3.3.3:

From Exercise 2.4.8, we find that the change of orbit length due to a modulating dipole kicker is given by

$$\Delta C = D(s_0)\,\theta(t) = D(s_0)\,\hat{\theta}\,\sin(\omega_{\rm m}t + \chi_0),$$

where $D(s_0)$ is the dispersion function at the dipole location, $\hat{\theta}$ is the maximum dipole kick angle, $\omega_{\rm m}$ is the modulating angular frequency, and χ_0 is an arbitrary initial phase. The modulating tune is $\nu_{\rm m} = \omega_{\rm m}/\omega_0$, where ω_0 is the angular revolution frequency.

1. The modulating dipole field produces an equivalent rf phase error

   $$\Delta\phi = h\Delta\theta = \frac{h}{R}\Delta C = \frac{2\pi h D(s_0)\hat{\theta}}{C}\sin(\omega_{\rm m}t + \chi_0) = \hat{\Delta\phi}\sin(\omega_{\rm m}t + \chi_0),$$

   where C is the circumference of the synchrotron, and h is the harmonic number.

2. Following the derivation of Chap. 3, Sec. III.4, the amplitude of the equivalent rf wave phase error is

   $$a = \hat{\Delta\phi}/2\pi\nu_{\rm m}.$$

   The amplitude of the equivalent rf wave phase error a is amplified as the modulation tune $\nu_{\rm m}$ becomes smaller.

3. Evaluate the effective rf modulation amplitude a for the accelerators listed in the table below, where C is the circumference, $\Delta B\ell$ is the integrated dipole field error, $f_{\rm mod}$ is the modulation frequency, D is the dispersion function at the dipole, γ is the Lorentz relativistic factor, and h is the harmonic number.

	IUCF Cooler	RHIC	MI	Recycler
C (m)	86.8	3833.8	3319.4	3319.4
$\Delta B\ell$ (Gm)	1	1	1	1
$f_{\rm mod}$ (Hz)	262	60	60	4
D (m)	4	1	1	1
γ	1.04796	24	21.8	9.5
h	1	342	588	1
a	1.85E-02	1.55E-04	3.91E-04	2.30E-05

3.5.2: For small amplitude synchrotron motion, the phase-space coordinates are (see Sec. II.3, Chap. 3)

$$\varphi = \hat{\phi}\cos(\omega_{\rm s}t + \chi), \quad \delta = -\frac{\nu_{\rm s}}{h|\eta|}\hat{\phi}\sin(\omega_{\rm s}t + \chi).$$

The phase-space ellipse of a particle becomes

$$\left(\frac{\delta^2}{\hat{\delta}}\right)^2+\left(\frac{\varphi}{\hat{\phi}}\right)^2=1,\qquad \frac{\hat{\delta}}{\hat{\phi}}=\left(\frac{eV|\cos\phi_{\rm s}|}{2\pi\beta^2Eh|\eta|}\right)^{1/2}=\frac{\nu_{\rm s}}{h|\eta|},$$

where $\hat{\delta}$ and $\hat{\phi}$ are maximum amplitudes of the phase-space ellipse. The phase-space area of the ellipse is $\pi\hat{\delta}\hat{\phi}$. Similarly, for a single bunch, we have

$$\left(\frac{\delta^2}{\hat{\delta}}\right)^2+\left(\frac{\varphi}{\hat{\theta}}\right)^2=1,\qquad \frac{\hat{\delta}}{\hat{\theta}}=\frac{\nu_{\rm s}}{|\eta|},$$

where $\hat{\delta}$ and $\hat{\theta}$ are maximum amplitudes of the phase-space ellipse. The phase-space area of the ellipse of a single bunch is $\pi\hat{\delta}\hat{\theta}$.

In the bunch rotation manipulation, the evolution of the beam bunch is

$$(V_1,\delta_0,\theta_0)\quad\xrightarrow{\text{adiabatic}}(V_2,\delta_1,\theta_1)\quad\xrightarrow{\text{non}-\text{adiabatic}}(V_1,\delta_1,\theta_1)\quad\xrightarrow{\text{rotate}}(V_1,\delta_2,\theta_2)$$

The phase-space area enclosed by the ellipse is invariant in linear synchrotron motion, i.e.

$$\pi\delta_0\theta_0=\pi\delta_1\theta_1$$

From the relations

$$\delta_0=\frac{\nu_{s2}}{\eta}\theta_0\qquad \delta_1=\frac{\nu_{s2}}{\eta}\theta_1$$

we get $\theta_1^2\nu_{s2}=\theta_0^2\nu_{s1}$. From (3.57), the rotation in the last step has the relations

$$\delta_2=\frac{\nu_{s1}}{\eta}\theta_1\qquad \theta_2=\frac{\eta}{\nu_{s1}}\delta_1$$

The final bunch length is

$$\theta_2=\frac{\eta}{\nu_{s1}}\delta_1=\frac{\nu_{s2}}{\nu_{s1}}\theta_1=\left(\frac{\nu_{s2}}{\nu_{s1}}\right)^{\frac{1}{2}}\theta_0=\left(\frac{V_2}{V_1}\right)^{\frac{1}{4}}\theta_0$$

From (3.57) and above, we find

$$\omega_0=0.5679\text{ MHz},\quad \theta_2=8.518\times10^{-5},\quad \theta_0=2.487\times10^{-4},\quad V_2=469.2\text{ (keV)}$$

The antiproton bunch has 0.15 ns bunch length. If the energy spread is $\Delta E=E_{\rm f}\cdot(\pm3\%)=0.534$ (GeV), the phase-space area of the antiprotons is 0.08 eV-s. Since the antiproton bunch has $6\times10^{10}\times10^{-5}=6\times10^5$ particles, the phase-space density is 7.5×10^6/(eV-s).

3.5.3: Formulas used in the calculation

$$\sigma_E^2=E^2\frac{C_q\gamma^2}{2\rho},\quad \hat{\delta}=\frac{1}{\beta^2E}\sigma_E,\quad \hat{\theta}=\left(\frac{\eta}{\nu_{\rm s}}\right)\hat{\delta},\quad \mathcal{A}=\pi\sigma_E\frac{\hat{\theta}}{\omega_0}=\pi\sigma_E^2\left(\frac{\eta}{\omega_0\beta^2E\nu_{\rm s}}\right),$$

where $C_q=3.83\times10^{-13}$ m.

	LEP	ALS	APS	NLC DR	BEPC	TRISTAN
C (m)	26658.9	196.8	1060	223	240.4	3018
Energy (GeV)	50	1.2	7.0	1.98	2.2	30.
ρ (m)	3096.2	4.01	38.96	4.35	10.35	246.5
V_{rf} (MV)	400	1.5	10	1.0	0.8	0.4
h	31320	328	1248	531	160	5120
γ_T	50.86	26.44	64.91	46.1	5.0	25.5
ν_x	76.2	14.28	35.22	23.81	6.18	36.8
ν_z	70.2	8.18	14.3	8.62	7.12	38.7
ϕ_{s}	26.5	1.75	33.0	18.0	14.0	46.6
Q_{s}	0.12	0.0097	0.0075	0.0044	0.019	0.107
$\sigma_\delta(10^{-4})$	7.7	5.13	9.61	8.13	5.86	1.75
σ_t (ps)	35.9	7.94	17.1	10.4	158.	34.4
$\mathcal{A}(10^{-4}$ eV-s)	43.4	0.154	3.61	0.525	6.39	0.605