{H}_0$, which is assumed to be in the $z$-direction, the equation of motion of the magnetization vector can be derived by considering the motion of the total number of magnetic dipoles $\boldsymbol{\mu}$ per unit volume which has a small angle $\theta$ with $\mathbf{H}_0$, as shown in Figure A.1.

Since the only field acting on $\boldsymbol{\mu}$ is $\mathbf{H}_0$, the torque exert on $\boldsymbol{\mu}$ is

$$
\mathbf{T} = \boldsymbol{\mu} \times \mathbf{H}_0
$$

(A.1)

Associated with the magnetic dipole $\boldsymbol{\mu}$ there is an angular momentum $\mathbf{J}$ given by

$$
\boldsymbol{\mu} = \frac{g \mu_0 e}{2m} \mathbf{J} = \gamma \mathbf{J},
$$

(A.2)

where $\mu_0$ is the magnetic permeability in vacuum, $g = 2$ is the $g$-factor for electrons, and $\gamma$ is called the gyromagnetic ratio given by $-2.21 \times 10^5 \text{[rad/sec/(A/m)]}$.

From Eq. (A.2), the torque can also be written as

$$
\mathbf{T} = \frac{1}{\gamma} \frac{d\boldsymbol{\mu}}{dt}
$$

(A.3)

Combining Eqs. (A.1) and (A.3) gives the equation of motion for a single dipole moment

$$
\frac{d\boldsymbol{\mu}}{dt} = \gamma (\boldsymbol{\mu} \times \mathbf{H}_0)
$$

(A.4)

Thus, the equation of motion for the total magnetization $\mathbf{M}_0$, the total number of dipole moments per unit volume, is

$$
\frac{d\mathbf{M}_0}{dt} = \gamma (\mathbf{M}_0 \times \mathbf{H}_0)
$$

(A.5)

In the microwave case, the total effective magnetic field consists of dc magnetic field $\mathbf{H}_0 = (0, 0, H_0)$ and the rf magnetic field $\mathbf{h} = (h_x, h_y, h_z)$

$$
\mathbf{H} = \mathbf{H}_0 + \mathbf{h}
$$

(A.6)
The total magnetization consists of the dc magnetization $M_0 = (0, 0, M_0)$ and the rf magnetization $m = (m_x, m_y, m_z)$

$$M = M_0 + m$$  \hspace{1cm} (A.7)

Eq. (A.5) can now be expanded to give

$$\begin{align*}
\frac{dm_x}{dt} &= m_y \gamma (H_0 + h_z) - h_y \gamma (M_0 + m_z) \\
\frac{dm_y}{dt} &= -m_x \gamma (H_0 + h_z) + h_x \gamma (M_0 + m_z) \\
\frac{dm_z}{dt} &= m_x \gamma h_y - m_y \gamma h_x
\end{align*}$$  \hspace{1cm} (A.8)

If the rf field is small compared to the dc field, the higher order terms of $m$ and $h$ are set to zero. The small signal approximation is therefore:

$$\begin{align*}
\frac{dm_x}{dt} &= m_y \gamma H_0 - h_y \gamma M_0 \\
\frac{dm_y}{dt} &= -m_x \gamma H_0 + h_x \gamma M_0 \\
\frac{dm_z}{dt} &\approx 0
\end{align*}$$  \hspace{1cm} (A.9)

Rewriting the last equations we have

$$\begin{align*}
\ddot{m}_x + \omega_0^2 m_x &= \mu_0 \omega_m \omega_0 h_x + \mu_0 \omega_m \dot{h}_y \\
\ddot{m}_y + \omega_0^2 m_y &= -\mu_0 \omega_m \dot{h}_x + \mu_0 \omega_m \omega_0 h_y \\
\dot{m}_z &\approx 0
\end{align*}$$  \hspace{1cm} (A.10)

where

$$\omega_m = -\frac{\gamma M_0}{\mu_0}, \hspace{1cm} \omega_0 = -\gamma H_0.$$  \hspace{1cm} (A.11)

