LATTICE MODELING AND APPLICATION OF INDEPENDENT COMPONENT ANALYSIS TO HIGH POWER, LONG BUNCH BEAMS IN THE LOS ALAMOS PROTON STORAGE RING

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Michael Snow, Ph.D.
To my loving parents.
Acknowledgments

As I finalize the revisions for my thesis, I am compelled to take a momentum to reflect on how I got to where I am. I have so many people to thank for their mentorship and guidance. I am just a small graduate student standing on the shoulders of very large giants.

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Lattice Modeling and Application of Independent Component Analysis to High Power, Long Bunch Beams in the Los Alamos Proton Storage Ring

The linear lattice properties of the Proton Storage Ring (PSR) at the Los Alamos Neutron Science Center (LANSCE) in Los Alamos, NM were measured and applied to determine a better linear accelerator model. We found that the initial model was deficient in predicting the vertical focusing strength. The additional vertical focusing was located through fundamental understanding of experiment and statistically rigorous analysis. An improved model was constructed and compared against the initial model and measurement at operation set points and set points far away from nominal and was shown to indeed be an enhanced model.

Independent component analysis (ICA) is a tool for data mining in many fields of science. Traditionally, ICA is applied to turn-by-turn beam position data as a means to measure the lattice functions of the real machine. Due to the diagnostic setup for the PSR, this method is not applicable. A new application method for ICA is derived, ICA applied along the length of the bunch. The ICA modes represent motions within the beam pulse. Several of the dominate ICA modes are experimentally identified.
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Chapter 1

Introduction

This thesis is the story of two experimental efforts conducted at the Proton Storage Ring (PSR) at the Los Alamos Neutron Science Center (LANSCE). The first of these narratives surrounds the efforts to obtain an improved linear model of the PSR. Accelerator models are very important for experimental studies. Models not only allow for predictive studies of the real accelerator, but also may be employed in simulations such as particle tracking. Models are also consulted during operations of the accelerator, most specifically for turn on and tuning procedures. Several measurements and beam manipulations in the PSR yield results not only after consultation of beam position measurements but also with input from a model. Thus, an improved linear model of the PSR affects many aspects from operations, to simulations, to experimental analysis. This first line of study occupies the bulk of this thesis in Chaps. 2 through 4. I report a detailed study of the measurement of the model predicted quantities of interest and the results from various experiment model improvement techniques.

The second part of my story and the material probably most of interest to you lies in Chap. 5. This narrative involves a newer analysis to accelerator physics, although
it has been a standard form of analysis on other fields such as telecommunications for quite some time. This method is called independent component analysis (ICA). ICA has been successfully applied to accelerators in the past[1, 2, 3], but the limitations of the PSR diagnostics force me to applied ICA in a different manner than previous.

Before I discuss further, I should comment on the style and audience of this thesis. I write this thesis for several different reasons and audiences. The first of which is of course as a requirement for graduation. Secondly, I write this thesis as a compilation of my work on the PSR at LANSCE, as a record for my coworkers and colleagues at LANSCE, should the need ever arise to review previous experiments. So, I included excruciating detail in the specifics of accelerator elements so that the facility may benefit from my experiences. Lastly, I write for posterity. I include very detailed analysis, results, and comments as a means of keeping things simple and understandable. Originally, I attempted to write this thesis such that my mother would understand it, but as that goal is mostly likely unobtainable (and definitely no offense to my mother) I have settled in hopes that a second year graduate student with a course at the accelerator school may understand and also learn from my experiences.

With that said, I should explain the overarching organization of this thesis and my story. After this rambling introduction, I begin at the beginning by introducing all of the pertinent fundamental accelerator physics that I will employ during my story, Sec. 1.1. While the introduction to accelerator physics section mimics those presented in Ref. [2] and especially Chap. 2 of Ref. [4], I have included supplemental derivations and left out a majority of the information. I include the introduction to accelerator physics as a reminder or maybe even a first time go through, but mostly for the completeness of my story.

After all of the equations in the introduction to accelerator physics section, I continue in, Sec. 1.2, by introducing the major players in my story: the LANSCE accelerator and the PSR. It is important for you to understand the operating condi-
tions of the PSR during experiments to get a full picture. The introductions finish lastly with the baseline linear model of the PSR in Sec. 1.3. Since the first narrative of this thesis is the metamorphosis of the baseline model to an improved model, it is crucial to understand what is included in the baseline model and what its short falls are. This also serves to motivate my study for an improve model.

I will employ several experimental methods as a means to improve, verify, and support the comparison of the linear model of the PSR. All but one of these experiments rely heavily on the beam position measurement in the PSR. The main diagnostic in the PSR, and the one of choice for these measurements, is the beam position monitor (BPM). Since the whole experimental effort rests on the beam position measurement in the PSR, it is necessary to fully understand this measurement device. Thus in Chap. 2, I diverge from the model improvement story line and recall a tale about the beam position measurement in the PSR. I relay a very fundamental and statistically rigorous study of the BPM measurement at what has become my preferred beam position measurement configuration.

In Chap. 2, I fully describe the beam position measurement, process, and method. I describe the hardware and software manipulations of beam signals and finally the off-line analysis. After this, I delve into the data acquisition errors of the beam position measurement, deriving methods to distinguish these errors from good measurements and postulating their origins for possible system reconfiguration mitigations. Once all of the data acquisition errors have been identified, I study the quality of the off-line analysis in the form of residuals along turn number to gain further insight into the beam position measurements. Now that I have fully explained the measurement, I yield the results of the beam position measurement and proceed to apply the measurement results in further calculations, not unlike what will be done in the experimental methods to improve the PSR model. Lastly, I digress even further to measure the pulse-to-pulse momentum variations of the PSR beam.
Now that the PSR model improvement initiative as been motivated and the measurement tool, the beam position measurement, is understood, it is time to discuss how this whole model improvement venture will be completed. The first goal is to establish an improved model of the PSR. This is done by comparing model predictions of certain quantities with measurement, and if a second model predicts these quantities better than the baseline model, then this second model must be an improved model. I chose to employ the betatron tune, betatron phase, betatron amplitude function, and the dispersion function as the quantities for comparison. The betatron tune and phase measurements were already explained in Chap. 2, so I only need to establish the betatron amplitude function and dispersion function measurement in Chap. 3. Chapter 3 is entitled *Supporting Measurements* because I explain the processes to obtain the quantities that will first serve to establish an improved model and then experimentally verify the improved model at other PSR operation set points. The first of these supporting measurements verifies that hysteresis of the PSR dipoles will not affect any of the model improvement measurements. Next, I compare two different methods to measure the beam momentum in the PSR in relation to the dispersion function and chromaticity measurements. From this comparison, I conclude the better method for use at the PSR. I also introduce a third method for fast momentum measurements using beam position data at a BPM in the high dispersion region of the transport after an initial calibration. The last supporting measurement is that of the betatron amplitude functions. I employ the standard quadrupole perturbation method in this measurement. Again, I cannot resist digressing from the supporting measurements and perform further calculations on the dataset from the betatron amplitude function measurement. As a means to test the gain of each beam position monitor, I apply the measured beta functions and the turn-by-turn beam position measurement to obtain the beam action at each beam position monitor. Lastly, the betatron amplitude function dataset is perfect for beam-based alignment, so I had
to perform that analysis as well as a means to gain further insight into the beam position measurement.

Finally, the endeavor to improve the model of the PSR comes to a happy ending in Chap. 4. In Chap. 4, I introduce the model improvement experiments. These are the experiments that will directly test certain components of the baseline model in order to obtain a better model of the PSR. The first of these model improvement measurements is an orbit response matrix measurement (ORM), Sec. 4.1. The ORM analysis tests the model quadrupoles against those inferred to be in the real machine via changes in the beam closed orbit (CO). The second model improvement measurement is a beam-based characterization of the fringe fields of the PSR extraction septa and is discussed in Sec. 4.2. The septum fringe field characterization is the most complicated experiment in this thesis. Here, I measure the dipole, quadrupole, and sextupole components of the septa fringe fields as observed by the circulating beam so that these fringe fields may be included in the improved model of the PSR. The last model improvement measurement is a ray tracing simulation through the 3D edge and fringe fields of the rectangular PSR bending dipoles, Sec. 4.3. The results of the ray tracing will be applied to constrain the focal length of the edge focusing in the improved model. Lastly, the improved linear model of the PSR is established and experimentally verified as the proper improved model in Sec. 4.4. And thus ends the epic of the improved PSR model. The second story line is much shorter. In Chap. 5, I introduce independent component analysis (ICA). I discuss the method of its analysis and indicate why it is better than other similar analysis methods such as principle component analysis (PCA). Next, I relate how ICA has been applied in the past to accelerators and give a quick example as to why this application is not possible in the PSR, Sec. 5.2. Thus, I motivate applying independent component analysis in a new way at the PSR: along the beam pulse. Because independent component analysis yields modes of motion that must be identified by the physicist, it is important to
simulate data for independent component analysis in order to identify which mode describes which motion. I describe the response of independent component analysis to simulated data and gain some knowledge about the limitations of the analysis in Sec. 5.3.2. Lastly, I conclude this short tale with ICA of real data from the PSR and the identification of several modes, which are experimentally verified.

Of course all of the highlights and important successes are restated with conflict and resolution in Chap. 6. So if the size of this thesis makes you, the reader, faint of heart, you may want to read the conclusions in Chap. 6 in order to select the sections of this thesis, which are of interest to you, to read.

1.1 Introduction to Accelerator Physics

In this section, I introduce all of the basic accelerator physics that you will need to follow and understand the discussions presented in this thesis. I present a thorough introduction for those readers not familiar with accelerator physics. Many discussions in this thesis reference equations in this section, especially at the beginning of derivations. So keep this section handy as a reference guide to the relevant physics.

1.1.1 Circular and linear accelerators

There are two types of accelerators: linear accelerators and circular accelerators. Linear accelerators (linac’s), as the names suggests, are a straight line of accelerator components. The beam only makes one pass through each element. Thus, the lattice functions and transport are not restricted to periodic solutions. This greatly reduces the number of resonances that can be encountered in operating a linac.

However, the Los Alamos Proton Storage Ring (PSR) is a circular machine operating with protons. Thus, the discussion in this introduction will be limited to
circular accelerators and positively single charged particles.

1.1.2 Curvilinear coordinate system

In a circular accelerator there is a theoretical trajectory around the ring that receives the design bend in each dipole and passes through the center of every element, but most importantly through the center of the magnetic quadrupole focusing elements. This theoretical trajectory is called the reference orbit.

Circular accelerators range in size from a few meters in circumference to 20 m circumferences such as the ALPHA machine at IUCEEM[5]; the PSR is a 90 m machine, and the extreme case is the LHC with a 27 km circumference. One might suspect that the obvious cylindrical coordinate system with longitudinal axis located at the center of the ring and normal to the plane of the reference trajectory might be employed. But this cylindrical coordinate system is not the preferred coordinate system because the radius from the origin to the reference orbit is often three to six orders of magnitude larger than the beam motion about the reference trajectory. Choosing a coordinate system that enables the physics to be written without this large radial term is necessary. The popular choice is to work in the Frenet-Serret coordinate system[4].

The Frenet-Serret coordinate system is a curvilinear coordinate system, Fig. 1.1. The curvilinear part of the coordinate system is in the longitudinal (s-axis) which travels along the reference trajectory of the accelerator. The positive horizontal axis (x-axis) is perpendicular to the longitudinal and points out. The positive vertical axis (y-axis) is perpendicular to both \( \hat{x} \) and \( \hat{s} \) and points up. So the Frenet-Serret coordinate system is a right handed coordinate system in \( (\hat{x}, \hat{s}, \hat{y}) \) space with \( \hat{x} \times \hat{s} = \hat{y} \).

The horizontal position in the Frenet-Serret coordinate system may be written in
1.1 Introduction to Accelerator Physics

terms of the ring centered cylindrical coordinate system,

\[ x(s) = r(s) - R_0(s), \]  

(1.1)

assuming that the vertical position does not contribute to the length of \( r \), which is true for all but the smallest circular accelerators because the radius is on the order of 10's of meters to kilometers and the vertical position is on the order of a few millimeters. Here \( R_0(s) \) represents the translation of the coordinate system from an origin at the center of the ring to the reference trajectory. However, sometimes it is useful to approximate the horizontal beam orbit \( x(s) \) as a circular orbit with a radius equal to the average of \( R_0(s) \) (\( \langle R(s) \rangle \)) or even \( \langle x(s) \rangle + \langle R(s) \rangle \) where \( \langle \ldots \rangle \) indicates the average value.

