Deposition of Particles under External Forces in Laminar Flow through Parallel-Plate and Cylindrical Channels

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A theoretical analysis of particle deposition kinetics onto walls of parallel-plate and cylindrical channels is presented. Rigorous transport equations are formulated by taking into account specific surface forces as well as external forces, e.g., gravity. By solving the transport equations numerically, the dimensionless mass transfer Sherwood number is determined as a function of various dimensionless parameters introduced such as Pe, Gr, Ad, and Dl, accounting for convection and diffusion, and for gravity, dispersion, and electrical double-layer interactions, respectively. The influence of attractive surface forces and gravity on the deposition kinetics is graphically presented and discussed. For large particles, i.e., about 1-μm diameter (Pe > 1), and for short distances from the point where deposition starts, a considerable increase in particle flux (up to an order of magnitude) is predicted over previous analytical values when strong attractive double-layer forces are present. For particles smaller than 0.1-μm diameter (Pe < 10^-4) our numerical results show that particle deposition rates may be successfully predicted by an analytical formula derived for particles of negligible size even in the presence of double-layer attractions (provided external forces are absent). Experimental results reported in the literature obtained under conditions of negligible gravity force are reinterpreted in terms of the present theory. A somewhat closer agreement with experimental data as compared to the analytical formula mentioned above is found in cases of strong double-layer attractions.

INTRODUCTION

Mass and energy transfer in parallel-plate and cylindrical channels has received considerable attention for a long time due to its great practical significance (1-5). Deposition of fine particles in such arrangements is of great importance because of the large variety of practical systems dealing with such processes: e.g., aerosol classification, formation of deposits in heat exchangers and pipelines, hydrodynamic field chromatography, thrombus formation in artificial organs, etc. A theoretical prediction of particle deposition kinetics in such systems may be very useful for design and optimization of processes such as deep-bed and electrofiltration, carrier flotation, dete-
of particle deposition kinetics when electrical double-layer forces played a significant role. This description was an extension of the surface force boundary layer approximation (SFBLA) developed and applied originally by Ruckenstein and Prieve (13) and Spielman and Friedlander (14) for spherical and cylindrical collectors. The underlying assumption of this approach is that the surface force effects are confined to a very small region close to the collector surface (in comparison to the extension of the diffusion boundary layer thickness) and can thus be incorporated into the boundary conditions which take the form of a first-order reaction at the collector surface. In this way fluid convection and surface force effects can be decoupled, largely simplifying the governing transport equations. The dimensionless reaction constant \( K \) is given by the formula (13)

\[
K = \frac{1}{\Delta} \int_{0}^{\Delta} \frac{1}{F_{i}(H)} \exp(\Phi/kT) - 1 \, dH, \tag{1}
\]

where \( \Delta \) is the surface force boundary layer thickness, \( F_{i}(H) \) is a universal hydrodynamic function accounting for Stokes' law correction near a bounding wall, \( H \) is the dimensionless particle collector separation, \( \Phi \) is the interaction energy, \( k \) is the Boltzmann constant, and \( T \) is the absolute temperature.

It was shown previously (15, 16) that such an assumption fails, however, in systems with no or very small energy barriers. Clearly, when \( \Phi < 0 \) (attraction) within the entire surface force boundary layer, we can deduce from Eq. [1] that \( K \) becomes negative, thus losing its physical interpretation. Moreover, for particles with dimensions of the order of micrometers and larger the SFBLA method yields deposition rates which are much too small compared to results obtained by solving the complete transport equation (15, 16), even if a large energy barrier is present. Similarly, in the presence of a secondary minimum this approach produces very incorrect results because it neglects secondary accumulation of particles in this region.

Another approach especially suitable for larger particles, i.e., when Brownian motion may be neglected, has been developed recently in our laboratory (17) and is based on the limiting trajectory concept; extensions to deposition in channels, taking into account specific surface forces and hydrodynamic corrections near the wall, have been made (18).

Recently, very interesting experimental results aimed at explaining the role of double-layer interactions in particle deposition in a parallel-plate channel were reported by Bowen and Epstein (19) for colloid-size silica particles (of radius 0.2–0.32 \( \mu \)m). However, their results were interpreted in terms of the SFBLA, an approach which seems not completely adequate.

Thus, the purpose of the present work is to solve the complete transport equations for the parallel-plate and cylindrical channels taking rigorously into account effects of surface interactions and external forces as well as the hydrodynamic particle–wall interactions. Reinterpretation of Bowen and Epstein’s results in terms of our theory is also attempted.

DEPOSITION IN A PARALLEL-PLATE CHANNEL

Let us consider a parallel-plate channel of depth \( 2b \) (see Fig. 1) through which a dilute suspension of spherical particles is flowing. The flow is assumed to be laminar and steady and the suspending fluid incompressible. It is further assumed that the suspension is stable and that all interactions between the particles can be neglected. The fluid velocity field and the transport equations are expressed relative to a fixed Cartesian coordinate system \( (x, y, z) \) having its origin at a point sufficiently far from the entrance region at a distance where the fluid
velocity profile may be assumed to be fully developed, i.e., parabolic (Fig. 1). Hence, the undisturbed fluid velocity field \( \mathbf{V} \) can be expressed as
\[
\mathbf{V} = \frac{3}{2} V_m \frac{z}{b} \left( 2 - \frac{z}{b} \right) \mathbf{i}_x, \tag{2}
\]
where \( V_m \) is the mean fluid velocity (a scalar quantity) and \( \mathbf{i}_x \) is the unit vector in the \( x \) direction.

If we assume that no deposition of particles occurs up to the point \( x = 0 \), the general continuity equation in the steady state, i.e., \( \partial n / \partial t = 0 \), in absence of sources and sinks can be written in the form
\[
\frac{\partial j_x}{\partial x} + \frac{\partial j_y}{\partial y} + \frac{\partial j_z}{\partial z} = 0, \tag{3}
\]
with
\[
\begin{align*}
    j_x &= -D_{\parallel} \frac{\partial n}{\partial x} + U_x n, \\
    j_y &= -D_{\parallel} \frac{\partial n}{\partial y} + U_y n, \\
    j_z &= -D_{\perp} \frac{\partial n}{\partial z} + U_z n, \tag{4}
\end{align*}
\]
where \( j_x, j_y, j_z \) are the respective components of the particle flux vector; \( D_{\parallel}, D_{\perp} \) are the tangential and normal components of the diffusion tensor which are in general not the same as a result of the presence of bounding walls; \( U_x, U_y, U_z \) are the components of the particle velocity vector deriving from all forces and torques acting on the particle; and \( n \) is the particle number concentration.