If the time dependence of the rf quantities is of the form $e^{j\omega t}$ a susceptibility tensor $[\chi]$ can be defined which relates the rf magnetization to the rf magnetic field

$$m = \mu_0 [\chi] h,$$  \hspace{1cm} (A.12)
where

\[
[\chi] = \begin{bmatrix}
\chi_{xx} & \chi_{xy} & 0 \\
\chi_{yx} & \chi_{yy} & 0 \\
0 & 0 & 0
\end{bmatrix},
\]

(A.13)

and

\[
\begin{aligned}
\chi_{xx} &= \chi_{yy} = \frac{\omega_m \omega_0}{-\omega^2 + \omega_0^2}, \\
\chi_{yx} &= -\chi_{xy} = \frac{j \omega_m \omega_0}{-\omega^2 + \omega_0^2}.
\end{aligned}
\]

(A.14)

The components of the susceptibility tensor have a singularity at \( \omega = \omega_0 = -\gamma H_0 \).

This is defined as the gyromagnetic resonance condition. At the resonance,

\[
H_0[A/m] = 28.47 f[\text{MHz}], \quad \text{or} \quad H_0[\text{Oe}] = \frac{f[\text{MHz}]}{2.80},
\]

(A.15)

which is the same as in [29].
Appendix B

Transverse Phase Space Matching

When the beam is injected from lattice 1 to lattice 2, the transverse lattice functions must be matched at the injection point to avoid beam dilution.

The ellipse defined by the lattice 2 is:

\[ \gamma y^2 y + 2\alpha y y' + \beta y'^2 = \epsilon \]  \hspace{1cm} (B.1)

If the injected beam phase ellipse has a mismatched optics

\[ \gamma_1 y^2 y + 2\alpha_1 y y' + \beta_1 y'^2 = \epsilon \]  \hspace{1cm} (B.2)

there will be an emittance growth or dilution of the injected beam.

B.1 Emittance Growth Due to Injection Mismatch

To calculate the emittance growth more directly, we use the normalized coordinates \((Y, P)\) of lattice 2. \((Y, P)\) are defined by the transformation \(B\)

\[ \begin{pmatrix} Y \\ P \end{pmatrix} = B \begin{pmatrix} y \\ y' \end{pmatrix} \text{, with } B = \begin{pmatrix} 1 \frac{\beta}{\alpha} & 0 \\ \frac{\alpha}{\beta} & \sqrt{\beta} \end{pmatrix} \]  \hspace{1cm} (B.3)
The ellipse of lattice 2 of Eq. (B.1) will become a circle

\[ Y^2 + P^2 = \epsilon \]  \hspace{1cm} (B.4)

By using \((y \ y')^T = B(Y \ P)^T\), the ellipse of the injected beam in Eq. (B.2) will have the form

\[
\left[ \gamma_1 \beta - 2\alpha \alpha_1 + \frac{\beta_1}{\beta} \alpha^2 \right] Y^2 + \frac{\beta_1}{\beta} Y P + \left[ 2\alpha_1 - 2\alpha \frac{\beta_1}{\beta} \right] Y P = \epsilon
\]  \hspace{1cm} (B.5)

Considering the Courant-Snyder parameter relation \(\beta \gamma = 1 + \alpha^2\), the injected ellipse is simplified as

\[ aY^2 + 2bYP + cP^2 = \epsilon \]  \hspace{1cm} (B.6)

where

\[
\begin{align*}
a &= \frac{\beta}{\beta_1} + \frac{(\alpha_1 \beta - \alpha \beta_1)^2}{\beta \beta_1} = \frac{1}{c} + \frac{\beta}{\beta_1} b^2 \\
b &= \frac{\alpha_1 \beta - \alpha \beta_1}{\beta} \\
c &= \frac{\beta_1}{\beta}
\end{align*}
\]  \hspace{1cm} (B.7)

with the relation \(ac = 1 + b^2\).