1.1.3 Magnetic field expansion

In the straight sections of the reference trajectory, where the Frenet-Serret coordinate system is not curvilinear because the reference trajectory is straight, an arbitrary transverse magnet field may be diagonalized in the Frenet-Serret coordinate system by rewriting the magnetic field as a sum of magnetic multipole moments. Each magnetic
1.1 Introduction to Accelerator Physics

Multipole moment is a term in the Taylor series expansion of the magnetic field around the reference trajectory. Thus, the dipole magnetic multipole is represented by the constant (zeroth derivative) in the Taylor series expansion. The second magnetic multipole moment is a quadrupole magnetic field which describes the slope of the magnetic field, the first derivative. The third term in the Taylor series expansion describes the second derivative of the magnet field at the reference trajectory or a sextupole magnet field, and so on.

If the skew multipole moments are also included in the Taylor series expansion (even though the skew multipoles are not applied in this thesis), one arrives at what is called the Beth representation of magnetic fields[4],

$$B_y + jB_x = B_0 \sum_{n=0}^{\infty} (b_n + ja_n)(x + jy)^n$$  \hspace{1cm} (1.2)

where $B_y$ and $B_x$ and the vertical and horizontal components of the magnetic field respectively, $j = -\sqrt{-1}$, and

$$a_n = \frac{1}{B_0 n!} \frac{\partial^n B_x}{\partial x^n} \bigg|_{x=y=0} \quad \text{and} \quad b_n = \frac{1}{B_0 n!} \frac{\partial^n B_y}{\partial x^n} \bigg|_{x=y=0}$$  \hspace{1cm} (1.3)

are respectively the skew and normal multipole coefficients. The derivatives of the magnetic fields in Eq. (1.3) are evaluated on the reference orbit. The normalization is chosen such that $B_0$ is equal to the main dipole strength so $b_0 = 1$. It is also a convenient shorthand to write the $n^{th}$ derivative of the magnetic field with respect to horizontal as $\frac{\partial^n B_y}{\partial x^n} \bigg|_{x=y=0} = B_n$.

From the Beth representation of the magnetic fields, the expansion of the normal horizontal and vertical magnetic fields to third order is

$$B_y = -B_0 + B_1 x + \frac{1}{2} B_2 (x^2 - z^2) \quad \text{and} \quad B_x = B_1 y + B_2 xy.$$  \hspace{1cm} (1.4)

The first three components of the magnetic field expansion are also called the dipole, quadrupole, and sextupole multipoles respectively.
The negative sign in front of $B_0$ in Eq. (1.4) is by convention. In the Frenet-Serret coordinate system plotted in Fig. 1.1, the positive horizontal axis points out, so it is necessary to have a bend in the negative horizontal direction to complete the orbit. Since the beam travels in the positive longitudinal direction, from the right hand rule, a negative vertical magnetic field is needed to bend the beam in the negative horizontal direction. Thus the vector part of the Lorentz force law is $\hat{s} \times -\hat{y} = -\hat{x}$.

There is no dipole term in the expansion of the horizontal magnetic fields because horizontal dipole magnetic field is considered a skew magnetic multipole. The Frenet-Serret coordinate system assumes that the reference trajectory is contained in a horizontal plane with no vertical bending. However, horizontal dipole magnetic fields are employed to steer the beam vertically. These are considered as horizontal dipole field errors and will be discussed in Sec. 1.1.9.

### 1.1.4 Momentum rigidity

When a singularly charged particle with momentum $p$ is immersed in a constant magnetic field (dipole field) of strength $B$ perpendicular to the particles direction of motion, the particle will travel in a circular orbit of radius $\rho$,

$$\rho = \frac{p}{Be}, \quad (1.5)$$

where $e$ is the positive charge of the electron. Rewriting Eq. (1.5) such that particle parameters are on the right-hand side of the equation and the machine parameters on the left-hand side yields,

$$B\rho = \frac{p}{e}. \quad (1.6)$$

Thus given any beam with momentum $p$ and charge $e$, there is a constant inverse relationship between the magnetic field strength and the bending radius. The $B\rho$ is called the momentum rigidity and or the magnetic rigidity. The magnetic rigidity
1.1 Introduction to Accelerator Physics

is typically reported in units of [Tm]. The $B\rho$ describes how hard it is to bend a beam. Beams with more momentum and less charge require stronger magnetic fields or larger bending radii. Ultimately, the angle bent in a circular accelerator must be $360^\circ$. It is this fact alone that dictates the circumference of a circular machine after the design particle momentum and bending dipole magnetic field strength have been chosen.

Because of the magnetic rigidity, the beam’s response to a magnetic field depends on both the strength of the field and the beam momentum. Thus, it is convenient to normalize the magnetic fields to the momentum rigidity. The shorthand applied to the normalized magnetic field components is $\frac{1}{B\rho} \frac{\partial B}{\partial s} = B_n = K_n$.

The design magnetic rigidity for the PSR is 4.8671 Tm.

### 1.1.5 Transverse equations of motion

The transverse motion of the beam is dictated by the magnetic field lattice of the accelerator. Dipole magnetic fields are needed to bend and steer the beam, while quadrupole magnetic fields are necessary to focus and confine the beam. Thus, the transverse equations of motion (EOM), which may be derived by either the general Hamiltonian for a charged particle in an electro-magnetic field or by the Lorentz force law resulting in Hill’s equation, are

\[
x''(s) - \frac{\rho(s) + x(s)}{\rho^2(s)} = \frac{B_d(s)}{B\rho} \frac{p_0}{p} \left( 1 + \frac{x(s)}{\rho(s)} \right)^2,
\]

\[
y''(s) = -\frac{B_x(s)}{B\rho} \frac{p_0}{p} \left( 1 + \frac{x(s)}{\rho(s)} \right)^2,
\]

where the derivatives are with respect to the longitudinal coordinate ($s$), $p_0$ is the design momentum of the accelerator, and $p$ is the beam or particle momentum.

The magnetic fields in Eqs. (1.7) and (1.8) may be expanded in Beth representation to the linear order in position, Eq. (1.4). If errors in the dipole and quadrupole
magnetic fields are also considered, the Beth expansion of the normal multipoles is

$$B_y = -B_{0_y} + \Delta B_{0_y} + B_1 x + \Delta B_1 x \quad \text{and} \quad B_x = -\Delta B_{0_x} + B_1 y + \Delta B_1 y,$$  

(1.9)

where $\Delta B_0$ and $\Delta B_1$ are dipole and quadrupole field errors respectively. Magnet field errors occur when the real magnet strengths deviate from the design values whether through power supply ripple/drift, incorrect set points operation, or magnet roll. Magnetic field errors are assumed to be small and treated perturbatively. The sign convention on the dipole field errors is such that positive errors produce kicks in the positive coordinate direction. From the right hand rule, a positive vertical dipole magnetic field is required to kick the beam in the positive horizontal direction, while a negative horizontal field is needed to kick the beam up in the positive vertical direction.

It is also convenient to describe the momentum difference between the beam and machine design as a ratio. The fractional momentum deviation between the beam momentum and design momentum is defined as,

$$\delta = \frac{p - p_0}{p_0} = \frac{\Delta p}{p_0}. \quad \text{(1.10)}$$

The fractional momentum deviation is small in most accelerators. The typical fractional momentum deviation for the beam injected into the PSR is a few $\times 10^{-4}$[6].

Introducing $\delta$, substituting the expansions for the magnetic fields into Eqs. (1.7) and (1.8), and keeping only linear terms in the position yields the following transverse EOM,

$$x''(s) + \left[ \frac{1 - 2\delta}{\rho^2(s)} - K_1(s) + K_1(s)\delta - \Delta K_1(s) \right] x(s) = \frac{\Delta B_{0_y}(s)}{B\rho} + \frac{\delta}{\rho(s)} \quad \text{(1.11)}$$

$$y''(s) + [K_1(s) - K_1(s)\delta + \Delta K_1(s)] y = -\frac{\Delta B_{0_x}(s)}{B\rho} \quad \text{(1.12)}$$

where, since the magnetic field errors are assumed to be perturbative, a magnetic field error multiplied the fractional momentum deviation or divided by the bending
radius square is considered higher order. So apparently, the beam undergoes pseudo-harmonic oscillations in both transverse dimensions.

The terms in the brackets of Eqs. (1.11) and (1.12) represent the focusing functions sometimes written as $K_x(s)$ and $K_y(s)$ or just $K(s)$. It is interesting to note that there is a horizontal focusing term dependent on the bending radius. This is a consequence of the curvilinear Frenet-Serret coordinate system. The $K_1$ term represents the linear focusing from the quadrupole fields. Any deviation from the design quadrupole field values will introduce a quadrupole field error in the lattice and is portrayed by the $\Delta K_1$ term. When the beam momentum deviates from the design momentum ($\delta \neq 0$), the $\frac{2\Delta}{p^2}$ term contributes the affect of different bending radii and thus different focusing around a bend, while the $K_1\delta$ term describes chromatic affects: particles with more momentum are more rigid and are focused less.

Lastly, there are two different types of driving terms in the harmonic oscillation due to the particular solutions. The first driving term is due to a dipole field error, $\Delta B_0$, which could result from beam steering or a dipole strength that deviates from design. The second driving term describes the change in offset motion as a function of momentum. This is known has dispersion.

While many orbits may satisfy the EOM, Eqs. (1.11) and (1.12), there is only one that repeats every revolution. This trajectory is called the closed orbit (CO) because unlike all other particle orbits, the CO closes on itself each time around the accelerator.

The solution of Eqs. (1.11) and (1.12) may be written as a sum of the homogeneous solution and the particular solutions,

$$x(s) = x_\beta(s) + x_{\Delta B_0}(s) + x_\delta(s), \quad (1.13)$$
$$y(s) = y_\beta(s) + y_{\Delta B_0}(s). \quad (1.14)$$

The particular solutions ($x_{\Delta B_0}(s)$ and $x_\delta(s)$) describe changes in the CO through
steering and dispersion, while the homogeneous solution solves for the oscillatory motion of the beam about the CO, $x_\beta(s)$. This pseudo-harmonic motion about the CO is called betatron motion because it was first discovered in the betatron accelerator.

The solutions to the EOM are presented in the following subsections. First, the on-momentum homogeneous equations of motion with no magnetic field errors are solved, Sec. 1.1.6. Then, additional terms are added to the focusing function of the homogeneous equations and solved: quadrupole field errors in Sec. 1.1.7 and chromatic affects in Sec. 1.1.8. Lastly, the particular solutions to the equations of motion from the driving terms due to a dipole field error and dispersion are presented in Secs. 1.1.9 and 1.1.10 respectively.

### 1.1.6 Betatron motion

Betatron motion describes the pseudo-harmonic transverse oscillations about the CO. Like a pendulum, the beam will oscillate about the minimum of the potential well. The transverse steepness of the well is dictated by the local magnetic fields, and the longitudinal shape of the potential well is determined by the magnetic lattice structure of the accelerator. The CO is the path that always lies at the minimum of the potential.

A beam with initial conditions not on the CO, will oscillate about the CO as it moves longitudinally about the accelerator. However, because the magnetic fields are not constant along the longitudinal, the transverse width of the potential well varies as a function of longitudinal position. This means that the amplitude of betatron oscillation also depends on longitudinal position.

The on-momentum homogeneous equations of motion governing the betatron os-
cillation are

\[ x''_\beta(s) + K_x(s)x_\beta(s) = 0, \quad \text{where} \quad K_x(s) = \frac{1}{\rho^2} - K_1(s), \quad (1.15) \]

and \[ y''_\beta(s) + K_y(s)y_\beta(s) = 0, \quad \text{where} \quad K_y(s) = K_1(s), \quad (1.16) \]

where \( K_x \) and \( K_y \) are called the horizontal and vertical focusing functions respectively.

The focusing functions describe the focusing properties of the magnetic fields, the width and steepness of the potential, everywhere in the accelerator. Positive focusing functions indicate beam focusing.

The EOM, Eqs. (1.15) and (1.16), can be solved separately for each individual accelerator element or solved continuously. This discussion will first examine the piecewise solutions to the EOM (Sec. 1.1.6 I) and then discuss the continuous solutions of betatron motion, Sec. 1.1.6 II.