According to (20) the components of the velocity vector can be formulated explicitly as follows:
\[
\begin{align*}
    U_x &= \frac{1}{\eta} K_{11}(K_{11} K_{11} - K_{11})^{-1} F_x + F_h(z)V_x, \\
    U_y &= \frac{1}{\eta} K_{11}(K_{11} K_{11} - K_{11})^{-1} F_y, \\
    U_z &= \frac{1}{\eta} K_{11} F_z, \tag{5}
\end{align*}
\]
where \( \eta \) is the fluid dynamic viscosity, \( K_{11}, K_{11} \) are the tangential and normal components of the particle translation resistance dyadic, \( K_{11} \) is the tangential component of the coupling dyadic, \( K_{11} \) is the tangential component of the rotation dyadic (20), \( F_x, F_y, F_z \) are the respective components of the external and surface force \( F \) acting on the particles, \( V_x = |\mathbf{V}| \) is the \( x \) component of the fluid velocity vector (which in this special case equals the fluid velocity field itself), \( F_h(z) \) is a hydrodynamic correction function accounting for the deviation of the velocity of the particle from the fluid motion as a result of hydrodynamic disturbances created by the presence of bounding walls.

In the derivation of Eq. \([5] \) external torques on a particle are assumed to be zero and all inertial effects are neglected because particle velocities relative to the fluid motion are such that the creeping-motion equations apply. Accordingly, all secondary inertial effects including transverse drift are assumed to be negligible. Equations \([3] - [5] \) constitute a three-dimensional partial dif-
ferential equation which can be reduced to a
two-dimensional form if we assume that the
y component of the external force is indepen-
dent of y and the width of the channel is
much larger than the diffusion boundary
layer thickness, thus edge effects can be ig-
nored; the surface interactions have obvi-
ously a z component only.

Equations [3]–[5] even in a two-di-
men-sional form are very inconvenient to handle
with numerical methods because of the great
number of physical variables involved. A
considerable simplification can be achieved
by an appropriate scaling of these equations.
Such a scaling, however, is by no means
unique; it depends on a somewhat arbitra-
ry choice of the characteristic scaling di-
mensio ns. For colloidal particles, when the
diffusion boundary layer thickness is com-
parable to the particle size, the particle radius
a was selected as scaling length (in the z
direction), analogously to the case of the ro-
tating disk, treated in (21). The longitudinal
distance is scaled by the channel half-depth
b. Such a scaling is especially well suited
when along the entire channel length the dif-
fusion boundary layer thickness remains
small compared to the channel dimensions
(this condition holds very well for particles
of colloidal size and larger and was also met
in Bowen and Epstein’s experiments (19)).

Thus, the dimensionless variables intro-
duced take the form

\[ H = \frac{z}{a} - 1, \]

\[ \bar{n} = \frac{n}{n_{\infty}}, \]

\[ A = \frac{a}{b}, \]

\[ Pe = \frac{3V_m a^3}{2b^2 D_{\infty}} = \frac{3A^2 Pe_{n}}{8}, \]

\[ A F = \frac{AF}{kT}, \]

where \( H \) is the dimensionless gap width,
\( n \) the dimensionless particle concentration,
\( n_{\infty} \) the particle number concentration in bulk
at the origin, \( Pe \) the dimensionless Pécelet
number, \( D_{\infty} \) the particle diffusion coefficient
in the bulk, \( Pe_{n} = 4V_m b/D_{\infty} \) the Pécelet num-
ber adopted in (19), and \( \bar{F} \) the dimension-
less external force.

Substituting these new variables into Eqs.
[1]–[5] yields

\[ \frac{\partial}{\partial x} \left[ -A^2 \bar{M}_n \frac{\partial \bar{n}}{\partial x} + Pe \bar{F} \bar{n}(H)(H + 1) \right] \times [2 - (H + 1)A] \bar{n} + \bar{M}_i \bar{F} \bar{n}A \bar{n} \]

\[ + \frac{\partial}{\partial H} \left[ \bar{K}_{i1} \left( - \frac{\partial \bar{n}}{\partial H} + \bar{F} \bar{n} \right) \right] = 0, \text{ [6]} \]

where \( \bar{x} = x/b \) and \( \bar{M}_n = 6\pi a K_{r} (K_{r}^{-1} K_{r}^{-1} - K_{r}^{-1})^{-1}, \bar{K}_{i1} = K_{i1}/6\pi a. \) Assuming that
\( A = a/b \ll 1, \) usually the case under exper-
imental conditions, then the term involving
\( A^2 \) in [6] can be neglected and the coef-
ficients \( \bar{M}_i, \bar{K}_{i1}, \) and \( F_{n}(H) \) can be
accurately approximated by the universal
hydrodynamic functions \( F_{n}(H), F_{i}(H), \)
and \( F_{n}(H), \) respectively, tabulated in (22–24).
This is due to the fact that it can be assumed
that the particles are effectively immersed
in a simple shear flow (with effective shear
rate \( 3V_m b \)) within the whole region where
the hydrodynamic corrections are signif-
ificant. Similarly, the hydrodynamic distur-
bance created by the second more distant
wall becomes unimportant. As a result of
these assumptions Eq. [6] becomes a para-
bolic-type partial differential equation (Eq.
[6] is an elliptical one), a fact which largely
simplifies numerical calculations.

Usually the collector surface is regarded
as a "perfect sink," as implied by the
boundary condition \( \bar{n} = 0. \) It was shown in
(26), however, that when a primary mini-
mum is at least about \(-20kT \) deep and when
particles are immobilized at collector sur-
faces due to the presence of specific tan-
gential interactions, particle flux values are
practically unaffected by changing this con-

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dition to $\tilde{n} = C$, where $C$ is an arbitrary positive constant much smaller, however, than $\exp(-\delta_{\text{min}}/kT)$, i.e., $\ll 10^8$. Under such conditions the relaxation time of particle accumulation within the primary minimum region becomes practically infinite, and the deposition process may be treated as irreversible and quasi-stationary.

However, in order to simplify numerical calculations, the usual boundary condition is applied to both the lower and the upper surfaces:

$\tilde{n} = 0$ for $H = \delta$  
$\tilde{n} = 0$ for $H = H_u$  
$x > 0$,  

$\delta$ being the dimensionless primary minimum distance.

