**USPAS UNM HW04 06/18/2009**

**2.5.1:** For a FODO cell, we have

$$\beta_{\rm F} = \frac{2L(1+\sin\frac{\Phi}{2})}{\sin\Phi} \qquad \beta_{\rm D} = \frac{2L(1-\sin\frac{\Phi}{2})}{\sin\Phi} \qquad \sin\frac{\Phi}{2} = \frac{L}{2f}.$$

Thus the chromaticities in thin lens approximation are

$$C_x \approx -\frac{N_{\rm cell}}{4\pi}\left[\frac{\beta_{\rm F}}{f} - \frac{\beta_{\rm D}}{f}\right] = -\frac{N_{\rm cell}}{4\pi}\frac{\sin\frac{\Phi}{2}}{\sin\Phi}\frac{4L}{f} = -\frac{N_{\rm cell}}{\pi}\tan\frac{\Phi}{2} = -\frac{\tan(\Phi/2)}{(\Phi/2)}\nu,$$

where $\nu = N\Phi/2\pi$ is the betatron tune, $\Phi$ is the phase advance per cell, and $N_{\rm cell}$ is the number of FODO cells.

**2.5.3:** Including sextupoles, the chromaticities are

$$C_x = -\frac{1}{4\pi}\oint \beta_x[K_x(s) - S(s)D(s)]ds = -\frac{1}{4\pi}\left[\beta_{\rm xF}\left(\frac{1}{f} - S_{\rm F}D_{\rm F}\right) + \beta_{\rm xD}\left(-\frac{1}{f} - S_{\rm D}D_{\rm D}\right)\right],$$

$$C_z = -\frac{1}{4\pi}\oint \beta_x[K_z(s) - S(s)D(s)]ds = -\frac{1}{4\pi}\left[\beta_{\rm zF}\left(-\frac{1}{f} + S_{\rm F}D_{\rm F}\right) + \beta_{\rm zD}\left(\frac{1}{f} + S_{\rm D}D_{\rm D}\right)\right],$$

Setting $C_x = 0$ and $C_z = 0$, we obtain

$$S_{\rm F} = \frac{1}{D_{\rm F}f} = \frac{1}{2f^2\theta}\frac{\sin\frac{\Phi}{2}}{1+\frac{1}{2}\sin\frac{\Phi}{2}}, \quad S_{\rm D} = -\frac{1}{D_{\rm D}f} = -\frac{1}{2f^2\theta}\frac{\sin\frac{\Phi}{2}}{1-\frac{1}{2}\sin\frac{\Phi}{2}}.$$

**2.6.2:** See Exercise 2.1.4

1. Using $y = x + jz$, the equation of particle motion in solenoid becomes

$$y'' - 2jgy' - jg'y = 0.$$

2. Define the particle coordinate in the rotating frame $\bar{y} = ye^{-j\theta(s)}$, where $\theta = \int_0^s g ds$, we find $\bar{y}'' + g^2\bar{y} = 0$.

3. The solutions of the above equation in the rotating frame are

$$(\bar{x} + j\bar{z}) = (\bar{x}_0 + j\bar{z}_0)\cos gs + (\bar{x}'_0 + j\bar{z}'_0)(1/g)\sin gs$$
$$(\bar{x}' + j\bar{z}') = -(\bar{x}_0 + j\bar{z}_0)g\sin gs + (\bar{x}'_0 + j\bar{z}'_0)\cos gs.$$

Then one can get $\bar{M}$ as shown in the problem where

$$(\,\bar{x} \quad \bar{x}' \quad \bar{z} \quad \bar{z}'\,)^\dagger = \bar{M}\,(\,\bar{x}_0 \quad \bar{x}'_0 \quad \bar{z}_0 \quad \bar{z}'_0\,)^\dagger$$

4. Transforming back to the original frame with $y = \bar{y}e^{i\theta}$, $y_0 = \bar{y}_0$, we define

$$\vec{\mathbf{r}} = (\, x \quad x' \quad z \quad z' \,)^{\dagger} \qquad \vec{\bar{\mathbf{r}}} = (\, \bar{x} \quad \bar{x}' \quad \bar{z} \quad \bar{z}' \,)^{\dagger}$$

and obtain

$$\vec{\bar{\mathbf{r}}} = \bar{M}\vec{\bar{\mathbf{r}}}_0 \qquad \vec{\mathbf{r}} = M\vec{\mathbf{r}}_0 \qquad \text{with} \qquad \vec{\bar{\mathbf{r}}}_0 = \vec{\mathbf{r}}_0,$$

i.e.

$$(x + jz) = (\bar{x} + j\bar{z})(\cos\theta + j\sin\theta) = (\bar{x}\cos\theta - \bar{z}\sin\theta) + j(\bar{x}\sin\theta + \bar{z}\cos\theta)$$

$$\vec{\mathbf{r}} = \begin{pmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & \cos\theta & 0 & -\sin\theta \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & \sin\theta & 0 & \cos\theta \end{pmatrix} \vec{\bar{\mathbf{r}}} = \bar{R}(\theta)\vec{\bar{\mathbf{r}}}.$$

So we get the intended result: $\vec{\mathbf{r}} = \bar{R}(\theta)\vec{\bar{\mathbf{r}}} = \bar{R}(\theta)\bar{M}\vec{\bar{\mathbf{r}}}_0 = \bar{R}(\theta)\bar{M}\vec{\mathbf{r}}_0$.