I Transfer matrices

The solutions to the betatron EOM are easy to solve for constant focusing functions. In the hard edge approximation, each element in the accelerator may be approximated to possess a constant focusing function. This can be thought of as solving the EOM piecewise as the focusing function may be approximated as being stepwise constant. The solutions for the betatron motion are

\[ x_\beta = \begin{cases} 
  as + b & \text{when } K(s) = 0 \\
  a \cos(\sqrt{|K|}s + b) & \text{when } K(s) \neq 0
\end{cases} \quad (1.17) \]

where \( K \) is either \( K_x \) or \( K_y \), and \( a \) and \( b \) are constants of integration to be determined by the initial conditions. Note that often the solution for negative \( K \) is written with a cosh because \( \cos(\sqrt{|K|}s + b) = \cosh(\sqrt{|K|}s + b) \) for \( K < 0 \). So, if the focusing function is positive, the beam converges; if the focusing function is zero, the beam diverges as a constant rate; and if the focusing function is negative, the beam exponentially diverges.
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The initial conditions are identified as the position and angle with respect to the longitudinal, $x$ and $x'$ respectively, at the beginning of the accelerator element. It is traditional to represent the propagation of motion through a constant focusing function element as a linear map,

$$\vec{x}(s_1) = M(s_1|s_0)\vec{x}(s_0), \quad (1.18)$$

where $M(s_1|s_0)$ is the transfer matrix for an accelerator element of length $L = s_1 - s_0$ employed to transport the initial phase space coordinates from the beginning of the element at location $s_0$ to the final phase space coordinates at the end of the element located at $s_1$.

There are four basic accelerator elements whose transfer matrices will be discussed in turn: drift space, quadrupole, sector dipole, and rectangular dipole.

A drift space is a straight section of beam pipe where the focusing function is zero because there is no focusing from magnetic fields and the bending radius is infinite. The phase space angle does not change in a drift space yielding a constant divergence in the position solely due to the initial angle. Effectively, the beam drifts in a drift space. The transfer matrix for a drift space of length $L$ is

$$M_{\text{Drift}} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}. \quad (1.19)$$

A quadrupole is a magnet that produces linear focusing fields but no bending, so $\rho$ is infinite. The focusing in the quadrupole acts on the phase space angle. However in long quadrupoles, the position also changes during the passage through the quadrupole. The transfer matrix for a quadrupole of length $L$ is

$$M_{\text{Quad}} = \begin{pmatrix} \cos(\sqrt{K}L) & \frac{1}{\sqrt{K}} \sin(\sqrt{K}L) \\ -\sqrt{K} \sin(\sqrt{K}L) & \cos(\sqrt{K}L) \end{pmatrix}, \quad (1.20)$$

where $K$ is the focusing function at the quadrupole, either $K_x$ or $K_y$, and a trigonometric function with negative $K$ may be written as the corresponding hyperbolic
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trigonometric function. If $K$ or $L$ are small, Eq. (1.20) may be written in the thin lens approximation,

$$M_{\text{Quad}} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix},$$

where $f = \lim_{L \to 0} \frac{1}{KL}$ is the focal length of the quadrupole. As in optics, a positive focal length is focusing. In the thin lens approximation, the quadrupole only changes the phase space angle via a point kick and does not modify the position.

![Figure 1.2: Pictorial representation of the reference trajectory's passage through a sector dipole](image)

A sector dipole is a bending magnet with longitudinal edges perpendicular to the path of the reference trajectory. A pure sector dipole only bends the beam horizontally. Thus, a sector dipole will focus horizontally because of the $\frac{1}{\rho^2}$ term in the horizontal focusing function due to the curvilinear coordinate system. Particles on the outer horizontal tracks trace through more dipole field and are bent more, while particles on the inner horizontal track experience less integral magnetic field and are bent less. This creates a focusing effect in the horizontal. A sector dipole appears as a drift space for vertical betatron motion due to the absence of gradient fields. The horizontal and vertical transfer matrices for a sector dipole with bend
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angle θ are

\[
\mathbf{M}_{\text{SecDipole}_x} = \begin{pmatrix}
\cos \theta & \rho \sin \theta \\
-\frac{1}{\rho} \sin \theta & \cos \theta 
\end{pmatrix}
\quad \text{and} \quad
\mathbf{M}_{\text{SecDipole}_y} = \begin{pmatrix}
1 & \rho \theta \\
0 & 1
\end{pmatrix}
\]  

(1.22)

where the length of the sector dipole is \( L = \rho \theta \).

The last basic transfer matrix is a rectangular dipole. The longitudinal edges of a rectangular dipole are not perpendicular to the reference trajectory. The reference orbit makes an angle to the normal of the longitudinal edge of the rectangular dipole equal to half the bending angle of the rectangular dipole. The edge angle of the magnet produces both horizontal and vertical magnetic fields which act as quadrupole fields. So, a rectangular dipole can be thought of as a sector dipole with thin lens quadrupoles on either side. Thus, the horizontal transfer matrix for a rectangular dipole may be written in terms of the quadrupole and sector dipole transfer matrices

\[
\mathbf{M}_{\text{RecDipole}_x} = \begin{pmatrix}
1 & 0 \\
\frac{1}{\rho} \tan \frac{\theta}{2} & 1
\end{pmatrix}
\begin{pmatrix}
\cos \theta & \rho \sin \theta \\
-\frac{1}{\rho} \sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
1 & 0 \\
\frac{1}{\rho} \tan \frac{\theta}{2} & 1
\end{pmatrix}
\]

\[
= \begin{pmatrix}
1 & \rho \sin \theta \\
0 & 1
\end{pmatrix}
\]  

(1.23)

The horizontal transfer matrix for a rectangular dipole appears similar to the transfer matrix for a drift space of length \( \rho \sin \theta \). In the transfer matrix equation of Eq. (1.23), the edge focusing of a rectangular dipole exactly cancels the focusing in the horizontal due to the bend and appears as a drift space in terms of the horizontal betatron motion. Physically, this is because the path length through a rectangular dipole, unlike a sector dipole, is independent of horizontal position.

The non-normal angle between the edge of the rectangular dipole magnet and the reference trajectory also acts to focus the vertical betatron motion. The vertical transfer matrix for a rectangular dipole magnet may also be written in terms of
quadrupole and sector dipole transfer matrices:

\[
\mathbf{M}_\text{RecDipole}_y = \begin{pmatrix}
1 & 0 \\
\frac{1}{\rho} \tan \left( \frac{\theta}{2} - \zeta \right) & 1
\end{pmatrix}
\begin{pmatrix}
1 & \rho \theta \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 \\
\frac{1}{\rho} \tan \left( \frac{\theta}{2} - \zeta \right) & 1
\end{pmatrix}
= \begin{pmatrix}
1 - \theta \tan \left( \frac{\theta}{2} - \zeta \right) & \rho \theta \\
\frac{1}{\rho} \tan \left( \frac{\theta}{2} - \zeta \right) \left[ 2 - \tan \left( \frac{\theta}{2} - \zeta \right) \right] & 1 - \theta \tan \left( \frac{\theta}{2} - \zeta \right)
\end{pmatrix}.
\] (1.24)

The edge focusing of the rectangular dipole imposes additional focusing in the vertical betatron motion.

\( \zeta \) is a correction term to the vertical focal length that takes into account the extent of the fringe fields[7] and the additional focusing due to those fringe fields,

\[
\zeta = \frac{q \kappa}{\rho} \sec \left( \frac{\theta}{2} \right) \left[ 1 + \sin^2 \left( \frac{\theta}{2} \right) \right]
\] (1.25)

where \( q \) is the full gap length of the dipole, \( \rho \) is the bending radius of the dipole, and \( \kappa \) is the fringe field integral,

\[
\kappa = \int_{-\infty}^{\infty} \frac{B_0 B_y(s) - B_y^2(s)}{g B_0^2} ds.
\] (1.26)

In Eq. (1.26), \( B_0 \) is the constant dipole field within the magnet, and \( B_y \) is the vertical component of the magnetic field integrated along the reference trajectory. Common
values for the fringe field integral for different end field geometries are posted in Tab. 1.1.

<table>
<thead>
<tr>
<th>Approximate Values for the Fringe Field Integral</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Hard Edge</td>
<td>0</td>
</tr>
<tr>
<td>Linear Field Decay</td>
<td>$\frac{1}{6}$</td>
</tr>
<tr>
<td>Clamped “Rogowski” Fringe Field</td>
<td>.4</td>
</tr>
<tr>
<td>Unclamped “Rogowski” Fringe Field</td>
<td>.7</td>
</tr>
<tr>
<td>“Square-Edged” Non-Saturating Magnet</td>
<td>.45</td>
</tr>
</tbody>
</table>

Table 1.1: Common approximations of the fringe field integral for different magnetic field decay geometries[8].

One particularly common lattice arrangement involves focusing and defocusing quadrupoles separated by a zero focusing elements, dipole or drift space. The focusing, zero focusing, defocusing, zero focusing lattice structure is called a FODO cell. The PSR lattice consists of 10 FODO cells.

It is common to multiply several transfer matrices together to obtain a transfer matrix for the periodicity of the accelerator. At the very least, the periodicity of a circular accelerator is one revolution. Thus, the multiplication of transfer matrices representing each element in the accelerator is called the one-turn transfer matrix. The CO is defined to be an eigenvector of the one-turn matrix with eigenvalue of one.

The one-turn matrix may be written most generally in the Courant-Snyder parameterization,

$$
M = \begin{pmatrix}
\cos \Phi + \alpha \sin \Phi & \beta \sin \Phi \\
-\gamma \sin \Phi & \cos \Phi - \alpha \sin \Phi
\end{pmatrix},
$$

(1.27)

where $\alpha$, $\beta$, and $\gamma$ are the Courant-Snyder parameters. The Courant-Snyder parameters may be propagated from one location in the accelerator to another if the transfer...
matrix is known between the two points,
\[
\begin{pmatrix}
\beta \\
\alpha \\
\gamma
\end{pmatrix}_1 =
\begin{pmatrix}
M_{11}^2 & -2M_{11}M_{12} & M_{12}^2 \\
-M_{11}M_{12} & M_{11}M_{22} + M_{12}M_{21} & -M_{12}M_{22} \\
M_{21}^2 & -2M_{21}M_{22} & M_{22}^2
\end{pmatrix}

\begin{pmatrix}
\beta \\
\alpha \\
\gamma
\end{pmatrix}_0,
\]
(1.28)
where \( M_{ij} \) is the element of \( M(s_1|s_0) \) in the \( i^{th} \) row and \( j^{th} \) column.

II Floquet’s transformation

The equations of betatron motion, Eqs. (1.15) and (1.16), may also be solved continuously. Since the focusing function \( K(s) \) is periodic, the popular method is to invoke Floquet’s theorem with solutions
\[
x_1 = \epsilon w(s)e^{j\phi(s)}, \quad \text{and} \quad x_2 = \epsilon w(s)e^{-j\phi(s)}
\]
(1.29)
where \( \epsilon \) is a constant solved by the initial conditions, \( w \) is the real valued amplitude function, and \( \phi \) is the phase function.

Floquet’s theorem is similar to recognizing that at a particular location in the accelerator, the equations of motion are the same as a harmonic oscillator, whose solutions are known to be a linear combination of a cosine and sine. The changing amplitude of the betatron oscillation along the accelerator is introduced into the solution by \( w \).

Substituting Eq. (1.29) into the betatron EOM (Eqs. (1.15) and (1.16)) yields constraints on both the phase and amplitude functions,
\[
w''(s) + K(s)w(s) - \frac{1}{w^3(s)} = 0
\]
(1.30)
\[
\phi'(s) = \frac{1}{w^2(s)}.
\]
(1.31)
Equations (1.30) and (1.31) are known as the betatron envelope and phase equations respectively.
The continuous solution to the betatron equations of motion may be combined and written in the form of a one-turn transfer matrix. This one-turn transfer matrix derived from the continuous solutions may be related to the Courant-Snyder parameterization of the one-turn matrix, Eq. (1.27). Such comparison results in the following relationships between the Courant-Snyder parameters and the amplitude function:

\[
\beta = w^2, \quad \alpha = -ww' = -\frac{\beta'}{2}, \quad \text{and} \quad \gamma = 1 + \frac{\alpha^2}{\beta}.
\] (1.32)

The Courant-Snyder parameter \( \beta \) is the square of the amplitude function. Thus, the beta function or betatron amplitude function, which is the Courant-Snyder parameter \( \beta \) as a function of \( s \), is related to the amplitude of the betatron oscillation around the accelerator. The alpha function describes the negative of the slope of the beta function with respect to the longitudinal coordinate. Lastly, the gamma function characterizes the divergence of the particle or beam.