In the normalized coordinates of lattice 2, the injected beam is still an ellipse rotating around the center of the lattice 2 beam circle as shown in Fig. B.1. The result is that at the injection point the beam envelope will oscillate at the frequency of twice the betatron tune. Because of the non-linearity effect, the final phase space area will increase to the size of dashed circle in Fig. B.1. The growth of the emittance is simply the ratio of the areas of dashed circle and the lattice 2 beam circle.

The radius of the dashed circle is the major axis of the injected beam ellipse. To find the major and minor axes of the injected beam ellipse, apply the rotation transformation [Appendix C] the injected beam ellipse in Eq. (B.6) now is
Figure B.1: Phase space mismatch at the injection point in normalized coordinates of lattice 2

\[ AY^2 + CP^2 = \epsilon \]  \hspace{1cm} (B.8)

where \( A, C \) are the roots of the equation

\[ u^2 - (a + c)u + 1 = 0 \]  \hspace{1cm} (B.9)

or:

\[ A, C = \frac{(a + c)}{2} \pm \sqrt{\left(\frac{a + c}{2}\right)^2 - 1} \]  \hspace{1cm} (B.10)

Define the parameter \( X_d \)

\[ X_d = \frac{a + c}{2} = \frac{1}{2} \left[ \frac{\beta^2 + \beta_1^2}{\beta \beta_1} + \frac{(\alpha_1 \beta - \beta_1 \alpha)^2}{\beta \beta_1} \right] \]

\[ = \frac{1}{2}(\gamma_1 \beta + \beta_1 \gamma - 2\alpha_1 \alpha) \]  \hspace{1cm} (B.11)
The major and minor axes of the ellipse $F_+, F_-$ are

$$F_\pm = \frac{1}{\sqrt{C_A}} = \frac{1}{\left[X_d \mp \sqrt{X_d^2 - 1}\right]^{\frac{1}{2}}}$$

$$= \left[ X_d \pm \sqrt{X_d^2 - 1} \right]^{\frac{1}{2}} \quad \text{(B.12)}$$

The injected ellipse is the same as the lattice 2 ellipse only when $F_+ = F_-$ or $X_d = 1$, i.e., beam injection is perfectly matched. $X_d$ is called mismatch factor. Otherwise, the emittance will grow by a factor of

$$f = F_+^2 = X_d + \sqrt{X_d^2 - 1} \quad \text{(B.13)}$$

### B.2 Emittance Growth Including Dispersion Mis-match

The mismatch of dispersion function produces additional emittance growth. Closed orbit for the particles with momentum deviation $\delta$ is

$$y = y_\beta + D\delta, \quad y' = y'_\beta + D'\delta \quad \text{(B.14)}$$

Dispersion function $D, D'$ causes a displacement of $(\delta y, \delta y') = (D\delta, D'\delta)$ in phase space. In the normalized phase space, the center of the lattice 2 beam circle will have a shift from origin $O$ to point $O'$ and the injected beam ellipse from $O'$ to $O''$, as shown in Fig. B.2. The coordinates of the positions $O'$ and deviations of $O''$ from $O'$ are:

$$O' : \quad Y = \frac{D\delta}{\sqrt{\beta}}, \quad P = \sqrt{\beta}D'\delta + \frac{\alpha}{\sqrt{\beta}}D\delta \quad \text{(B.15)}$$

$$O'' : \quad \Delta Y = \frac{\Delta D\delta}{\sqrt{\beta}}, \quad \Delta P = \sqrt{\beta}\Delta D'\delta + \frac{\alpha}{\sqrt{\beta}}\Delta D\delta \quad \text{(B.16)}$$
Figure B.2: Phase space mismatch including dispersion mismatch at
the injection point in normalized coordinates of lattice 2