The boundary, or rather quasi-initial, condition at the origin is

$\tilde{n} = n_0(H)$ at $\tilde{x} = 0$,  

and it describes the inlet particle concentration distribution in the suspension which may be a given function of the $H$ variable. In previous papers concerning particle deposition within the parallel-plate channel, e.g., in (19), a third "symmetry" boundary condition was assumed at the channel midline, i.e.,

$$
\left. \frac{\partial \tilde{n}}{\partial H} \right|_{H = (b/a) - 1} = 0.
$$

However, when an arbitrary external force (e.g., gravity) is acting, such a condition cannot be assumed a priori (and fulfilled) without analyzing the magnitude of this force and the extension of the diffusion boundary layer.

Assume now that the force acting on particles consists of (i) a net gravity and buoyancy force $F_g$ described by the formula

$$
F_g = \frac{4}{3} \pi a^3 (\rho_p - \rho_f) g,
$$

where $\rho_p$, $\rho_f$ are the particle and fluid densities, respectively, and $g$ is the gravity acceleration vector; (ii) the dispersion London-van der Waals interaction $F_1$, given by the formula

$$
F_1 = -\frac{A_{123}}{6a} \times [f(H,\lambda) - f(H_u - H, \lambda)]|_{H_u},
$$

where $A_{123}$ is the Hamaker constant for interactions of a particle 1 through a medium 2 with a planar surface 3; $\lambda = \lambda/a$ is a dimensionless parameter accounting for the retardation effect ($\lambda$ is the characteristic wavelength of the radiation absorption band). The second term in Eq. [11] denotes interactions with the second more distant wall. In the present paper only the retardation case is treated and the function $f(H,\lambda)$ is approximated by the formula given in (27), i.e.,

$$
f(H,\lambda) = \frac{\lambda(\lambda + 2qH)(\lambda + qH)^2}{(\lambda + qH)^2},
$$

where $q = 11.116$; (iii) the electrical double-layer forces, expressed by the following approximate formula (28):

$$
F_e = 4\pi \varepsilon_0 \varepsilon_1 \xi_2 \tau [g(H,\tau) - g(H_u - H, \tau)]|_{H_u},
$$

where $\varepsilon$ is the dielectric constant of the liquid and $\varepsilon_0$ the permittivity of a vacuum; $\varepsilon_1$, $\varepsilon_2$ are the zeta potentials of the particle and the channel surface, respectively; $\tau = \kappa a$, $\kappa = (2e^2c^2/\varepsilon_0 kT)^{1/2}$ is Debye’s reciprocal double-layer thickness, $e$ the electron charge, $i$ the ion valency, and $c$ the ion number concentration. For zeta potentials smaller than 60 mV and for large value of $\tau$, double-layer interactions can be approximated by the formula (28)

$$
g(H,\tau) = \exp(-\tau H)[1 + \exp(-\tau H)].
$$

This expression becomes inaccurate for separations smaller than the double-layer thickness. However, this deviation is unimportant for oppositely charged particle and collector surfaces (when only attraction
forces occur) as proved by numerical calculations in which more accurate formulas

\[
\frac{\partial n}{\partial \vec{x}} (PeF_\delta(H)(H + 1)[2 - (H + 1)A] + Gr \cos \theta \sin \phi F_\delta(H)A)
\]

\[
+ \frac{\partial}{\partial H} \left[ F_\delta(H) \left[ - \frac{\partial n}{\partial H} - Ad[f(H,\hat{k}) - f(H - H_u, \hat{k})] \hat{n}
\right. \right.
\]

\[
+ \left. Dl\tau \left[ g(H,\tau) - g(H - H_u, \tau) \right] \hat{n} - Gr \cos \phi \right] \right] = 0, \quad [15]
\]

where \( Ad = A_{123}/(6kT) \) is the dimensionless adhesion number, \( Dl = 4\pi \varepsilon_0 \zeta_\varepsilon \zeta_a a/kT \) is the dimensionless double-layer number, \( Gr = 2\Delta \rho \rho_a/(9\eta D_a) \) is the dimensionless gravity number (\( \Delta \rho = \rho_a - \rho_b \)), \( \phi \) is the angle between the direction of gravity and the \( z \) axis, \( \theta \) is the angle between the direction of the component of the gravity vector in the \( xy \) plane and the direction of the \( x \) axis (see Fig. 1). The boundary conditions [7] apply also to Eq. [15]. From the boundary conditions \( \hat{n} = 0 \) for \( H = \delta \) and \( H = H_u \) it follows that \( \partial \hat{n}/\partial \vec{x} = 0 \) at the channel surfaces. Hence Eq. [15] may be reduced to

\[
\frac{\partial}{\partial H} \left( F_\delta(H) \frac{\partial \hat{n}}{\partial H} \right)_\delta = 0. \quad [16]
\]

Integration of this equation yields

\[
\left[ F_\delta(H) \frac{\partial \hat{n}}{\partial H} \right]_\delta = C_1 = Sh_1(\hat{x}, Pe, A, Gr, Ad, \hat{\lambda}, Dl, \tau) \quad [17]
\]

and

\[
\left( F_\delta(H) \frac{\partial \hat{n}}{\partial H} \right)_{\delta} = C_2 = Sh_2(\hat{x}, Pe, A, Gr, Ad, \hat{\lambda}, Dl, \tau), \quad [18]
\]

where \( Sh_1, Sh_2 \) are the dimensionless mass transfer Sherwood numbers for the lower and upper channel plates, respectively (which equal the dimensionless particle flux at these points). They are local quantities and depend on the longitudinal coordinate \( \hat{x} \); \( Sh_1 = Sh_2 \) only in the special case when \( Gr = 0 \).

The mass transfer Sherwood number averaged over the whole surface of the channel plate may be defined as

\[
\bar{Sh}(Pe, A, Gr, Ad, \hat{\lambda}, Dl, \tau, L) = \frac{b}{L} \int_0^L \left[ Sh_1(\hat{x}, Pe, A, Gr, Ad, \hat{\lambda}, Dl, \tau) \right] d\hat{x}, \quad [19]
\]

where \( L \) is the channel length and \( \bar{L} = L/b \).

A similar equation holds for the \( \bar{Sh}_2 \) value.