From Eq. (1.31) the betatron phase is then defined as

\[
\phi(s) = \int_{s_0}^{s} \frac{ds}{\beta(s)},
\] (1.33)

where \( s_0 \) is the longitudinal position in the accelerator defined as having phase zero. It follows that the betatron phase advance for one turn is

\[
\Phi = \oint \frac{ds}{\beta(s)}.
\] (1.34)

The one-turn phase advance describes the amount of betatron oscillation undergone in one revolution around the machine. It is useful to define what is called the betatron tune, which describes the number of betatron oscillations performed in one turn

\[
\nu = \frac{\Phi}{2\pi} = \frac{1}{2\pi} \oint \frac{ds}{\beta(s)},
\] (1.35)

where \( \nu \) is the betatron tune, and \( \Phi \) is the one-turn betatron phase advance.
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Because the focusing function $K(s)$ is different for both horizontal and vertical EOM, there is a set of Courant-Snyder parameter functions for each dimension. This also means that the horizontal and vertical betatron motions are different yielding different one-turn phase advances and betatron tunes in each direction.

An arbitrary transfer matrix between any two points in the accelerator may be derived by substituting the relationships of the Courant-Snyder parameters with the betatron amplitude and phase functions into the general solutions of the Floquet transformation, Eq. (1.29),

$$
M(s_1|s_0) = \begin{pmatrix}
\sqrt{\beta_1} \cos \mu_{0\to1} + \alpha_0 \sin \mu_{0\to1} & \sqrt{\beta_0 \beta_1} \sin \mu_{0\to1} \\
-\frac{1 + \frac{\alpha_0}{\sqrt{\beta_0 \beta_1}}}{\sqrt{\beta_0 \beta_1}} \sin \mu_{0\to1} + \frac{\alpha_0 - \alpha_1}{\sqrt{\beta_0 \beta_1}} \cos \mu_{0\to1} & \sqrt{\frac{\alpha_0}{\beta_1}} (\cos \mu_{0\to1} - \alpha_1 \sin \mu_{0\to1})
\end{pmatrix},
$$

where the indices 0 and 1 indicate values at the initial and final position position respectively, and $\mu_{0\to1} = \phi(s_1) - \phi(s_0)$ is the betatron phase advance between $s_0$ and $s_1$.

The constant of integration, $\epsilon$, in Eq. (1.29) is a constant of the motion defined by the initial conditions and the Courant-Snyder functions,

$$
\epsilon = 2J = \gamma x^2 + 2\alpha xx' + \beta x'^2
$$

where $J$ is the action.

Finally, pulling everything together, the continuous solution to the equations of motion may be written

$$
x_\beta(s) = \sqrt{2J \beta(s)} \cos(\phi(s) + \phi_0),
$$

where the action and initial phase ($J$ and $\phi_0$) are determined by the initial conditions. The solution may also be written in action-angle variables

$$
x_\beta(s) = \sqrt{2J \beta(s)} \cos(\nu \theta(s) + \phi_0),
$$
where $\theta$ is the angle variable ranging between 0 and $2\pi$ in one revolution. So, the actual amplitude of the betatron oscillation along the accelerator is

$$A(s) = \sqrt{2J\beta(s)}, \quad (1.40)$$

and the frequency of the betatron oscillation is equal to the betatron tune.

### 1.1.7 Consequences of quadrupole field errors

The additional terms in the focusing functions of the EOM, Eqs. (1.11) and (1.12), act to perturb the focusing. The first of these discussed in this thesis is the $\Delta K(s)$ term due to a quadrupole field error.

A quadrupole field error is introduced into the focusing lattice of the accelerator when one of the quadrupole strengths varies from the design value. There are two first order effects of a quadrupole field error: a shift in the betatron tune (discussed in Sec. 1.1.7 I), and a modulation to the betatron amplitude function also known as beta beat[4], Sec. 1.1.7 II. In the following derivations of the tune shift and beta beat due to a quadrupole error, the quadrupole error is assumed to be perturbative.

Assume there is a quadrupole field error located at longitudinal position $s_1$. The quadrupole error adds an additional focusing element to the lattice at $s_1$. In the thin lens approximation or in the limit that the strength of the quadrupole error is small, the quadrupole field error yields a transfer matrix,

$$m(s_1) = \begin{pmatrix} 1 & 0 \\ -\oint k(s_1)ds_1 & 1 \end{pmatrix}, \quad (1.41)$$

where $kds$ is the strength of the perturbing quadrupole field error and the integral is over the circumference of the ring but only non-zero at the location of the quadrupole error ($s_1$) as shown in Fig. 1.4.
I Betatron tune shift due to quadrupole field error

The betatron tune shift can be obtained by calculating the difference in the trace of the perturbed ($\tilde{M}$) and unperturbed ($M$) one-turn transfer matrices. The perturbed one-turn transfer matrix is the one-turn matrix that includes the quadrupole error at $s_1$, while $M$ can be thought of as the design one-turn transfer matrix. $M$ is defined in Eq. (1.27). The perturbed one turn transfer matrix at $s_0$ can be written in terms of the unperturbed one turn transfer matrix and the quadrupole error,

$$\tilde{M}(s_0) = M(s_0|s_1)m(s_1)M(s_1|s_0), \quad (1.42)$$

where $M(s_1|s_0)$ and $M(s_0|s_1)$ are the transfer matrices from longitudinal positions $s_0$ to $s_1$ and $s_1$ to $s_0$ respectively. In this notation, the transfer matrices can only propagate the beam forward, so there is an inherent addition of the circumference length ($C$) when circling the machine: i.e. when starting at $s_0$, $M(s_0|s_1) = M(s_0 + C|s_1)$.

Working through the matrix multiplication in Eq. (1.42) yields the $11$ and $22$ matrix elements of the perturbed transfer matrix. The difference in the $11$ and $22$
matrix elements between the perturbed and unperturbed transfer matrices is

\[ \Delta M_{11} = \tilde{M}_{11} - M_{11} = -\oint k(s_1) \beta(s_1) \sin(\mu_{0\to1})[\cos(\mu_{0\to1}) + \alpha(s_0) \sin(\mu_{0\to1})] ds_1 \]  

(1.43)

\[ \Delta M_{22} = \tilde{M}_{22} - M_{22} = -\oint k(s_1) \beta(s_1) \sin(\mu_{0\to1})[\cos(\mu_{1\to0}) + \alpha(s_0) \sin(\mu_{1\to0})] ds_1, \]  

(1.44)

where \( \mu_{0\to1} = \phi_1 - \phi_0 \) is the positive phase advance from \( s_0 \) to \( s_1 \), \( \mu_{1\to0} = \Phi + \phi_0 - \phi_1 \) is the positive phase advance from \( s_1 \) to \( s_0 \), and \( \phi_0 \) and \( \phi_1 \) are the betatron phases at \( s_0 \) and \( s_1 \) respectively.

The change in the betatron phase advance for one revolution due to a quadrupole field error can be calculated by comparing the traces of the perturbed and unperturbed one-turn transfer matrices,

\[ \text{tr}(\tilde{M}) - \text{tr}(M) = \Delta M_{11} + \Delta M_{22}, \quad \text{or} \]

(1.45)

\[ 2 \cos \tilde{\Phi} - 2 \cos \Phi = -\oint k(s_1) \beta(s_1)[\sin(\mu_{0\to1}) \cos(\mu_{1\to0}) + \sin(\mu_{1\to0}) \cos(\mu_{0\to1})] ds_1, \]  

(1.46)

where \( \tilde{\Phi} \) is the perturbed betatron phase advance for one revolution. Recognizing that the terms in the brackets can be condensed to the unperturbed one-turn phase advance yields

\[ \cos \tilde{\Phi} - \cos \Phi = -\frac{\sin \Phi}{2} \oint \beta(s_1) k(s_1) ds_1. \]  

(1.47)

Since this derivation assumes that \( k(s_1) \) is a perturbative error, it must be that the change in the one-turn phase advance is also small, \( \Delta \Phi = \tilde{\Phi} - \Phi \ll 1 \), and the small angle approximation may be applied. The change in the one revolution betatron phase advance can be written,

\[ \Delta \Phi = \frac{1}{2} \oint \beta(s_1) k(s_1) ds_1. \]  

(1.48)
Remembering the betatron tune is defined in Eq. (1.35), the shift in the betatron tune due to a quadrupole field error is dependent on the unperturbed beta function at the location of the quadrupole error and the strength of the perturbation. The betatron tune shift due to a quadrupole field error is
\[
\Delta \nu = \frac{1}{4\pi} \oint \beta(s_1) k(s_1) ds_1.
\] (1.49)

II Beta function modulation due to quadrupole field error

The beta function modulation induced by a quadrupole error can be found by calculating the difference in the $12$ matrix element of the perturbed and unperturbed one-turn transfer matrices. Working through the matrix multiplication of Eq. (1.42) yields the difference in the $12$ matrix element as,
\[
\tilde{M}_{12} - M_{12} = \Delta M_{12},
\] (1.50)

\[
\tilde{\beta}(s_0) \sin \tilde{\Phi} - \beta(s_0) \sin \Phi = -\beta(s_0) \oint k(s_1) \beta(s_1) \sin \mu_{1\rightarrow 0} \sin \mu_{0\rightarrow 1} ds_1,
\] (1.51)

where $\tilde{\beta} = \beta + \Delta \beta$ is the perturbed beta function and $\Delta \beta$ is assumed to be small because the quadrupole error is perturbative. Rewriting $\tilde{\Phi} = \Phi + \Delta \Phi$, applying the small angle approximation, and solving for the change in the beta function at $s_0$ in terms of the unperturbed beta function yields
\[
\Delta \beta(s_0) \sin \Phi = -\beta(s_0) \oint \beta(s_1) k(s_1) \sin \mu_{1\rightarrow 0} \sin \mu_{0\rightarrow 1} ds_1 - \beta(s_0) \Delta \Phi \cos \Phi.
\] (1.52)

Substituting Eq. (1.48) for $\Delta \Phi$ and keeping only the terms linear in $\Delta \Phi$ and $\Delta \beta$, Eq. (1.52) becomes
\[
\Delta \beta(s_0) \sin \Phi = -\frac{\beta(s_0)}{2} \oint \beta(s_1) k(s_1) \cos(\mu_{1\rightarrow 0} - \mu_{0\rightarrow 1}) ds_1.
\] (1.53)

Rewriting Eq. (1.53) in general for any $s_0$ results in the beta function modulation everywhere in the ring due to a quadrupole field error at $s_1$,
\[
\frac{\Delta \beta(s)}{\beta(s)} = -\frac{1}{2 \sin \Phi} \oint \beta(s_1) k(s_1) \cos(\mu_{s_1\rightarrow s} - \mu_{s\rightarrow s_1}) ds_1.
\] (1.54)
Thus a quadrupole error will also change the beta function everywhere in the accelerator. The relative change in the betatron amplitude function at $s$ due to a quadrupole field error at $s_1$ depends on the beta function at $s$ and at the location of the quadrupole error, the strength of the perturbation, and the difference in the positive phase advance to and from $s$ and the location of the quadrupole error.

### 1.1.8 Chromaticity

The two terms left in the focusing functions of the EOM, Eqs. (1.11) and (1.12), represent chromatic effects acting on off-momentum particles, particles with momentum not equal to the design momentum. Chromaticity describes the change in the betatron tune versus particle or beam momentum,

$$
\mathcal{C} = \frac{d\nu}{d\delta},
$$

(1.55)

where as defined in Eq. (1.10), $\delta = \frac{\Delta p}{p_0}$ is the fractional particle or beam momentum deviation from the design momentum. Particles with higher momentum are more rigid and are focused less in the focusing fields than the on-momentum particles, decreasing the betatron tune. Likewise, slower particles spend more time in the focusing fields and are focused more, increasing the betatron tune. This effect is portrayed in the EOM by $K(s)\delta$. In addition, particles with less momentum are easier to bend because they have a smaller magnetic rigidity than particles with greater momentum. This effect yields different slightly path lengths for particles of different momentum and is represented in the equations of motion by the $\frac{2\delta}{p^2}$ term.