As a result, the space phase will increase from the original ellipse to the dashed circle shown in Fig. B.2. The radius of the circle is the largest distance from $O'$ to the ellipse boundary. Appendix D shows the procedure to derive the radius of the dashed circle. However, it is always simple and safe to estimate that the radius of the new phase space is less than $(F_+ + O'O')$. Finally the emittance growth factor will be

$$ f \leq [F_+ + O'O']^2 = \left[ F_+ + \sqrt{(\Delta Y)^2 + (\Delta P)^2} \right]^2 $$

$$ = \left[ \sqrt{X_d + \sqrt{X_d^2}} - 1 + \delta \sqrt{\frac{(\Delta D)^2}{\beta}} + \left( \sqrt{\beta \Delta D' + \frac{\alpha}{\sqrt{\beta}} \Delta D} \right) \right]^2 \quad \text{(B.17)} $$
B. Transverse Phase Space Matching
Appendix C

Rotation Transformation of An Ellipse

The general equation of an ellipse is in the form
\[ ax^2 + 2bxy + cy^2 = 1 \]  \hspace{1cm} (C.1)

Define
\[ X = \begin{pmatrix} x \\ y \end{pmatrix}, \quad \sigma = \begin{pmatrix} a & b \\ b & c \end{pmatrix} \]  \hspace{1cm} (C.2)

The ellipse then is
\[ X^T \sigma X = 1 \]  \hspace{1cm} (C.3)

The ellipse in the coordinates \( X = (x \ y) \) can be transformed to the upright orientation in the coordinates \( X_1 = (x_1 \ y_1) \) by the rotation transformation \( \Theta(\theta) \)
\[ X = \Theta(\theta)X_1, \quad \text{with} \quad \Theta(\theta) = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \]  \hspace{1cm} (C.4)
The ellipse in Eq. (C.1) becomes

\[ Ax_1^2 + Cy_1^2 = 1 \]  \hspace{1cm} (C.5)

To derive the relations between \( A, C \) and the original coefficients \( a, b, c, d \), use the transformation \( X = \Theta(\theta)X_1 \) in Eq. (C.3)

\[ X_1^T \cdot \Theta^T(\theta)\sigma\Theta(\theta) \cdot X_1 = X_1^T \sigma_1 X_1 = 1 \]  \hspace{1cm} (C.6)

The coefficient matrix of the new ellipse is

\[ \sigma_1 = \Theta^T(\theta)\sigma\Theta(\theta) \]

\[ = \begin{pmatrix} a \cos^2 \theta + c \sin^2 \theta - b \sin 2\theta & \frac{1}{2}(a - c) \sin 2\theta + b \cos 2\theta \\ \frac{1}{2}(a - c) \sin 2\theta + b \cos 2\theta & a \sin^2 \theta + c \cos^2 \theta + b \sin 2\theta \end{pmatrix} \]  \hspace{1cm} (C.7)

The condition for the upright orientation is \( (\sigma_1)_{12} = 0 \). So

\[ \frac{1}{2}(a - c) \sin 2\theta + b \cos 2\theta = 0 \]  \hspace{1cm} (C.8)

The rotation angle \( \theta \) is decided by

\[ \cot 2\theta = \frac{c - a}{2b} \]  \hspace{1cm} (C.9)

The new coefficients

\[ A = (\sigma_1)_{11} = a \cos^2 \theta + c \sin^2 \theta - b \sin 2\theta \]

\[ = \frac{1}{2}[(a + c) - (c - a) \cos 2\theta] - b \sin 2\theta \]

\[ = \frac{1}{2}(a + c) - \frac{1}{2} \frac{(c - a)^2 + 4b^2}{2b} \sin 2\theta \]  \hspace{1cm} (C.10)

\[ C = \frac{1}{2}(a + c) + \frac{1}{2} \frac{(c - a)^2 + 4b^2}{2b} \sin 2\theta \]  \hspace{1cm} (C.11)

