ICA: Data mining for BPM turn-by-turn data

- The turn-by-turn beam position signal is a combination of various source signals.

$$x_i(t)=\sum_j a_j s_j(t)+n_j(t)$$ For the i'th BPM

or $$\mathbf{x}(t)=\mathbf{A}\mathbf{s}(t)+\mathbf{n}(t)$$ $\mathbf{A}$ is the mixing matrix

The source signals include betatron motion, synchrotron motion and others (e.g., ground motion, power supply ripples, etc.)

m BPMs and T turns

Form a matrix of the BPM data

$$\mathbf{x}=\begin{pmatrix} x_1(1) & x_1(2) & \cdots & x_1(T)\\ x_2(1) & x_2(2) & \cdots & x_2(T)\\ \vdots & \vdots & \ddots & \vdots\\ x_m(1) & x_m(2) & \cdots & x_m(T)\end{pmatrix}$$

- The source signals are assumed to be narrow-band with non-overlapping spectra, so their un-equal time covariance matrices are diagonal.

$$\langle \mathbf{s}(t)\mathbf{s}(t+\tau)^T\rangle = \mathrm{diag}[\rho_1(\tau),\rho_2(\tau),\ldots,\rho_n(\tau)]$$

Or
$$\mathbf{C}_x(0)\equiv\langle\mathbf{x}(t)\mathbf{x}(t)^T\rangle=\mathbf{A}\mathbf{C}_s(0)\mathbf{A}^T+\sigma^2\mathbf{I}$$
$$\mathbf{C}_x(\tau)\equiv\langle\mathbf{x}(t)\mathbf{x}(t+\tau)^T\rangle=\mathbf{A}\mathbf{C}_s(\tau)\mathbf{A}^T,\ \tau\neq 0$$

The mixing matrix A diagonalizes the un-equal time sample covariance matrices simultaneously.

- Diagonalize the equal-time covariance matrix (data whitening)

D_1,D_2 are diagonal

Set to remove noise

$$\mathbf{C}_x(0)=[\mathbf{U}_1,\mathbf{U}_2]\begin{bmatrix}\mathbf{D}_1 & \\ & \mathbf{D}_2\end{bmatrix}[\mathbf{U}_1,\mathbf{U}_2]^T \quad \text{with} \quad 0\le\max(\mathbf{D}_2)<\lambda_c\le\min(\mathbf{D}_1)$$

Construct an intermediate "whitened" data matrix

$$\mathbf{z}=\mathbf{D}_1^{-\frac{1}{2}}\mathbf{U}_1^T\mathbf{x}=\mathbf{V}\mathbf{x}$$ which satisfies $$<\mathbf{z}\mathbf{z}^T>=\mathbf{I}$$

This is the PCA-based MIA. Matrix z is just the temporal patterns of MIA modes.

- Jointly diagonalize the un-equal time covariance matrices of matrix z of selected time-lag constants.

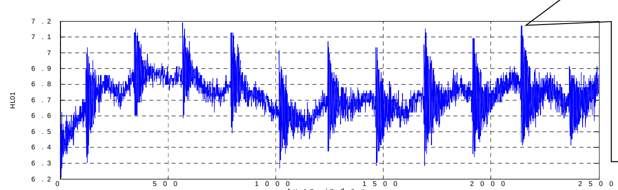
$$\mathbf{C}_z(\tau)=\mathbf{W}\mathbf{C}_s(\tau)\mathbf{W}^T \quad \text{for} \quad \tau=\{\tau_i\,|\,i=1,2,\cdots,k\}$$

Then
$$\mathbf{s}=\mathbf{W}^T\mathbf{V}\mathbf{x} \quad \text{and} \quad \mathbf{A}=(\mathbf{U}_1\mathbf{D}_1^{\frac{1}{2}})\mathbf{W}$$

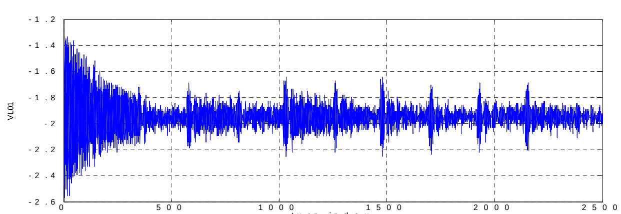
The columns of A (spatial vectors) and corresponding rows (temporal vectors) of s are the resulting modes.

* The second order blind identification (SOBI) algorithm of A. Belouchrani, et al.

Measurements of turn-by-turn data have traditionally being used to measure and model accelerators. Employing the independent component analysis (ICA), we were able to measure the betatron and synchrotron tunes, betatron amplitude functions, dispersion functions for the entire ramping cycle (submitted for publication).



The amplitude of oscillation is about 0.4 mm; Notice the bursts due to the pinger, which is fired about every 225 turns.



An impression of raw data; 2500 turns from turn 1; at location L01