Natural chromaticity describes the change in the tune due to the linear focusing effects of the dipoles and quadrupoles with respect to a fractional momentum change. The change in the tune due to a perturbation in the focusing function was calculated in Eq. (1.49). The focusing field error in Eq. (1.49) may be approximated as

$$
\Delta K(s) \approx -K(s)\delta.
$$

(1.56)
The minus sign describes the chromatic effect of less focusing for particles with more momentum. Combining Eqs. (1.49) and (1.56) and rearranging, the natural chromaticity may be written in terms of the focusing function, \( K \),

\[
\mathcal{C}_n = -\frac{1}{4\pi} \oint \beta(s) K(s) ds.
\] (1.57)

For some accelerators, the natural chromaticity can be large leading to a large spread in the betatron tune due to the momentum spread of the beam. However, sextupole magnets may be employed to correct the chromaticity. Substituting \( x = x_\beta + D\delta \) for the position in the magnetic field of a sextupole, \( B_y = \frac{1}{2} B_2 (x^2 - y^2) \) and \( B_x = B_2 xy \), yields an effective quadrupole focusing term which is by definition linear in the beam position,

\[
\Delta K(s)_{\text{Sextupole}} = \mp K_2(s) D(s) \delta,
\] (1.58)

where \( K_2(s) \) is the normalized second derivative of the magnetic field and the \( \pm \) indicates plus for horizontal and minus for vertical focusing functions which is derived from the fact that a horizontally focusing quadrupole field is a vertically defocusing field. The minus sign in the definition of the sextupole strength is by convention so that a positive sextupole strength is horizontally focusing. Including the change in the focusing function due to a sextupole field in Eq. (1.49) via Eq. (1.56), the chromaticity may be written as

\[
\mathcal{C} = -\frac{1}{4\pi} \oint \beta(s) (K(s) \mp K_2(s) D(s)) ds.
\] (1.59)

The conversion from \( \pm \) to \( \mp \) in the sextupole component of the linear focusing reflects the convention set up for Eq. (1.56), particles with larger momentum have smaller tunes.
1.1 Introduction to Accelerator Physics

1.1.9 Dipole field error

With the solutions to the EOM for every term in the focusing function discussed, it is time to solve the particular solutions of the EOM. Whereas the solutions previously discussed described betatron oscillation about the CO, the particular solutions to the EOM detail the propagation of the CO around the accelerator. If there were no particular solutions to the EOM, the CO would lie on the reference trajectory.

The effects of steering and the dipole field error ($\Delta B_0$) are evaluated in this subsection, while the affects of dispersion on the CO are examined in Sec. 1.1.10.

A dipole field error is introduced when the integrated dipole field departs from the design value. There are two first order consequences of a dipole field error. First, the dipole field error will induce an oscillation in the CO, and second, the path length of the beam around the machine will change due to the induced dipole oscillation. The dipole field error leads to a non-zero right-hand side in Hill’s equation,

$$x''_\Delta B_0(s) + K_x(s)x_\Delta B_0(s) = \frac{\Delta B_y(s)}{B_0\rho}$$  \hspace{1cm} (1.60)

where, $K$ is the periodic focusing function depending on the lattice, and $\Delta B_y$ is the vertical dipole field error. The vertical Hill’s equation is easily obtained from Eq. (1.60) by exchanging the $x$’s for $y$’s and vice versa.

The particular solution to Eq. (1.60) describes a dipole oscillation in the CO due to the dipole field error. The particular solution for a single dipole error at $s_0$ may be solved via a greens function method[4],

$$x_\Delta B_0(s) = G(s, s_0)\theta(s_0) = \frac{\sqrt{\beta(s_0)\beta(s)}}{2\sin \pi \nu} \cos(\pi \nu - \mu_{s_0 \rightarrow s})\theta(s_0),$$  \hspace{1cm} (1.61)

where $G(s, s_0)$ is the Green’s function for Hill’s equation, $\beta$ is the betatron amplitude function, $\nu$ is the betatron tune, $\mu_{s_0 \rightarrow s}$ is the positive betatron phase advance from the longitudinal location of the dipole error ($s_0$) to $s$, and

$$\theta(s_0) = \int \frac{\Delta B_y(s_0)ds_0}{B_0\rho}$$  \hspace{1cm} (1.62)
is the integrated and normalized dipole kick strength.

The path length of the circumference around the machine may be written
\[ C = \oint \sqrt{\left(1 + \frac{x(s)}{\rho(s)}\right)^2 + x'(s)^2 + y'(s)^2} ds, \]  
(1.63)
where \( x \) and \( x' \) are the horizontal position and angle with respect to the reference trajectory, \( y' \) is the vertical angle with respect to the reference trajectory, \( \rho \) is the bending radius, and the integral is over the entire circumference of the ring. Expanding the square root because \( \rho \) is large and the position and angle coordinates are small leads to
\[ C \approx C_0 + \oint \frac{x(s)ds}{\rho(s)} + \ldots, \]  
(1.64)
where \( C_0 \) is the circumference of the reference orbit.

Thus the change in the circumference due to the introduction of a dipole field error is
\[ \Delta C = C - C_0 = \frac{\sqrt{\beta(s_0)\theta(s_0)}}{2\sin \pi \nu} \oint \frac{\sqrt{\beta(s)}}{\rho(s)} \cos(\pi \nu - \mu_{s_0 \rightarrow s}) ds = D(s_0)\theta(s_0), \]  
(1.65)
where the dispersion function \( (D) \) is introduced and discussed in the next subsection.

### 1.1.10 Dispersion

Considering particles with momentums different than the design momentum not only introduces chromatic effects which perturb the focusing function, it also introduces a particular solution to the EOM. Since the chromatic affects were discussed in Sec. 1.1.8, they will be ignored here.

The EOM for the off-momentum CO is
\[ D''(s) + K(s)D(s) = \frac{1}{\rho(s)}, \]  
(1.66)
where \( D \) is the dispersion function and \( \rho \) is the bending radius.
The dispersion function is a lattice function describing the motion of off-momentum particles. Just like twirling a mass on a string, where if it is spun faster the mass swings further out in orbit, in a circular accelerator particles with different momentum have different COs. Thus, the CO of an off-momentum particle may be written as

\[ x_{CO}(s) = x_{\Delta B_0}(s) + D(s)\delta, \]  

where \( s \) is the longitudinal coordinate along the accelerator, \( x_{\Delta B_0} \) is the CO for the on-momentum (synchronous) particle including steering affects, and as before \( \delta = \frac{\Delta p}{p_0} = \frac{p-p_0}{p_0} \) is the fractional momentum deviation of an off-momentum particle with momentum \( p \) and the design momentum \( p_0 \). Following from Eq. (1.67), the dispersion is defined as,

\[ D(s) = \frac{\partial x_{CO}(s)}{\partial \delta}. \]  

Notice that for straight sections where \( \rho \) is infinite, Eq. (1.66) yields EOM for the dispersion function that are the same as the betatron EOM. Thus, the dispersion function propagates along the drift spaces and quadrupoles via the transfer matrices derived in Sec. 1.1.6 I, Eqs. (1.19) and (1.20). There is only a particular solution to the dispersion function EOM around a bend. When \( \rho \) is not infinite, the dispersion function equation mirrors the equation of motion for a dipole field error, Eq. (1.60). So the dispersion function propagates like betatron motion but receives a dipole kick of strength \( \theta_0(s) = \delta \oint_{\rho(s)} \frac{dx}{\rho(s)} \) at each bend.

The dispersion function may be solved by summing the contributions from all “dipole kicks” where \( \rho \) is finite,

\[ D(s) = \frac{\sqrt{\beta(s)}}{2\sin \pi \nu} \int_s^{s+C} \frac{\sqrt{\beta(l)}}{\rho(l)} \cos(\pi \nu - \mu_{l \rightarrow s}) dl, \]  

where \( l \) is a dummy variable integrating about the accelerator, and \( \mu_{l \rightarrow s} \) is the positive phase advance from longitudinal location \( l \) to \( s \).
The affects of the kicks on the dispersion function from the bends may be described in matrix form for a sector dipole as

\[
\begin{pmatrix}
D_2 \\
D'_2 \\
1
\end{pmatrix} =
\begin{pmatrix}
\cos \theta & \rho \sin \theta & \rho(1 - \cos \theta) \\
-\frac{1}{\rho} \sin \theta & \cos \theta & \sin \theta \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
D_1 \\
D'_1 \\
1
\end{pmatrix}
\]

(1.70)

where \(D'\) is the derivative of the dispersion function with respect to the longitudinal and the indices 1 and 2 represent quantities at the beginning and end of the dipole respectively.

The off-momentum orbit will have a different circumference than the on-momentum trajectory, which may be calculated by substituting Eq. (1.67) into Eq. (1.64),

\[
\Delta C = C_\delta - C_0 = \int \frac{x(s)ds}{\rho(s)} = \delta \int \frac{D(s)ds}{\rho(s)},
\]

(1.71)

where \(C_0\) and \(C_\delta\) are the on and off-momentum circumferences respectively. The fractional change in the circumference with respect to the fractional momentum deviation is defined as the momentum compaction factor,

\[
\alpha_c = \frac{p_0}{C_0} \frac{\Delta C}{\Delta p} = \frac{1}{C_0} \frac{d\Delta C}{d\delta} = \frac{1}{C_0} \int \frac{D(s)ds}{\rho(s)},
\]

(1.72)

where the integral is along the reference orbit of the ring[4].

Because the off-momentum particle moves along a trajectory with a different path length than the reference orbit and at a slightly different speed, the revolution period of the off-momentum particle will also deviate from the synchronous particle. Since the period is a function of the beam’s orbit circumference and its velocity, the fractional time delay or change in period may be written

\[
\frac{\Delta T}{T_0} = \frac{\Delta C}{C_0} - \frac{\Delta v}{v_0},
\]

(1.73)

where the index 0 indicates a design value. Writing the fractional momentum deviation \((\delta)\) in terms of the fractional velocity deviation

\[
\delta = \frac{\Delta p}{p_0} = \frac{1}{\gamma_0 m v_0} (\gamma_0 m \Delta v + \gamma_0^3 m \beta_0^3 \Delta v) = \gamma_0^2 \frac{\Delta v}{v_0},
\]

(1.74)
where $\gamma_0$ and $\beta_0$ are the on-momentum Lorentz factors. Plugging $\delta$ into Eq. (1.73) yields

$$\frac{\Delta T}{T_0} = \frac{\Delta C}{C_0} \frac{p_0}{\Delta p} \delta - \frac{1}{\gamma_0^2} \delta = \left( \alpha_c - \frac{1}{\gamma_0^2} \right) \delta, \quad \text{or} \quad (1.75)$$

$$\delta = \left( \alpha_c - \frac{1}{\gamma_0^2} \right)^{-1} \frac{\Delta T}{T_0}, \quad (1.76)$$

where $\delta$ is now related as a function of the fractional deviation in the revolution period which may be measured by time of flight (TOF).

### 1.2 PSR Operations

The Proton Storage Ring (PSR) at the Los Alamos Neutron Science Center (LANSE) is really an accumulator ring. The PSR is a 90 m circumference circular accelerator with broken 10-fold FODO cell symmetry. The lattice symmetry is broken in section 0 (top center of Fig. 1.5) by the injection merging magnet RIBM09 and in section 1 by the C-magnets which replaced a single common-type PSR bender during the direct $\bar{H}$ injection upgrade. A PSR section is defined by a OFOD cell, starting with a bending dipole, horizontally focusing quadrupole, drift space, horizontally defocusing quadrupole. PSR sections 2-9 start with what is called a common 36 degree bending dipole. This terminology is employed to distinguish these dipoles from the dipole in section 0 (SRBM01), whose current is operated 100 A below the common PSR benders. SRBM01 only bends the beam 32.8° in order to “make room” in bend angle for the merging magnet, RIBM09, which bends the beam 6.8°. The remaining angle to close the circle is provided by the two C-magnets SRBM11 and SRBM12 both of which bend the beam 16.2°. Thus, the lattice symmetry is broken by four magnets taking the place of two common PSR bends in the injection region of the PSR.
All 12 PSR bender are rectangular dipoles. There are 20 quadrupoles in the PSR: 10 focusing and 10 defocusing. Every section except section zeros has a vertical corrector magnet (a small vertically bending dipole) located just upstream of the defocusing quadrupole for maximum affect.

