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4.1.5: For an isomagnetic machine, the energy loss per revolution is

$$U_0 = C_\gamma \beta^3 E_0^4 R \left\langle \frac{1}{\rho^2} \right\rangle = C_\gamma E_0^4 R \left\langle \frac{1}{\rho^2} \right\rangle = C_\gamma E_0^4/\rho,$$

where $C_\gamma = 8.85 \times 10^{-5}$ m/(GeV)^3.

For LEP, the bending radius is $\rho = 3096.2$ m. we find that the dipole field at 55 GeV is 0.05925 T, and the energy loss per revolution is $U_0 = 0.2615$ GeV. At 100 GeV, the magnetic field is $B = 0.1077$ T, and the energy loss per revolution is 2.86 GeV, i.e. about 3%.

If the magnetic field for LEP at 55 GeV were 0.5 T, then bending radius would have been 366.92 m, and the energy loss per revolution would be 2.21 GeV. If the LEP were to operate at 100 GeV with a bending radius of 366.92 m, the energy loss would be 24.1 GeV. This is the reason why the circumference of LEP is so large for minimizing the energy loss due to the synchrotron radiation.

At $\rho = 3094.2$ m, the energy loss per turn is equal to its E, i.e.

$$E = 8.85 \times 10^{-5} E^4/\rho.$$

If the beam energy is $E = 327$ GeV, particle will radiate all its energy in one revolution.

4.3.1: The dispersion function, in thin lens approximation, at the cener of the dipole is

$$\begin{pmatrix} D \\ D' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2}L & L\theta/8 \\ 0 & 1 & \theta/2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ -\frac{1}{2f} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} D_{\rm F} \\ 0 \\ 1 \end{pmatrix},$$

where L and θ are the length and the bending angle of the dipole in the half cell, f is the focal length of the quadrupole, and $D_{\rm F}$ is the dispersion function at the center of the quadrupole.

1. Thus the dispersion function at the center of the dipole is

$$D = \frac{L\theta(1 - \frac{1}{8}\sin^2\frac{\Phi}{2})}{\sin^2(\Phi/2)}, \quad D' = -\frac{\theta}{\sin(\Phi/2)},$$

   where Φ is the phase advance per cell and L is the half cell length.

2. The betatron amplitude function at the midpoint is

$$\beta = \frac{L}{\sin\Phi}\left(2 - \sin^2\frac{\Phi}{2}\right), \qquad \alpha = \frac{1}{\sin\frac{\Phi}{2}}, \qquad \gamma = \frac{2\sin\frac{\Phi}{2}}{L\cos\frac{\Phi}{2}}.$$

   Thus the $\mathcal{H}$-function at the center of the dipole is

$$\mathcal{H} = \frac{\rho\theta^3}{\sin^3(\Phi/2)\cos(\Phi/2)}(1 - \frac{3}{4}\sin^2\frac{\Phi}{2} + \frac{1}{32}\sin^4\frac{\Phi}{2}).$$

3. Since the $\mathcal{H}$-function is invariant in straight section, the average of $\mathcal{H}$-function, with Simpson's rule, is

$$\langle \mathcal{H} \rangle = \frac{1}{6}(\mathcal{H}_{\rm F} + 4\mathcal{H}_{\rm midpoint} + \mathcal{H}_{\rm D}) = \left[\frac{1 - \frac{3}{4}\sin^2(\Phi/2) + \frac{1}{48}\sin^4(\Phi/2)}{\sin^3(\Phi/2)\cos(\Phi/2)} \right] \rho\theta^3 = \mathcal{F}\rho\theta^3.$$

The number in brackets is the $\mathcal{F}$ factor, where the numerator depends slightly on the dipole configuration. The $\mathcal{F}$ vs the phase advance of the FODO cell is shown in Fig. 4.12.