From the rotation angle equation (C.9),

\[ \sin 2\theta = \pm \frac{1}{\sqrt{1 + \cot^2 \theta}} = \pm \frac{2b}{\sqrt{(c - a)^2 + 4b^2}} \]  \hspace{1cm} (C.12)
So

\[
A, C = \frac{1}{2} \left[ (a + c) \pm \sqrt{(c - a)^2 + 4b^2} \right]
\]

\[
= \frac{1}{2} \left[ (a + c) \pm \sqrt{(a + c)^2 - 4} \right]
\]  \quad \text{(C.13)}

This means \( A, C \) are the roots of the equation

\[
u^2 - (a + c)u + 1 = 0 \]  \quad \text{(C.14)}
Appendix D

Maximum Distance From the Origin to An Off-center Ellipse

As shown in Fig. D.1, the general equation for an ellipse located at $O''(\Delta x, \Delta y)$. 

**Figure D.1:** Mismatch in normalized phase space
is

\[ a(x - \Delta x)^2 + 2b(x - \Delta x)(y - \Delta y) + c(y - \Delta y)^2 = 1 \]  

(D.1)

Use the parameters \( \rho, \theta \),

\[ x - \Delta x = \rho \cos \theta, \quad y - \Delta y = \rho \sin \theta \]  

(D.2)

The ellipse becomes

\[ \rho^2 \left[ a \cos^2 \theta + 2b \sin \theta \cos \theta + c \sin^2 \theta \right] = 1 \]  

(D.3)

or

\[ \rho^2 \left[ \frac{1}{2}(a + c) + \frac{1}{2}(a - c) \cos 2\theta + b \sin 2\theta \right] = 1 \]  

(D.4)

The distance \( D \) from a point on the ellipse to the origin is

\[ D^2 = L^2 + \rho^2 + 2L\rho \cos(\theta - \theta_0) \]  

(D.5)

where \( L \) and \( \theta_0 \) are defined by

\[ L = \overline{OO'} = \sqrt{(\Delta x)^2 + (\Delta y)^2}, \quad \tan \theta_0 = \frac{\Delta y}{\Delta x} \]  

(D.6)

The condition for maximum \( D \) is

\[ \frac{d(D^2)}{d\theta} = 2\rho \rho' + 2L \rho' \cos(\theta - \theta_0) - 2\rho L \sin(\theta - \theta_0) = 0 \]  

(D.7)

From Eq. (D.4)

\[ \rho' = \rho^3 \left[ (a - c) \sin 2\theta - 2b \cos 2\theta \right] \]  

(D.8)

Then the optimum angle \( \theta_m \) is decided by

\[ \rho_m^2 \left[ \rho_m + L \cos(\theta_m - \theta_0) \right] \cdot \left[ (a - c) \sin 2\theta_m - 2b \cos 2\theta_m \right] - L \sin(\theta_m - \theta_0) = 0 \]  

(D.9)
where
\[
\rho_m^2 \left[ \frac{1}{2}(a + c) + \frac{1}{2}(a - c) \cos 2\theta_m + b \sin 2\theta_m \right] = 1 \quad \text{(D.10)}
\]

Solving the above two equations for \( \rho_m \) and \( \theta_m \) and inserting them to Eq. D.5, one can find the maximum distance.

For a special case, when the ellipse center is at the origin, \( L = 0 \), the optimum angle is
\[
\rho_m^3 [(a - c) \sin 2\theta_m - 2b \cos 2\theta_m] = 0 \quad \text{(D.11)}
\]
or
\[
\cot 2\theta_m = \frac{a - c}{2b} \quad \text{(D.12)}
\]
The maximum distance \( D_m \) is
\[
D_m = \rho_m = \left[ a \cos^2 \theta_m + 2b \sin \theta_m \cos \theta_m + c \sin^2 \theta_m \right]^{-\frac{1}{2}}
\]
\[
= \left[ \frac{1}{2}(a + c) + \frac{1}{2} \frac{(c - a)^2 + 4b^2}{2b} \sin 2\theta_m \right]^{-\frac{1}{2}}
\]
\[
= \frac{1}{2} \left[ (a + c) - \sqrt{(a + c)^2 - 4} \right]^{-\frac{1}{2}} \quad \text{(D.13)}
\]
which is the same result as that of the rotation transformation.
D. Maximum Distance From the Origin to An Off-center Ellipse
Appendix E