The LANSCE linac supplies the PSR with 800 MeV kinetic energy chopped H− (one proton with two electrons) beam. As the H− beam is injected into the PSR it encounters a stripper foil. Beam collisions with the stripper foil convert the H− beam to a H+ or proton beam. The protons are then circulated about the PSR while more beam is injected each turn. The linac beam is chopped so that beam is injected into the PSR only during part of the traversal of a turn. The typical production pattern width for injection is 290 ns. The beam revolution period in the PSR is about 358 ns. Thus, injecting a pattern width of 290 leaves a 70 ns gap of no beam in the PSR. As the first turn of injected beam completes its first revolution in the PSR, it
returns to the position of the foil at the same time that the second turn of injected beam reaches the foil, is stripped from H− to protons, and stacked on top of the first injected turn. This accumulation process continues for 625 µs (~1800 turns), then the extraction kickers are fired during the gap, and the beam is extracted from the PSR in a single turn. The extracted PSR production beam is 290 ns long (~72 m) and ~5 µC. This intense, long bunched beam is transported to the 1L target (1 left) for spallation neutron production for the moderated target at the Lujan Center.

The design revolution frequency of the PSR is the 72.07 subharmonic of the drift tube linac (DTL) 201.25 MHz frequency, 2.7924 MHz also called the “moving” 2.8 MHz. It is called the “moving” 2.8 MHz because of the phase shift from turn-to-turn which causes it to appear to be moving against the 201.25 MHz. This picture is opposed to the 2.8 MHz frequency, which does not shift phase against the 201.25 MHz because it is an integer 72 subharmonic of the 201.25 MHz DTL frequency.

There are three types of pulse structures within the operations of the PSR. The smallest of these is what is called the micropulses. The micropulses are created by the accelerating fields of the linac. The micropulses are separated by 5 ns (the period of 201.25 MHz), although the micropulses themselves are less than 100 ps long. The next largest pulse structure is called a minipulse. A minipulse is the equivalent of one turn of injection into the PSR. The length of the minipulse is determined by the injection pattern width. For production where the pattern width is 290 ns, there are 58 micropulses in a minipulse. The last and largest pulse structure is called the macropulse. The macropulse is an entire machine cycle. During production the macropulse is typically 625 µs long. Note that the PSR compresses one 625 µs long macropulse into a stacked pulse the length of a minipulse, 290 ns.
1.3 Baseline Model

There are three accelerator models of the PSR described in this thesis. The first of which is what is named the baseline model. This model is called the baseline model because it represents the state of the model at the beginning of the model improvement endeavor. The predictions of the baseline model are baseline predictions where the predictions from an improved model should enhance some model predictions but maintain at least the same quality as the baseline model for all predicted quantities.

The baseline model is basically an extension of F. Neri’s DIMAD deck, psrdimad.txt\cite{9}, which in turn was a continuation of D. Johnson’s SYNCH deck written during the direct H− injection upgrade. So the baseline model is based on a PSR model that was at least 8 years old prior to the beginning of this model improvement exercise. Thus, several modifications and extensions augmented F. Neri’s model to construct the baseline model for this study.

The first complication was to assign the proper quadrupole magnet mapping data to the correct quadrupole. F. Neri’s model contains current to gradient length conversions in the form of a fourth order polynomial fit to the magnet mapping data described in Ref.\cite{10} and recorded for posterity in Ref.\cite{11}. The problem is that the magnet mapping data is labeled by quadrupole position in the PSR and not by magnet name or property number. It was also known that some of the quadrupoles had been relocated since the creation of F. Neri’s model and the start of this investigation, let alone the time between the PSR commissioning (1986) and D. Johnson’s SYNCH model in 2000. Thus, the first modification of F. Neri’s model was to assign the quadrupole current to gradient length fits to the proper quadrupole by quadrupole name and then assign that quadrupole to the correct location in the PSR.

Following in the footsteps of D. Johnson, F. Neri defined the edge focusing of the rectangular PSR dipoles as thin lens quadrupoles with focal lengths $\frac{\rho}{\tan \beta}$ as in
Eq. (1.23) for the horizontal, where $\beta$ is the edge angle of the rectangular bender. However, the effects of the fringe field focusing is not included in the vertical and Eq. (1.24) is applied as the vertical transfer matrix with $\zeta = 0$. The baseline model models the rectangular dipoles as rectangular dipoles with both edge and fringe field focusing where the gap height and fringe field integral parameters defined in the baseline model.

Aside from the two modifications of F. Neri’s model mentioned above, several additions were made to F. Neri’s model to construct the baseline model. The first of these is that the locations of the dipoles and quadrupoles are determined from the 2006 alignment data[12]. The positions of the other elements in the lattice (the vertical correctors, the injection merging magnet, and the foil) are determined by a combination of methods such as physical tape measurements and previous PSR models including S. Cousineau’s[13], T. Spickermann’s, and F. Neri’s.

The vertical corrector magnets are also included in the baseline model. Constant current to kick conversations from Ref. [11] are assigned to the 7” and 11” vertical correctors. Although the PSR does not have dedicated horizontal corrector magnets, zero length horizontal correctors are placed at the center of each of the PSR dipoles so that the baseline model may be compatible for use in an orbit response matrix analysis, Sec. 4.1. Beam position monitors (BPMs) are also included in the baseline model for this same reason. As in the PSR, the model BPMs are placed 18 cm upstream of the center of each quadrupole.

Lastly, the baseline model is capable of reading magnet current and shunt input from a Save Accel data file. The Save Accel data file is a text file output by the controls system which captures the state of the accelerator.

The predictions of the baseline model will be set as the standard for the improved model. The baseline model tune predictions compared with measurement are shown in Tab. 1.2. While the baseline model predicts the horizontal tune well, the baseline
model vertical tune prediction is not good at all. A model prediction of the vertical
tune with error of .05 is just unacceptable. There is some over-focusing in the vertical
in the baseline model. There must be a vertical focusing element in the real machine
that is not included in the baseline mode, or the baseline model may not be properly
handling a vertical focusing element. This is the motivation for an improved model
of the PSR. The improved model of the PSR must produce a better vertical tune
prediction.

<table>
<thead>
<tr>
<th></th>
<th>Measured</th>
<th>Model</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal</td>
<td>3.1915 ± 4×10^{−4}</td>
<td>3.1973</td>
<td>−5.8×10^{−3}</td>
</tr>
<tr>
<td>Vertical</td>
<td>2.1979 ± 3×10^{−4}</td>
<td>2.2451</td>
<td>−4.7×10^{−2}</td>
</tr>
</tbody>
</table>

Table 1.2: The baseline model predicted betatron tunes compared with measurement. The error in
the model prediction is defined as measured minus model.

The poor vertical tune prediction of the baseline model was expected because it existed in F. Neri’s DIMAD deck. There were two popular methods to correct for the model’s vertical tune prediction[6]. F. Neri adjusted the edge angles of PSR dipoles by about .8° to obtain the proper tunes. The other method involved modifying the focusing and defocusing quadrupole strengths by a percent or so. Both of these methods were quick fixes of changing the favorite model parameter to obtain the proper vertical tune. No experiments were preformed to verify that either method was the correct modification.

Interestingly, the baseline model does a fairly good job at predicting the measured betatron amplitude functions (Sec. 3.4) and the measured dispersion function, Sec. 3.3. Since the baseline model predicted beta functions agree with measurement, one might be lead to believe that the main source of focusing in the PSR (the quadrupoles)
is handled properly. Another way to view the baseline model’s performance compared to measurement is to compare the betatron phase at each BPM. (The results of the betatron phase measurement are discussed in Sec. 2.6.3). Figure 1.6 plots the difference in the measured and model betatron phases referenced to the BPM 2, the first BPM after the point of injection. Observe the systematic mistreatment of the phase advance in the vertical between BPMs starting at BPM 5. This systematic mishandling of the vertical phase starts in section 2 of the PSR right after the model encounters the first common PSR bender. This may be an indication of where the additional vertical focusing is located for model improvement.

The constant slope in the difference between the measured and model vertical phase is an indication that the vertical focusing error in the model is not due to a single element (like a single bad quadrupole) but a mistreatment of several like-elements which combine to form the systematic mistreatment of the vertical phase shown in Fig. 1.6.
There are three possible sources of additional vertical focusing in the PSR: the quadrupoles, the fringe and leakage fields of the PSR extraction septa, and the edge focusing of the rectangular dipoles. I will employ a different beam-based measurement in investigation of each of these possible sources of additional vertical focusing. I will measure the real quadrupole strengths in the PSR by means of an orbit response matrix (ORM) analysis, Sec. 4.1. To characterize the fringe fields of the extraction septa, I will expand the integrated magnetic field encountered by the beam in Beth’s expansion and measure the dipole, quadrupole, and sextupole components of the fringe field from the beam response, Sec. 4.2. To nail down the focal length of the edge focusing of the rectangular dipoles, I will trace parallel rays through a 3D magnetic field simulation of the PSR benders, Sec. 4.3.

The beam-based measurements rely heavily on the quality of the beam position measurement in the PSR. Thus, I dedicate an entire chapter to understanding the beam position measurement, Chap. 2.
Chapter 2

A RingScan Reproducibility Measurement

You can easily imagine for any thesis in experimental physics that at least one chapter should be devoted to the measurement device and the experimental setup. You could also expect in such a discussion detailed statistical analyses regarding the quality and reproducibility of the measurement including descriptions of the random measurement errors and the systematic errors. One might even go so far as to catalog the types of data acquisition errors encountered during the experiment and the performance of each individual instrument employed as a diagnostic. Well, this is my chapter on the measurement device: “Everything you wanted to know about the PSR BPMs but were too afraid to ask.”

The beam position monitors (BPMs) are the main diagnostic in the PSR. During operations and tuning, the BPMs are employed to provide information on the CO, betatron tune, and injection offset into the PSR. The BPMs can measure the location of the beam for many beam passages and output turn-by-turn position data. A controls software program may be executed to coordinate the consecutive data acquisition of
$N$ turns of beam position data at each of the PSR BPMs. The name of this BPM program is RingScan, thus the title of this chapter.

The RingScan program records both the horizontal and vertical beam position for $N$ turns at a BPM during a macropulse, and then it records $N$ turns of position data at the next BPM during the next macropulse. This continues until data is collected at each of the PSR BPMs. The number of turns recorded at each BPM ($N$) is a user input to the RingScan program. One RingScan then provides $N$ turns of data at each of the 20 PSR BPMs in 20 pulses or 40 scans. A scan is composed of $N$ turns of either horizontal or vertical turn-by-turn BPM data at a single BPM, and remember that a macropulse is one machine cycle where beam is injected and extracted from the PSR.

Aside from the fast current monitor SRWC41, the BPMs are the only diagnostic employed in the series of experiments detailed in this thesis chronicling the improvement of the PSR model. Thus, it is important for us to understand the accuracy and precision of the BPM measurement and any limitations there may be. Not only should I do this for the collective BPM measurement, but each individual BPM should be scrutinized. I chose to characterize the RingScan measurement through extensive statistical analysis of a reproducibility measurement.

In this chapter, I start the discussion with the grueling details of how the beam signal is processed by the data acquisition hardware and software, Sec. 2.1. I also catalog the coefficients and equations wielded in the manipulation of the signal processing and ultimately estimate the intrinsic position measurement error of each BPM. From there, I introduce the measurement setup for this RingScan reproducibility experiment and all PSR model improvement experiments in Sec. 2.2 and motive the analysis of the turn-by-turn BPM data, which is fit to a cosine wave, Sec. 2.3. I also provide procedures for obtaining the initial guesses for the nonlinear cosine wave fits and show that a maximum likelihood (ML) error analysis may be applied to such fits. In Sec.
2.4, I digress in order to catalog a compilation of 9 distinctly different data acquisition errors. I also discuss the method for identifying outliers in the fitted parameters and the procedure for removing scans with either data acquisition errors or outlying fitted parameters from the data set. Next, I discuss the quality of the cosine wave fitting through an analysis of the fitting residuals as a function of turn number along the scan, (Sec. 2.5). Finally, I will share the results of this reproducibility measurement in Sec. 2.6. This encompasses the measurement spread and fitting errors on the fitted parameters, the single turn BPM measurement error, the quality of the cosine wave fits, and comparison with results from the central control room (CCR) BPM program. The RingScan reproducibility measurement is further extended in calculations of the injection offset in Sec. 2.7. I discuss the reproducibility and measurement spread of the injection offset calculations and compare the results with injection offset measured by the CCR BPM program. Lastly, I also wish to share with you a calculation of the magnitude of the pulse-to-pulse momentum variations due to fluctuation in the output energy of the linac, Sec. 2.8.