Finally, the overall mass transfer number is defined as

\[
\bar{Sh}(Pe, A, Gr, Ad, \hat{\lambda}, Dl, \tau, L)
\]

\[
= (1/2)(\bar{Sh}_1 + \bar{Sh}_2). \quad [20]
\]

For \( Pe/A^3 = Pe_b \ll 1 \), neglecting surface and external forces as well as hydrodynamic corrections, the local mass transfer number can be expressed as (3)

\[
Sh_1 = Sh_2 = \frac{1}{\Gamma(4/3)} \left( \frac{2Pe}{9\bar{\alpha}} \right)^{1/3}. \quad [21]
\]

This analytical formula was used for testing the accuracy of the numerical solution.

The local value of the normal component of the particle flux vector is, according to Eq. [17] and the definitions of the dimensionless variables \( \hat{n} \) and \( H \), given by

\[
\hat{j}_z = -Sh_1 D_a n_\alpha/a \quad \text{for} \quad H = \delta \quad [22]
\]

and

\[
\hat{j}_z = Sh_2 D_a n_\alpha/a \quad \text{for} \quad H = H_u. \quad [23]
\]

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Because of the quasi-steady-state assumption made previously, the amount of particles deposited onto an infinitesimal surface element \( dS = d\vec{d} \) (where \( d \) is the channel width) after time \( t \) can be expressed by the relation

\[
N = |J_x|dSt = Sh_1(\hat{x},Pe,A,Gr,Ad,\hat{\lambda},Dl,x) \\
\times \frac{D_\infty n_\infty}{a} dSt. \quad [24]
\]

An analogous equation holds for the second channel plate.

From this formula we can see that the value of \( N \) depends, among other things, on the \( \hat{x} \) coordinate; thus the particle coating density differs at various distances from the inlet point.

The total amount of particles deposited within the channel is

\[
N_t = \overline{Sh} \frac{D_\infty n_\infty}{a} \overline{Lt} = \overline{Sh} \frac{D_\infty n_\infty}{2ab} V_c t, \quad [25]
\]

where \( V_c \) is the channel volume.

Equations [24] and [25] retain their validity as long as the particles captured on the collector surface do not disturb deposition of particles arriving at the primary minimum, i.e., for very low coating density (small \( N \) value).

From Eqs. [17] and [18] we can see that determination of the Sherwood numbers or the amount of particles deposited is possible if we know the dimensionless particle concentration gradient \( \partial n/\partial H \) at the channel walls. Due to the complexity of the coefficients in Eq. [15] this equation can only be solved by numerical methods.

It is interesting to note that Eq. [24] and the analysis presented hitherto apply also directly to the problem of particle deposition onto a smooth surface from a suspension undergoing simple shear flow, e.g., \( V = Gz \), where \( G \) is the shear rate, by letting \( A \to 0 \). The Péclet number is accordingly defined as

\[
Pe = \frac{Ga^3}{2bD_\infty}. \quad [26]
\]

![Fig. 2. Schematic representation of a cylindrical channel. The fluid velocity field is fully developed at \( z = 0 \). No deposition of particles occurs for \( z < 0 \).](image)

All remaining parameters retain their form unchanged.

**DEPOSITION IN A CYLINDRICAL CHANNEL**

It is convenient to express the transport equation for the cylindrical channel relative to a cylindrical frame of reference \((r,\theta,z)\) having its origin at the channel center sufficiently far from the entrance region (see Fig. 2). The fluid velocity field is assumed to be fully developed at the point \( z = 0 \) and in terms of this frame of reference can be expressed as

\[
V = 2V_m \left[ 1 - \left( \frac{r}{R} \right)^2 \right] z, \quad [27]
\]

\( R \) being the cylinder radius.

Introducing the following dimensionless variables

\[
\tilde{n} = \frac{n}{n_\infty}, \\
A = \frac{a}{R}, \\
H = \frac{R - r}{a} - 1, \\
\tilde{z} = \frac{z}{R},
\]

\[
Pe = \frac{2V_m a^3}{R^2 D_\infty} \approx Pe_n A^3,
\]

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\[ Pe_n = 2V_m R/D, \]
\[ \tilde{F} = \frac{Fa}{kT}, \]

the continuity equation

\[
\frac{1}{[1 - (H + 1)A]} \frac{\partial}{\partial H} \left[ K[H][1 - (H + 1)A] \left( -\frac{\partial \tilde{n}}{\partial H} + \tilde{F}_n \tilde{n} \right) \right] \\
+ \frac{1}{[1 - (H + 1)A]} \frac{\partial}{\partial \theta} \left[ M[H] \left( -A^2 \frac{\partial \tilde{n}}{\partial H} + A\tilde{F}_\theta \tilde{n} \right) \right] \\
+ \frac{\partial}{\partial \tilde{z}} \left[ M[H] \left( -A^2 \frac{\partial \tilde{n}}{\partial \tilde{z}} + A\tilde{F}_z \tilde{n} \right) \right] + PeF_\eta (H + 1)[2 - (H + 1)A] \tilde{n} = 0. \]

This equation as it stands is a three-dimensional elliptic partial differential equation and unlike Eq. [6], derived for the parallel-plate channel, cannot in principle be reduced to the two-dimensional form except for the case of vanishing external forces. However, when \( a \ll R \) \((A \ll 1)\), the tangential and azimuthal diffusion becomes negligible in comparison to radial diffusion and as a result the force terms which involve a multiplication by \( A \) in [29] can be omitted. Hence, making use of the formulas for the gravity, dispersion, and electrical double-layer forces, Eq. [29] becomes

\[
\frac{1}{[1 - (H + 1)A]} \frac{\partial}{\partial H} \left[ F_1(H)[1 - (H + 1)A] \left( -\frac{\partial \tilde{n}}{\partial H} - Gr \cos \theta \cos \phi \tilde{n} \right) \\
- Adf(H,\tilde{z}) \tilde{n} + Dl \tau g(H,\tau) \tilde{n} \right] + PeF_\eta (H + 1)[2 - (H + 1)A] \frac{\partial \tilde{n}}{\partial \tilde{z}} = 0, \]

where the \( Ad, Dl, Gr \) parameters are the same as those defined previously for the parallel-plate channel. In this case, however, particle concentration and particle flux still depend on the \( \theta \) coordinate because of the presence of the term \( Gr \cos \theta \sin \phi \tilde{n} \) in Eq. [30]. For \( A \ll 1 \) this dependence is so slight that, as mentioned previously, the azimuthal diffusion may be completely neglected.

The boundary condition for Eq. [30] is

\[ \tilde{n}(H,\theta,\tilde{z}) = 0 \quad \text{for} \quad H = \delta, \quad \tilde{z} > 0, \]
\[ 0 < \theta < \pi. \]

This is the perfect sink boundary condition at the cylindrical channel surface, analogous to the parallel-plate channel. The quasi-initial condition takes the form

\[ \tilde{n}(H,\theta,0) = f(H), \]

where \( f(H) \) denotes the particle concentration distribution of the suspension at the point \( \tilde{z} = 0 \), assumed to be a function of the \( H \) coordinate only.