Spin Transfer Matrices of CIS Extraction Line

E.1 Matrix for Each Element

Dipole 1 and 2:

\[
T_d(\theta_2) = \begin{pmatrix}
\cos \frac{G \gamma \theta_2}{2} - i \sin \frac{G \gamma \theta_2}{2} & 0 \\
0 & \cos \frac{G \gamma \theta_2}{2} + i \sin \frac{G \gamma \theta_2}{2}
\end{pmatrix}
\]

(E.1)

\[
T_d(\theta_1) = \begin{pmatrix}
\cos \frac{G \gamma \theta_1}{2} - i \sin \frac{G \gamma \theta_1}{2} & 0 \\
0 & \cos \frac{G \gamma \theta_1}{2} + i \sin \frac{G \gamma \theta_1}{2}
\end{pmatrix}
\]

(E.2)

Solenoid 1 and 2:

\[
T_s(\chi_2) = \begin{pmatrix}
\cos \frac{\chi_2}{2} & - \sin \frac{\chi_2}{2} \\
\sin \frac{\chi_2}{2} & \cos \frac{\chi_2}{2}
\end{pmatrix}
\]

(E.3)
\[ T_s(\chi_1) = \begin{pmatrix} \cos \frac{\chi_1}{2} & -\sin \frac{\chi_1}{2} \\ \sin \frac{\chi_1}{2} & \cos \frac{\chi_1}{2} \end{pmatrix} \] (E.4)

### E.2 Total Matrix

Matrix of D1 ans S1:

\[ T_{d1s1}(\theta_1, \chi_1) = T_d(\theta_1) \cdot T_s(\chi_1) \]

\[ = \begin{pmatrix} \cos \frac{\chi_1}{2} \left( \cos \frac{G \gamma \theta_1}{2} - i \sin \frac{G \gamma \theta_1}{2} \right) & -\cos \frac{\chi_1}{2} \left( \cos \frac{G \gamma \theta_1}{2} - i \sin \frac{G \gamma \theta_1}{2} \right) \\ \left( \cos \frac{G \gamma \theta_1}{2} + i \sin \frac{G \gamma \theta_1}{2} \right) & \cos \frac{\chi_1}{2} \left( \cos \frac{G \gamma \theta_1}{2} + i \sin \frac{G \gamma \theta_1}{2} \right) \end{pmatrix} \] (E.5)

Matrix of D1 ans S1:

\[ T_{d2s2}(\theta_2, \chi_2) = T_d(\theta_2) \cdot T_s(\chi_2) \]

\[ = \begin{pmatrix} \cos \frac{\chi_2}{2} \left( \cos \frac{G \gamma \theta_2}{2} - i \sin \frac{G \gamma \theta_2}{2} \right) & -\cos \frac{\chi_2}{2} \left( \cos \frac{G \gamma \theta_2}{2} - i \sin \frac{G \gamma \theta_2}{2} \right) \\ \left( \cos \frac{G \gamma \theta_2}{2} + i \sin \frac{G \gamma \theta_2}{2} \right) & \cos \frac{\chi_2}{2} \left( \cos \frac{G \gamma \theta_2}{2} + i \sin \frac{G \gamma \theta_2}{2} \right) \end{pmatrix} \] (E.6)

Total Matrix:

\[ T_{d2s2d1s1} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} = T_{d2s2}(\theta_2, \chi_2) \cdot T_{d1s1}(\theta_1, \chi_1) \]

\[ = T_d(\theta_2) \cdot T_s(\chi_2) \cdot T_d(\theta_1) \cdot T_s(\chi_1) \] (E.7)