So please enjoy “Everything you wanted to know about the PSR BPMs but were too afraid to ask.”

2.1 The Beam Position Measurement

The BPMs are the main diagnostic in the PSR. A BPM is a collection of four stripline electrodes located inside the beam pipe and situated top, bottom, left, and right. The BPMs in the PSR have diameters of 4 and 6 inches. The PSR BPMs are tuned to the 201.25 MHz longitudinal frequency of the beam, imposed by the drift tube linac (DTL) accelerating structure. The striplines are thus a 201.25 MHz

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\[\text{Much of this section references Ref. [14]. Follow Fig. 2.1 for a pictorial view of the discussion on the BPM measurement.}\]
quarter wavelength long, \(\sim 37\) cm. Each stripline is terminated on the downstream end, and the stripline signal propagates along cables attached to the upstream end. There are 18 BPMs in the PSR. Each BPM is located inside a quadrupole with BPM center about 18 cm upstream of the magnet center. Two of the PSR quads do not have BPMs; SRQU41 holds an electron cloud detector, and part of the first extraction kicker runs through SRQF71.

Data is also collected at one additional very important BPM during experiments requiring beam momentum measurements. LDPM03 is a 2.8'' BPM in the high dispersion region of the transport from the linac to the PSR.

The signals from the four electrodes of each BPM travel to a mechanical matrix switch (MUX). There are four MUXs, one for each of the four BPM electrodes (top, bottom, left, and right). The MUX selects from which BPM data will be processed and recorded. So if the BPM program is instructed to collect data from the fifth BPM, all of the MUXs select BPM number five, and the top, bottom, left, and right signals from the fifth BPM are transported through the MUX. The MUX allows for a single horizontal and a single vertical digitizer to process data from all of the PSR BPMs, but not simultaneously. Data from only one BPM may be analyzed per macropulse,
allowing the MUX time to switch between machine cycles.

After the MUX, the signals pass to the analog front end (AFE). The AFE consists of two sets of analog front end electronics to analyze the horizontal and vertical signals separately and simultaneously. They filter, convert from amplitude modulation to phase modulation (AM to PM), and combine the signals outputting a voltage related to the ratio of power deposited on opposite BPM electrodes. The output voltage of the AFE can range between $V_{AFE} = \pm 4$ V. This voltage is then sent past two 6 dB attenuators, which reduce the output voltage of the AFE by a factor of 2, to the analog to digital converter (ADC) to be digitized.

The ADC digitizes the output voltage from the AFE with 12-bit resolution between $V_{ADC} = \pm 2$ V. Thus, the AFE output voltage can be written as a function of the digitized value, $d$,

$$V_{AFE} = \left( \frac{2d}{d_{max}} - 1 \right) V_{max} \quad (2.1)$$

where $V_{max} = 4$ V is the maximum output voltage of the AFE and $d_{max} = 2^{12} = 4096$ is the maximum digitized point number. The intrinsic error in the BPM measurement due to this digitization will be discussed later in the section.

The ADC is triggered to digitize data by a beam present and a 201.25 MHz intensity trigger, both of which are produced when the beam signal intensity or the 201.25 MHz signal intensity from the bottom BPM electrode surpasses a threshold value respectively. During production, all \(\sim 1800\) turns the beam is in the PSR are digitized because beam is present and the newly injected beam has the 201.25 MHz structure imposed by the DTL. To increase the data acquisition rates of the ADC, there is a pipeline storage before the digitized voltages are saved to the ADC buffer. The pipeline holds three digitized values and is flushed by the software before the buffer is read to the input-output controller (IOC) by digitizing three more values of random noise long after the beam has been extracted from the PSR.
2.1 The Beam Position Measurement

The IOC is where the digitized voltages are reverted back to positions based on models of AM-PM conversion and how power is deposited on the BPM electrodes for charge offset from the center of the beam pipe[15, 16]. Although all turns of digitized voltages are sent to the IOC, the IOC only applies the calculation to the first $N$ turns of data. The number of turns to be analyzed is a user input for the RingScan program.

The first calculation performed by the IOC is to convert the digitized value $d$ to an AFE output voltage by applying Eq. (2.1). Then the digitized voltage is converted to a power ratio,

$$R = a_1 \log\{\tan[a_2(V_{AFE} + a_3) + \pi/4]\},$$  \hspace{1cm} (2.2)

where $R$ is the ratio of power deposited on opposing BPM electrodes by the beam, $V_{AFE}$ is the output voltage from the AFE, and $a_1$ $a_2$ $a_3$ are coefficients of the AFE calibrated by fitting a known power ratio input with measured $V_{AFE}$ voltage output. Because there is a different AFE to process the horizontal and vertical signals, there are two sets of $a_i$ coefficients. The values of $a_1$ and $a_3$ for an ideal AM-PM conversion are respectively 20 and 0. Table 2.1 shows the values of $a_1$, $a_2$, and $a_3$ applied in the BPM program as of August 2009. These values can be found in ringBpmCvtInit[17].

<table>
<thead>
<tr>
<th>The AFE Calibrations Applied in Eq. (2.2)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>AFE</td>
<td>$a_1$ [dB]</td>
</tr>
<tr>
<td>Horizontal</td>
<td>11.207</td>
</tr>
<tr>
<td>Vertical</td>
<td>10.9906</td>
</tr>
</tbody>
</table>

Table 2.1: The calibration coefficients of the horizontal and vertical AFEs as of August 2009. The calibration coefficients are applied in the BPM program to calculate the power ratio given a digitized voltage from the ADC, Eq. (2.2).
2.1 The Beam Position Measurement

The Geometric Constants Applied in Eq. (2.3)

<table>
<thead>
<tr>
<th>BPM Type</th>
<th>( c_1 ) [mm/dB]</th>
<th>( c_2 ) [mm/dB(^3)]</th>
<th>( c_3 ) [mm/dB(^3)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.8''</td>
<td>1.0204</td>
<td>-1.710\times10^{-4}</td>
<td>7.210\times10^{-4}</td>
</tr>
<tr>
<td>4''</td>
<td>1.4737</td>
<td>-2.700\times10^{-4}</td>
<td>1.110\times10^{-3}</td>
</tr>
<tr>
<td>6''</td>
<td>2.1797</td>
<td>-3.830\times10^{-4}</td>
<td>2.140\times10^{-3}</td>
</tr>
<tr>
<td>Horizontal Diamond</td>
<td>2.9922</td>
<td>-5.259\times10^{-4}</td>
<td>2.939\times10^{-3}</td>
</tr>
<tr>
<td>Vertical Diamond</td>
<td>3.3192</td>
<td>-5.834\times10^{-4}</td>
<td>3.260\times10^{-3}</td>
</tr>
</tbody>
</table>

Table 2.2: The geometric coefficients applied in the BPM program as of August 2009 to calculate the beam centroid position from a power ratio, Eq. (2.3).

After the IOC calculates the power ratio, it converts the power ratio to the position of the beam centroid,

\[
x = c_1 R_x + c_2 R_x^3 + c_3 R_x R_y^2 - x_{\text{Offset}},
\]

where \( x \) is the position of the beam centroid, \( R_x \) is the power ratio in the same direction as the position (horizontal or vertical), \( R_y \) is the power ratio from the other dimension (this introduces a certain amount of coupling in the BPM measurement), \( x_{\text{Offset}} \) is a calibration describing the misalignment of the electric center of the BPM with the geometric center of the BPM[18] (which may be different than the misalignment of the BPM’s electronic center with the magnetic center of the quadruple in which it rests), and \( c_1, c_2, c_3 \) are geometric coefficients relating the power ratio between two opposite stripline electrodes and the beam position. There are four sets of these \( c_i \) coefficients employed by the BPM program for the PSR BPMs, one for the 4'' BPMs, one for the 6'' BPMs, and two for the diamond type BPMs because the horizontal and vertical geometries are not symmetric in the diamond BPM case. Table 2.2 lists the \( c_i \)’s currently employed in the BPM program. Table 2.3 lists the
### Table 2.3: The calibration coefficient offsets used as of August 2009 for Eq. (2.3) in the BPM program to calculate the beam centroid position from a power ratio, and the BPM type for each BPM. The * indicates a missing BPM, which still has a calibration offset and a BPM type assigned to it in the BPM program.

<table>
<thead>
<tr>
<th>BPM</th>
<th>Horizontal [mm]</th>
<th>Vertical [mm]</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>LDPM03</td>
<td>0.000</td>
<td>0.000</td>
<td>2.8''</td>
</tr>
<tr>
<td>SRPM01</td>
<td>0.406</td>
<td>−0.194</td>
<td>4''</td>
</tr>
<tr>
<td>SRPM02</td>
<td>0.000</td>
<td>0.000</td>
<td>6''</td>
</tr>
<tr>
<td>SRPM11</td>
<td>−2.030</td>
<td>0.130</td>
<td>6''</td>
</tr>
<tr>
<td>SRPM12</td>
<td>0.042</td>
<td>−0.090</td>
<td>4''</td>
</tr>
<tr>
<td>SRPM21</td>
<td>0.229</td>
<td>−0.398</td>
<td>4''</td>
</tr>
<tr>
<td>SRPM22</td>
<td>−0.100</td>
<td>0.122</td>
<td>4''</td>
</tr>
<tr>
<td>SRPM31</td>
<td>0.067</td>
<td>0.578</td>
<td>4''</td>
</tr>
<tr>
<td>SRPM32</td>
<td>−0.676</td>
<td>0.258</td>
<td>4''</td>
</tr>
<tr>
<td>SRPM41</td>
<td>−0.176</td>
<td>0.292</td>
<td>4''</td>
</tr>
<tr>
<td>SRPM42*</td>
<td>0.292</td>
<td>0.222</td>
<td>4''</td>
</tr>
<tr>
<td>SRPM51</td>
<td>−0.116</td>
<td>0.333</td>
<td>4''</td>
</tr>
<tr>
<td>SRPM52</td>
<td>−0.848</td>
<td>−0.177</td>
<td>4''</td>
</tr>
<tr>
<td>SRPM61</td>
<td>0.229</td>
<td>0.151</td>
<td>4''</td>
</tr>
<tr>
<td>SRPM62</td>
<td>0.247</td>
<td>0.566</td>
<td>4''</td>
</tr>
<tr>
<td>SRPM71*</td>
<td>0.000</td>
<td>0.000</td>
<td>4''</td>
</tr>
<tr>
<td>SRPM72</td>
<td>0.344</td>
<td>0.112</td>
<td>4''</td>
</tr>
<tr>
<td>SRPM81</td>
<td>1.140</td>
<td>−0.310</td>
<td>D</td>
</tr>
<tr>
<td>SRPM82</td>
<td>0.034</td>
<td>0.0428</td>
<td>4''</td>
</tr>
<tr>
<td>SRPM91</td>
<td>−0.230</td>
<td>0.230</td>
<td>D</td>
</tr>
<tr>
<td>SRPM92</td>
<td>0.251</td>
<td>−0.135</td>
<td>4''</td>
</tr>
</tbody>
</table>
2.1 The Beam Position Measurement

$x_{\text{Offset}}$'s currently set in Eq. (2.3) and the BPM type for each BPM in the PSR. The values for the $c_i$'s can be found in `ringBpmDefs.h[17].

The beam positions calculated by Eq. (2.3) are saved in the IOC buffer and read out by the Experimental Physics and Industrial Control System (EPICS). When the RingScan program is run, it simultaneously measures horizontal and vertical turn-by-turn BPM data in this manner, cycling through the MUX one BPM per macropulse starting with SRPM01 and consecutively selecting the 20 BPMs that it thinks exists (18 real and 2 that do not really exist but data is still collected) ending with SRPM92 and saves the turn-by-turn BPM data to a time-stamped text file, `ringscan_timestamp'. The data in the time-stamped files are analyzed for the RingScan reproducibility measurement.

In order to understand the limitations of the RingScan measurement, it is of interest to know the intrinsic resolution of the BPM. For this study, the BPM intrinsic resolution is defined as the uncertainty due to the discrete digitization of the AFE output voltage by the ADC. The discrete nature of the measurement at small length scales limits the precision of the BPM measurement. Thus, the statistical random error on the average of distributions with dimensions of length (the fitted amplitude and offset parameters introduced in Sec. 2.3) cannot be less than the intrinsic BPM resolution because at this level the discrete nature of the digitization dominates the uncertainty.