In the case of a cylindrical channel the local mass transfer Sherwood number depends on both the longitudinal and the angular coordinates, \( z \) and \( \theta \). In order to obtain an average surface mass transfer number Eq. [30] must be solved for a sufficiently large number of \( \theta \) values. The angular coordinate-averaged mass transfer number can be defined as follows:
Similarly, the overall mass transfer number is given by the formula

\[ \overline{Sh}(\frac{\pi}{2}, Pe, A, Gr, Ad, \lambda, D, L) = \frac{b}{\pi L} \int_0^{\pi/2} \int_0^{\pi/2} \overline{Sh}(\theta, \frac{\pi}{2}, Pe, A, Gr, Ad, \lambda, D, L) \, d\theta \, dz \, d\phi \]  

[34]

and is independent of both \( \theta \) and \( z \).

NUMERICAL PROCEDURE FOR SOLVING THE TRANSPORT EQUATIONS

In order to solve Eqs. [15] and [30] together with the associated boundary conditions [7]-[8] and [31]-[32] the implicit weighted average Crank-Nicolson scheme was applied. For convenience Eqs. [15] and [30] can be rewritten as

\[ \frac{\partial n}{\partial \xi} = \alpha(H) \frac{\partial^2 n}{\partial H^2} + \beta(H) \frac{\partial n}{\partial H} + \gamma(H) \bar{n} \]  

[35]

and

\[ \frac{\partial \bar{n}}{\partial \xi} = \tilde{\alpha}(H) \frac{\partial^2 \bar{n}}{\partial H^2} + \tilde{\beta}(H, \theta) \frac{\partial \bar{n}}{\partial H} + \tilde{\gamma}(H, \theta) \bar{n}, \]  

[36]

where the functions \( \alpha, \beta, \gamma, \tilde{\alpha}, \tilde{\beta}, \) and \( \tilde{\gamma} \) can be found straightforward from Eqs. [15] and [30].

Similar equations (without the double-layer interaction terms) were formulated and solved numerically in (29) and (30) for the spherical and rotating disk collectors.

Because of the presence of terms describing the surface interactions which vary extremely rapidly as \( H \to \delta \) or \( H \to \infty \), using \( H \) as a primary variable resulted in a poor accuracy of the numerical solution. Thus, a special transforming function was used to "compress" the step size close to the primary minimum and to extend it when \( H \) approached the channel central line. In this way a good adjustment to the physical nature of the problem under consideration was attained. The transformation can be expressed as

\[ u = \ln \left[ \frac{H - \delta}{p} + 1 \right], \]  

[37]

or

\[ H = \delta + p(\exp u - 1) \]  

for \( H < \frac{b}{a} - 1, \)

i.e., for the midpoint, and

\[ u = u_u - \ln \left[ \frac{H_u - H}{p} + 1 \right] \]  

for

\[ H > \frac{b}{a} - 1, \]  

[38]

where

\[ u_u = 2 \ln \left[ \frac{(b/a) - 1 - \delta}{p} + 1 \right], \]

\( p \) being an arbitrary positive constant.

For the cylindrical channel the singularity occurring in \( \tilde{\gamma}(H, \theta) \) was avoided by fixing the upper integration limit at a very small distance from the channel center. This did not affect the computed flux values.

Because in all calculations presented in this paper the condition \( A \ll 1 \) was met and the diffusion boundary layer was much smaller than the channel transverse dimension, it was convenient to use the symmetry boundary condition Eq. [9] and thus reduce by half the number of points required in the \( \xi \) domain. The particle flux at the channel walls and the local value of the Sherwood number were computed by a subroutine evaluating the first derivative, i.e., \( \partial \bar{n}/\partial H \) at the origin. Application of the central derivative for approximating the \( \partial \bar{n}/\partial H \) term produced especially accurate results. Since the interaction potential attains very large negative values (attraction) for \( H \to \delta \) the calculated mass transfer rates (flux values) were insensitive to the selection of the \( \delta \) parameter (primary minimum separation).
when changed within the range of $10^{-3}$ to $2 \times 10^{-2}$. For convenience, in all calculations presented in this paper the value of $\delta$ was taken as $10^{-2}$. The step size in the $x(\bar{z})$ domain was changed automatically, being very small close to the origin and increasing as the longitudinal distance increased. The criterion of the step change was the relative difference between the solution obtained by using two different step sizes. If this difference was smaller than a given very small number $\varepsilon_r$ (usually $\varepsilon_r < 10^{-9}$) the step size was increased by a selected factor.

RESULTS AND DISCUSSION

In the computations special attention was focused upon the role of specific surface interactions in particle deposition onto the channels’ walls. Only the cases of attractive interactions were considered, i.e., oppositely charged particle and channel surfaces. No attempt was undertaken in these computations to examine the influence of repulsive interactions, i.e., the presence of an energy barrier on particle deposition kinetics. In all cases studied, i.e., for $10^{-4} < Pe < 10^1$ and $10^{-2} < \bar{x} < 10^3$, the diffusion boundary layer thickness was of the order of particle dimensions even at the outlet region because of the small value of the $A$ parameter selected, i.e., $10^{-3}$. This value corresponds to a particle of 1-$\mu$m diameter and a channel depth of 2 mm, which is close to the experimental conditions in (19). Numerical computations revealed that changing $A$ within the range $10^{-2}$ to $10^{-4}$ (keeping $Pe$ constant) did not significantly influence the particle flux at the channel walls or the mass transfer Sherwood number. Because in all cases the diffusion boundary layer was practically confined to the immediate vicinity of the channel walls, the lower and upper plates of the channel (lower and upper relative to the direction of gravity) may be treated as being fully separated by a homogenous suspension with a uniform particle number concentration equal to $n_x$. In the presence of gravity the diffusion boundary layer was thinner at the lower channel plate (and consequently the particle flux there was larger) as compared to the upper plate at which the diffusion layer thickness was greater and the particle flux smaller.