With each element of \( T_{d2s2d1s1} \):
\[
T_{11} = \frac{\cos \chi_1}{2} \cos \frac{\chi_2}{2} \left( \cos \frac{G \gamma \theta_1}{2} - i \sin \frac{G \gamma \theta_1}{2} \right) \left( \cos \frac{G \gamma \theta_2}{2} - i \sin \frac{G \gamma \theta_2}{2} \right) \\
- \left( \cos \frac{G \gamma \theta_1}{2} + i \sin \frac{G \gamma \theta_1}{2} \right) \left( \cos \frac{G \gamma \theta_2}{2} - i \sin \frac{G \gamma \theta_2}{2} \right) \sin \frac{\chi_1}{2} \sin \frac{\chi_2}{2}
\]

(E.8)

\[
T_{12} = -\cos \frac{\chi_2}{2} \left( \cos \frac{G \gamma \theta_1}{2} - i \sin \frac{G \gamma \theta_1}{2} \right) \left( \cos \frac{G \gamma \theta_2}{2} - i \sin \frac{G \gamma \theta_2}{2} \right) \sin \frac{\chi_1}{2} \\
- \cos \frac{\chi_1}{2} \left( \cos \frac{G \gamma \theta_1}{2} + i \sin \frac{G \gamma \theta_1}{2} \right) \left( \cos \frac{G \gamma \theta_2}{2} - i \sin \frac{G \gamma \theta_2}{2} \right) \sin \frac{\chi_2}{2}
\]

(E.9)

\[
T_{21} = \cos \frac{\chi_2}{2} \left( \cos \frac{G \gamma \theta_1}{2} + i \sin \frac{G \gamma \theta_1}{2} \right) \left( \cos \frac{G \gamma \theta_2}{2} + i \sin \frac{G \gamma \theta_2}{2} \right) \sin \frac{\chi_1}{2} \\
+ \cos \frac{\chi_1}{2} \left( \cos \frac{G \gamma \theta_1}{2} - i \sin \frac{G \gamma \theta_1}{2} \right) \left( \cos \frac{G \gamma \theta_2}{2} + i \sin \frac{G \gamma \theta_2}{2} \right) \sin \frac{\chi_2}{2}
\]

(E.10)

\[
T_{22} = \cos \frac{\chi_1}{2} \cos \frac{\chi_2}{2} \left( \cos \frac{G \gamma \theta_1}{2} + i \sin \frac{G \gamma \theta_1}{2} \right) \left( \cos \frac{G \gamma \theta_2}{2} + i \sin \frac{G \gamma \theta_2}{2} \right) \\
+ \left( \cos \frac{G \gamma \theta_1}{2} - i \sin \frac{G \gamma \theta_1}{2} \right) \left( \cos \frac{G \gamma \theta_2}{2} + i \sin \frac{G \gamma \theta_2}{2} \right) \sin \frac{\chi_1}{2} \sin \frac{\chi_2}{2}
\]

(E.11)
Bibliography


# Curriculum Vitae

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## Education

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<td>Indiana University, Bloomington</td>
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<tr>
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## Membership and Awards

- Member of the American Physics Society since 1993;

## Professional Experience

US Particle Accelerator School, Austin, TX (Teaching Assistant, 1/98)  
Indiana University Cyclotron Facility (Research Associate, 5/94 - present)  
  - Cooler Injector Synchrotron Project (Supported by National Science Foundation and Indiana University)  
  - Cooler beam dynamics experiment (Supported by National Science Foundation and U.S. Department of Energy)  
Indiana University, Bloomington (Associate Instructor, 8/93-5/94)  
Beijing Nuclear Instrument Factory, China (Engineer, 7/91 - 8/93)  
Peking University, Beijing, China (Research Associate, 9/88 - 7/91)  
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