In the calculations the inlet particle concentration distribution was approximated by the formula

$$\bar{n} = \exp\{-s(H_0 - H)\} \quad \text{for} \quad H \leq H_0$$

and

$$\bar{n} = 1 \quad \text{for} \quad H > H_0.$$ [39]

When $H_0 \to \delta$ the usually applied quasi-initial condition (14) can be obtained, i.e., $\bar{n} = 1$ for all $H$. On the other hand when $s \to \infty$ the particles were initially absent within the region confined between $\delta$ and $H_0$. Several runs were performed for various values of $s$ and $H_0$ indicating that particle concentration distribution and particle flux were not much affected at remote distances from the origin, i.e., for $\bar{x} > 0.1$, even if $H_0$ was changed between $10^{-2}$ and 1 and $s$ between 0 and 50. At shorter distances the particle flux increased as $H_0 \to \delta$ and $s \to 0$.

The influence of specific surface interactions on the local mass transfer rate (Sh1 value) is presented in Fig. 3 for Péclet numbers varying from $10^{-4}$ to $10^3$. In this figure results are presented both for distances close to the inlet point, i.e., for $\bar{x} = 10$, and for remote distances, i.e., $\bar{x} = 10^3$. For a 1-$\mu$m-diameter particle this corresponds to a distance from the origin of 1 and 100 cm, respectively. Calculations were carried out for (i) $Ad = 0.04$ and (ii) $Ad = 0.4$, corresponding to Hamaker constants of $10^{-21}$ and $10^{-20}$ J at room temperature, respectively. The effect of the double-layer attractions on the particle deposition rate is also shown in this figure by selecting the case (iii) of $Di = 4 \times 10^5$ and $\tau = 5$. This corresponds approximately to a 1-$\mu$m-diameter particle with a zeta potential of $-70$ mV (the wall
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3. Dependence of the local mass transfer Sherwood number $Sh_1$ on the Péclet number $Pe$ for $Gr = 0$ (no sedimentation). $\lambda = 0.3, A = 10^{-3}$. (1) $\bar{x} = 10, Ad = 0.4, DI = -4 \times 10^3, \tau = 5$; (2) $\bar{x} = 10, Ad = 0.4, DI = 0$; (3) $\bar{x} = 10, Ad = 0.04, DI = 0$; (4) $\bar{x} = 10^3, Ad = 0.4, DI = -4 \times 10^3, \tau = 5$; (5) $\bar{x} = 10^3, Ad = 0.4, DI = 0$; (6) $\bar{x} = 10^3, Ad = 0.04, DI = 0$; $\bar{x} = x/b$ for the parallel-plate channel and $\bar{x} = z/R$ for the cylindrical one.

zeta potential being 70 mV) suspended in water at room temperature. We can see from this figure that changing the magnitude of the surface forces between these wide limits, i.e., between cases (i) and (iii), causes a rather limited increase in the particle flux value, i.e., of about 30% for $Pe = 10^{-2}$ and 12% for $Pe = 10^{-4}$ ($\bar{x} = 10^3$). However, for Péclet numbers larger than 1 and for small distances from the origin, the influence of specific surface interactions manifests itself more dramatically; the particle flux increases by a factor of 3 for $Pe = 1$ ($\bar{x} = 10$) and almost an order of magnitude for $Pe = 10$.

According to the definition of the Péclet number we can conclude that the effect of surface forces is especially important for larger particles, i.e., for $a > 1 \mu m$ and for intense flows. In these cases the diffusion boundary layer thickness becomes comparable to distances over which surface interactions play a dominant role. This is especially true at short distances from the origin where the diffusion boundary layer is still undeveloped. In Figs. 4–6 the dependence of the mass transfer number $Sh_1$ on the dimensionless longitudinal distance $\bar{x}$ (or $z$ for the cylindrical channel) for $Pe = 1, A = 10^{-3}$, and $Gr = 0$. (1) $Ad = 0.4, \lambda = 0.3, DI = -4 \times 10^3, \tau = 5$; (2) analytical approximation, i.e., $Sh_1 = 0.678 \bar{x}^{-1/2}$ (Eq. [21]); (3) $Ad = 0.04, DI = 0, \lambda = 0.3$.

DEPOSITION OF PARTICLES

FIG. 7. Effect of the double-layer thickness on the dependence of $Sh_1$ on $\tilde{x}$ (or $\tilde{z}$ for the cylindrical channel) for $Pe = 0.1$, $\Lambda = 10^3$, $Gr = 0$, $Ad = 0.4$, $\lambda = 0.3$, $Dl = -4 \times 10^3$. (1) $\tau = 1$; (2) $\tau = 5$; (3) $\tau = 25$; (4) $\tau \to \infty$.

$x(\tilde{z})$ is plotted for various selected $Pe$ values (and for $Gr = 0$). We can observe from these figures that the effect of surface forces increases considerably as $\tilde{x}$ decreases. Thus, for $Pe < 1$ and $\tilde{x} < 10$ the analytical approximation, i.e., Eq. [21] used in [19] for the interpretation of experimental results, cannot be used, even for a crude estimation of the particle deposition rate.

In Fig. 7 the influence of the double-layer thickness (expressed by $\tau$) on the particle flux is illustrated by selecting the case $Pe = 0.1$ and $Gr = 0$ (no sedimentation). Although, in principle, the formula used for describing the double-layer interactions becomes inaccurate for small values of $\tau$ (curve 1 on Fig. 7) it was found that the use of more sophisticated and accurate formulas only slightly influences the results obtained, due to the small value of the partial derivative $\partial Sh_1/\partial \tau$. Moreover, because Hamaker constants are usually not better known than within an order of magnitude, selecting a value for $A_{12}$ for interpretation of experimental results introduces an error much larger than the one arising from the use of an approximate formula for the description of the double-layer interactions.

We can see from Fig. 7 that a change of $\tau$ between 1 and 25 causes a 60% decrease in particle flux for $\tilde{x} = 10^2$ and changes it by a factor of about 7 for $\tilde{x} = 1$. We observe again a pronounced effect of surface forces on particle deposition kinetics at distances close to the origin. All results presented hitherto apply directly to the circular channel as well, because the computations have shown that the flux values for the cylindrical channel differ only by 0.5–1% from those obtained for the parallel-plate channel which are presented in Figs. 3–6.

Results of numerical calculations of particle flux when gravity acts are presented in Figs. 8 and 9 for the parallel-plate channel. These results may equally well be used for interpretation of particle deposition in a uniform external electric field provided it generates a constant ($H$-coordinate independent) force on the particles.

In Fig. 8 the mass transfer number $Sh_1$ (at the point $\tilde{x} = 10^3$) is shown as a function of Péclet number for $Gr = 0.1$ and $-0.1$. In the second case the normal component of the gravity vector was directed outwards from the lower channel plate. For the upper wall this component was directed of course toward the wall (thus there $Gr = 0.1$). From this figure we observe that the surface forces cause a 35% increase in $Sh_1$ (for $Gr = 0.1$) when going from $Ad = 0.04$ and $Dl = 0$ to

FIG. 8. Dependence of $Sh_1$ on $Pe$. Effect of gravity force on the deposition in a parallel-plate channel (or cylindrical at the point $\theta = 0$) for $\tilde{x} = 10^3$, $A = 10^3$.

(1) $Gr = 0.1$, $Ad = 0.4$, $\lambda = 0.3$, $Dl = -4 \times 10^3$, $\tau = 5$; (2) $Gr = 0.1$, $Ad = 0.04$, $\lambda = 0.3$, $Dl = 0$; (3) $Gr = 0$, $Ad = 0.4$, $\lambda = 0.3$, $Dl = -4 \times 10^3$, $\tau = 5$; (4) $Gr = 0$, $Ad = 0.04$, $\lambda = 0.3$, $Dl = 0$; (5) $Gr = -0.1$, $Ad = 0.04$, $\lambda = 0.3$, $Dl = -4 \times 10^3$, $\tau = 5$; (6) $Gr = -0.1$, $Ad = 0.04$, $\lambda = 0.3$, $Dl = 0$.

Ad = 0.4 and $Dl = -4 \times 10^2$. This effect, unlike the case of no sedimentation, is equally well pronounced for very small values of $Pe$. Note that for negative values of $Gr$ the particle flux becomes practically negligible for $Pe = 5 \times 10^{-2}$ and consequently the particle deposition rate falls to zero at the channel plate at which the gravity force acts outwards.

In Fig. 9 the relationship between $Sh_1$ and the longitudinal distance $\bar{x}$ is presented in the case of strong sedimentation. We can see from this figure that the influence of surface interactions is more pronounced for larger values of $Gr$, e.g., the particle flux at $\bar{x} = 10^9$ ($Pe = 10^{-2}$) is about four times larger for $Ad = 0.4, Dl = -4 \times 10^3, \tau = 5$ (strong double-layer attractions) than for $Ad = 0.04$ and $Dl = 0$ (no double-layer interactions).

An increase of the $Sh_1$ value with the longitudinal distance $\bar{x}$ (curve 1 in Fig. 9) can be attributed to an accumulation of particles in the vicinity of channel surfaces as a result of sedimentation. Within the region where the accumulation occurs, particle concentration exceeds considerably the bulk value and tends to increase as $\bar{x}$ attains larger values. This can be seen in Fig. 10 where the concentration profiles at various distances from the origin are presented for $Pe = 10^{-2}$ and $Gr = 1$.

Numerical calculations revealed also that a change of the $H_0$ parameter in Eq. [39] within the range $10^{-2}$ to $2 \times 10^{-1}$ (i.e., the extension of the region where particles are depleted) did not influence the overall mass transfer number and only slightly affected the concentration profiles at distances $\bar{x} < 0.1$.

In Fig. 11 the mass transfer number $Sh_1$ (particle dimensionless flux) is presented for the cylindrical channel as a function of the angular position $\theta$ measured relative to the direction of the component of the gravity vector lying in the $r, \theta$ plane, at various distances from the point where the deposition starts, i.e., $\bar{z} = 0$. It can be observed from this figure that for distances close to the origin the particle flux depends very little on the angular coordinate $\theta$. As the longitudinal distance increases, however, the particle flux falls rapidly in the region where the gravity force acts outwards from the channel surface (i.e., for $\theta > 90^\circ$) due to a considerable increase in the diffusion boundary layer thickness. Thus, the amount of particles deposited, in contrast to the parallel-plate channel, depends in this case on both the longitudinal and the transverse
coordinates. Due to symmetry, \( Sh_1(\theta) = Sh_1(-\theta) \).

It is interesting to note that in contrast to the rotating disk (21, 16) or spherical collectors (29), in the case of parallel-plate and cylindrical channels the geometrical interception effect plays no significant role even for larger values of the \( Pe \) parameter (i.e., for larger particles). This is due to the absence of a normal component of the fluid velocity which could transfer particles toward the channels’ walls. Consequently, for larger values of \( Pe \) the dependence of \( Sh \) on \( Pe \) can be roughly expressed as \( Sh_1 \propto Pe^{1/3} \) (modified by presence of surface interactions) whereas for the rotating disk this relationship is \( Sh \propto Pe \). Moreover, for larger values of \( Pe \) (particle dimensions larger than 1 \( \mu \)m) the particle flux is, in the case of parallel-plate and cylindrical channels, much more affected by specific surface interactions than it was in the case of the rotating disk and spherical collectors, especially for distances close to the origin. It seems possible that because of the absence of a normal component of the fluid velocity vector, a removal of deposited particles from the channels’ surfaces may be easier than from the surface of a rotating disk.

**COMPARISON WITH EXPERIMENTAL RESULTS**

Recently, Bowen and Epstein (19) have published experimental data on deposition of negatively charged silica particles onto smooth walls of parallel-plate channels coated with various polymeric substances bearing both positive and negative charges. These well-designed experiments are of great value because they present a very rare attempt to investigate quantitatively the role of specific surface interactions (especially double-layer attractions) in particle deposition from flowing suspensions onto smooth surfaces. For the determination of particle surface concentration an indirect method was used, based on measurements of radioactive signals generated by isotopically marked particles deposited onto the channel surface. This method possesses certain disadvantages, some of which are inherent in the use of any indirect method in general and some can be attributed to the specificity of the experimental arrangements used in (19). They measured the isotopic activity of the deposited particles and not the amount of particles deposited on a given surface area which can be obtained from direct microscopic observations. Extremely good homogeneity of the radioactive marker distribution among particles and elimination of all aggregates must be assured to achieve reliable results. Moreover, presence of a background radiation, originating from the flowing suspension, decreases the precision of this method. Results of Bowen and Epstein’s measurements taken from their original work (19) (for the case of oppositely charged particles and surfaces) are summarized in Table I. They show that the precision of the radioactive method was limited. For instance, runs II-1 and II-10 differ by only 10% in double-layer thickness and by about 10% in flow velocity (these parameters are expected to have little effect.

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on the particle deposition rate when changed between these limits) but the deposition rate is more than 100% larger in the second case, accounting for the difference in suspension concentration. Similar, although less distinctive effects can be observed when comparing other runs. Such a dramatic change of results by slightly varying physical parameters shows the limitation of the method. Nevertheless, their data are valuable and a comparison between our theoretical predictions and their experimental results may be of interest. In Table I results of Bowen and Epstein's experiments are given together with results obtained from the analytical equation [21] (adopted by these authors for interpretation of their results) and our predictions. The theoretical results are presented as a ratio of the experimentally determined particle flux value to that computed by numerical methods (a direct comparison of particle flux is impossible because in (19) the particle concentration of the suspension is not explicitly given). It can be seen from this comparison that our theory is able to account, at least partially, for experimentally observed particle deposition rates exceeding considerably those values predicted by Eq. [21] (see last two columns of Table I), in cases of strong electrical double-layer interactions. In these experiments the gravity effect was practically eliminated because of the vertical orientation of the channel. The

### Table I

<table>
<thead>
<tr>
<th>RUN</th>
<th>( \bar{J}_e ) (mV)</th>
<th>( \bar{J}_a ) (mV)</th>
<th>( x ) (cm)</th>
<th>( V_a ) (cm/sec)</th>
<th>( \rho_a ) (g/cm³)</th>
<th>( D_i ) ( \times 10^{-5} )</th>
<th>( \tau ) (sec)</th>
<th>( \bar{J}_e^2 ) ( \times 10^{-4} )</th>
<th>( \bar{J}_a^2 ) ( \times 10^{-4} )</th>
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<td>2.15</td>
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- **Note**: \( T = 298^\circ K, \epsilon = 78.8, \rho_a = 2.18 \text{ g/cm}^3, b = 0.0425 \text{ cm}, A_{el} = 10^{-51} \text{ J (assumed).}
- **Run I, \( a = 0.2 \mu m; \) Run II, \( a = 0.325 \mu m; \) Run III, \( a = 0.3 \mu m.\)**
- \( \bar{J}_e \) is the experimental flux value.
- \( \bar{J}_a \) is the flux value calculated from Eq. [21].
- \( \bar{J}_e^2 \) is the flux value predicted by using present theory.
deviation between experimental results and theory remains in the case of small double-layer thickness. As discussed above, this could be due to experimental error, but in part also to removal of particles from surfaces by the liquid stream. It was already shown in (26) that the existence of strong attractive surface forces acting normal to the wall did not guarantee the formation of a stable contact between particle and collector surface. A tangential component of the specific force must exist to prevent particle motion parallel to the surface. Such interactions may occur as a result of particle or collector surface roughness or surface heterogeneity or deformability. A seemingly paradoxical conclusion may be drawn that the smoother the surface the more likely particles are able to be removed from it. From the comparison summarized in Table I a negative correlation between particle flux (or amount of particles deposited after a given time onto a selected surface area) and the mean fluid velocity (or shear rate at the channel surface) may indeed be deduced. It seems that further, more precise, experiments on particle deposition kinetics within parallel-plate channels are required, preferably carried out using direct methods enabling one to observe the sticking of particles to the collector surface in statu nascendi and to measure the particle surface density distribution. A study aimed at measuring the role of an external force, e.g., gravity or electric forces, would be of great interest from both a theoretical and a practical point of view.

CONCLUDING REMARKS

Numerical solutions of the complete transport equations for the parallel-plate and cylindrical channels indicate that the particle flux may be strongly influenced by the presence of attractive surface interactions, i.e., dispersion and electrical double-layer forces, when particle and channel surfaces are oppositely charged. This effect is especially well pronounced for distances close to the origin and for large values of the Péclet number. This is due to the fact that under such conditions the diffusion boundary layer thickness becomes comparable to distances over which surface interactions dominate. Accordingly, the effect of surface interactions diminishes as $Pe \to 0$ and $x \to \infty$, i.e., for small particles and for distances remote from the origin where the diffusion layer thickness becomes large.

Thus, for $Pe \lesssim 10^{-3}$ (i.e., for particles smaller than about 0.2 µm in diameter) the numerical results prove that the analytical formula [21] (which is strictly valid for particles of negligible size in the absence of specific or external forces) may be successfully used for predicting particle deposition rates. Even for relatively strong double-layer attractions the deviation between exact results and those obtained by using Eq. [21] does not exceed 10%. Similarly, for particles smaller than 0.2–0.1 µm in diameter the influence of gravity becomes negligible. For larger particles, however, especially when the orientation of channel walls is perpendicular to the direction of gravity, Eq. [21] cannot be used for estimation of the particle flux, not even as a crude approximation, giving results much too small for the lower channel surface and much too large for the upper one. For larger particles, i.e., of about 1 µm in diameter, the influence of specific interactions on particle deposition rates remains important even at remote distances from the origin.

For the cylindrical channel when gravity and buoyancy forces are present, the particle flux and thus the amount of particles deposited depends not only on the longitudinal coordinate but also on the angular coordinate $\theta$; this dependence is more pronounced for larger distances from the origin.

In contrast to the rotating disk or spherical collectors the interception effect is unimportant for the parallel-plate and cylindrical collectors even for larger particles because of the lack of a normal component of the fluid velocity vector. Consequently, the specific surface interactions play a much
more significant role in determining particle deposition rates within parallel-plate and cylindrical channels as compared to the rotating disk or spherical collectors. For the same reasons removal of deposited particles from the channel surfaces seems to be easier than from the rotating disk surface.

Strictly speaking, our theoretical calculations predict only the amount of particles captured in the primary minimum region in a given time interval. The amount of particles deposited, i.e., those particles which are able to form a permanent contact with the collector surface, may be smaller due to limited adhesion between the particle and the surface. This adhesion force is determined by a number of factors difficult to define, such as surface charge heterogeneity, particle and collector surface roughness and deformabilities.

A comparison of our theoretical predictions with existing experimental results of Bowen and Epstein (19), although in cases of strong double-layer attractions showing a somewhat better agreement between theory and experiment compared to the use of Eq. [21], reveals also that in some cases the theory predicts particle deposition rates larger than observed experimentally. This can be attributed to a limited accuracy of the experimental procedure and in part to removal of deposited particles as a result of a weak tangential component of the adhesion force. Although in (19) some experiments were carried out which show that removal of previously deposited particles was negligible, it cannot be excluded that this occurred at the very moment of contact between particle and collector; possibly only those particles which were able to “find” a favorable surface area preventing their further tangential motion became effectively captured. A direct observation of the particle deposition in situ nascenti could provide a detailed insight into the mechanisms of deposition. This is of interest from both a theoretical and a practical point of view.

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