The intrinsic resolution of the BPM can be found by calculating the slope of the position as a function of digitized value. It will be assumed for simplicity that the beam is centered in the other dimension so the coupling term in Eq. (2.3) is zero. The position as a function of digitized values may be obtained by combining Eqs. (2.1),
(2.2), and (2.3),

\[ x(d) = c_1 \left( \log_{10} \{ \tan[a_2(8d/d_{max} - 4) + \pi/4] \} \right) 
+ c_2 \left( \log_{10} \{ \tan[a_2(8d/d_{max} - 4) + \pi/4] \} \right)^3 - x_{\text{offset}}. \]  

(2.4)

A plot of the position as a function of AFE output voltage is shown in Fig. 2.2. The complex values in the position occur when the tangent functions in Eq. (2.4) yield a negative value. It is interesting that only about one third of the digitized values yield a real valued position. This does not seem to be very good stewardship of the bit depth of the ADC. According to Fig. 2.2, the most extreme real valued positions that BPM 1 (SRPM01x) is able to measure is ±40 mm.

Figure 2.2: (Color) The position calculated from the output voltage of the AFE for each digitized value in the ADC at BPM 1, SRPM01x. Blue circles indicate real valued positions, and the green circles represent the real part of complex valued positions.

The position resolution of the BPM is the slope of the real valued positions in Fig. 2.2 with respect to digitized value. As expected from the AM-PM conversion, the position resolution is not constant across the diameter of the BPM. However, the position resolution in the center of the BPM is better than a constant resolution across the diameter of the BPM because the slope is shallower. The resolution in the
2.1 The Beam Position Measurement

The center of the BPM is fairly constant by the AM-PM conversion. The slope of the real valued positions in Fig. 2.2 can easily be found by calculating the derivative of Eq. (2.4),

\[
\frac{dx(d)}{dd} = c_1 \left( \frac{8a_1a_2 \sec^2[a_2(8d/d_{max} - 4) + \pi/4]}{d_{max} \ln(10) \tan[a_2(8d/d_{max} - 4) + \pi/4]} \right)
\]

\[
+ 3c_2 \left( \log\left\{ \tan[a_2(8d/d_{max} - 4) + \pi/4] \right\} \right)^2 \left( \frac{8a_1a_2 \sec^2[a_2(8d/d_{max} - 4) + \pi/4]}{d_{max} \ln(10) \tan[a_2(8d/d_{max} - 4) + \pi/4]} \right).
\]

Equation (2.5) yields the resolution of the BPM position measurement in units of millimeters per digitized value. The position resolution of BPM 1 (SRPM01x) is plotted versus position in Fig. 2.3. Both Figs. 2.2 and 2.3 are not exactly symmetric about zero voltage or zero position. This is because the calibration offsets \(x_{\text{Offset}}\)'s were included in Eq. (2.4).

![Resolution of BPM 1 vs Horizontal Position](image)

**Figure 2.3:** The intrinsic position resolution of BPM 1 (SRPM01x) as a function of position in the BPM.

The minimum position resolution of BPM 1 (SRPM01x) is .01531 mm. All like-type BPMs yield the same minimum BPM position resolution for each dimension because in Eqs. (2.4) and (2.5) the \(a_i\)'s are coefficients of the AFE’s and only depend
on whether the BPM is horizontal or vertical and because the $c_i$’s are geometric coefficients and only depend on the size and shape of the BPM. Table 2.4 summarizes the minimum BPM position resolution for the PSR BPMs. For the statistics of the RingScan measurement, the intrinsic BPM resolution and the limiting precision obtainable by the BPM measurement is defined as the minimum position resolution of the BPM due to the least count of the digitization of the ADC.

<table>
<thead>
<tr>
<th>BPM Type</th>
<th>Horizontal [mm]</th>
<th>Vertical [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.8″</td>
<td>0.01060</td>
<td>0.01034</td>
</tr>
<tr>
<td>4″</td>
<td>0.01531</td>
<td>0.01494</td>
</tr>
<tr>
<td>6″</td>
<td>0.02263</td>
<td>0.02209</td>
</tr>
<tr>
<td>Diamond</td>
<td>0.03108</td>
<td>0.03365</td>
</tr>
</tbody>
</table>

Table 2.4: The minimum PSR BPM position resolutions based on the AFE and geometric coefficients in Tabs. 2.1 and 2.2 respectively.

Even though Fig. 2.3 plots the horizontal intrinsic position resolution of the BPM 1, something may be said about two vertical positions of interest. First, the typical production vertical injection offset is 17 mm in position. This relates to $\sim$1 mm intrinsic measurement error, and second, the typical vertical first turn position at BPM 2 (SRPM02y) is 22 mm, corresponding to a $\sim$2 mm intrinsic BPM error. These intrinsic errors in the BPM measurement can greatly reduce the accuracy of the injection offset measurement for production.

## 2.2 RingScan Measurement Setup

The PSR was setup in the same manner for all of the PSR model improvement and verification experiments discussed in this thesis.

The PSR was setup for single shot accumulation mode. In this mode, only the first turn of the accumulation cycle is injected into the ring. This one turn of beam
circulates \( \sim 1800 \) times as in a normal machine cycle and is extracted from the PSR to the tune up beam stop (TUBS) or to the first target left (1L target), which services the Lujan center with spallation neutrons during production. To maximize the data acquisition rates of the RingScan program, the machine cycle repetition rate for the PSR beam is set to 20 Hz.

Single shot beam is chosen for the RingScan measurements because it is the most simple. There are no added complications to filling the phase space with other turns of injected beam. Like a single particle, the single shot beam executes coherent betatron motion about the CO due to the injection offset, so it is very easy to measure the CO and the betatron tune from the turn-by-turn position data.

The vertical bump magnets that are energized during production to allow phase space painting in the vertical were turned off. Not turning off the vertical bump magnets would cause the CO to change during the beam position measurement. The harmonic buncher, which serves to keep the beam bunched longitudinally during production, was also turned off. With the buncher off, the beam does not rotate in longitudinal phase space, so the only mechanism to wash out the 201.25 MHz longitudinal structure of the beam is momentum spread in the micropulses. Remember that the BPMs are triggered off of the 201.25 MHz intensity of the beam, meaning that more turns of data may be taken for slower decohering longitudinal structure.

The CO was centered because this is where the BPM measurement is most linear. The orbit control (SRMP999) was the knob of choice for the CO correction. The beam was injected near-on-axis. Near-on-axis indicates small injection offsets of only a few millimeters. The injection offsets for the PSR model improvement experiments were typically \( \sim [-.72 \text{ mm}, .31 \text{ mrad}] \) in the horizontal and \( \sim [1.99 \text{ mm}, .3 \text{ mrad}] \) in the vertical. Injecting so near to the longitudinal axis of the CO limits the amplitude of the betatron oscillations. This should keep the beam positions within the calibrated linear measurement region of the BPMs, avoid beam scraping and
BPM signal saturation, and allow for significant dipole kicks and momentum changes during the model improvement and verification experiments.

Lastly, the beam energy was corrected to match the design energy. The energy of the beam was centered by minimizing the time of flight delay (TOF) for 1000 turns between the circulating beam and the imposed (design) PSR revolution frequency measured with cursors on an oscilloscope of the fast current monitor trace (SRWC41) referenced to the “moving” 2.8 MHz (the 72.07 subharmonic of the 201.25 MHz DTL frequency) revolution frequency of the PSR. The phase of the last module in the linac (Mod48) was shifted to correct for the energy. It was sometimes necessary to recenter the CO after the energy correction, and then again to correct the energy in order to converge on the centered CO at the design momentum.

Forty turns of RingScan data can be obtained with the PSR setup in this manner. This is very good as normal operating conditions only yield between 25 and 30 turns of beam position data. Once the beam leaves the linac, the 201.25 MHz longitudinal frequency structure starts to wash out due to momentum spread in the micropulses. Since the BPMs are tuned to the 201.25 MHz longitudinal structure of the beam, the BPMs cannot measure the beam position after several turns because of the decay in the 201.25 MHz intensity. Injecting the beam near-on-axis allows for more turns of RingScan data to be collected because the amplitudes of betatron oscillation are much smaller compared to production, where the vertical injection offset is ~17 mm, negating any effect of large amplitude nonlinearities in the frequency spread.

For this reproducibility measurement, 101 RingScans were executed at the same PSR configuration. All of the RingScans should give the same result since all measurements were taken with the PSR in the same state, but there exists measurement variation and error which are of interest. It took about 7.5 minutes to take the 101 RingScans measurements. Results from this experiment (fitting, fitting errors, and measurement spread) are stated in the Sec. 2.6, and particular measurement errors
(non-sinusoidal scans due to data acquisition errors) are discussed in the Sec. 2.4.

2.3 RingScan Data Analysis

If the beam position is measured at a particular location in the ring for several turns (like at a BPM), it will be observed that the beam is undergoing betatron or harmonic motion at that location with a frequency $2\pi \nu$ (where $\nu$ is the tune) and with amplitude related to the beta function at that location in the ring and the action ($J$), which is determined by the injection offset, Eq. (1.40). The beam motion measured at the longitudinal position of the BPM is purely harmonic oscillation because the longitudinal coordinate is fixed in Hill’s equation for betatron motion (Eqs. (1.15) and (1.16)), the focusing function is constant. Thus, the turn-by-turn RingScan data may be fit to a cosine wave.

The output of the RingScan measurement is $N$ turns of position data at each BPM. In the case of this reproducibility measurement, there are 40 points of turn-by-turn beam position data for each BPM. However for this study and the model improvement experiments, the beam position is not the information that is of the most interest; the parameters from the cosine wave fit are the most important: the CO and tune for their role in ORM and beta function measurements, the phase advance between BPMs for constraining model parameters and for calculating the injection offset, and the amplitude whose relative values at different BPMs are the same as the ratios of the beta functions at those BPMs because $J$ is constant, Eq. (1.40).

To extract the above parameter information from the turn-by-turn RingScan data, the BPM data is fit to a cosine wave, Eq. (2.6). The difference in choice between a sine wave and cosine wave is only a shift in the phase, however it is more convenient to choose a cosine wave because this phase is the same that is used in propagating particle motion which is the procedure used to obtain the injection offset using the
2.3 RingScan Data Analysis

RingScan data, Eq. (1.39).

\[ x_n = A \cos(2\pi \nu n + \phi) + O_{\text{offset}} \]  

(2.6)

Aside from extracting the CO and tune \((O_{\text{offset}}\) and \(\nu\) respectively) from the turn-by-turn BPM data \((x_n, \text{where } n \text{ ranges from 1 to the number of turns of RingScan data taken, } N)\), the amplitude and phase information \((A \text{ and } \phi)\) can also be obtained. The consistency of these four fitting parameters (the spread in their distribution for all 101 RingScans) will be the measure of reproducibility of the RingScan measurement. A typical scan is shown in Fig. 2.4.

Figure 2.4: (Color) A typical scan from the RingScan reproducibility dataset. This is BPM 5 (SRPM21x) scan 2 with the 40 turns of position data. The blue circles are RingScan data, and the green squares are the cosine wave fit. The red line is the value of the extracted CO from the cosine wave fit of Eq. (2.6). The SSR/DOF for this scan is very good, .026 mm\(^2\).

When the turn-by-turn BPM data is fit with the fitting function described in Eq. (2.6), it is assumed that there is no nonlinear motion induced by higher order multipole magnets (such as sextupole and octupole magnets), or nonlinearities in the beam position measurement. This assumption is justified because the beam is injected near-on-axis where the nonlinearities due to the higher order multiple magnets and
the BPM measurement are smallest, and it will be shown that the fit of BPM data to Eq. (2.6) is good, further proving this assumption. However, nonlinearities in the beam position measurement are observed in BPM data when the beam is injected at production injection offsets ($\sim [-3 \text{ mm}, .6 \text{ mrad}]$ in the horizontal and $\sim [17 \text{ mm, } 3 \text{ mrad}]$ in the vertical). The study of how the BPM measurement saturates has not been completed and will not be discussed in this thesis.

The cosine wave is fit to the turn-by-turn BPM data by executing a nonlinear least squares method. The Matlab function *lsqnonlin* is employed. The sum of squares of residuals per degree of freedom (SSR/DOF) is defined as the goodness of fit quality factor,

$$\text{SSR/DOF} = \frac{\sum_n (x_n - f(n; \vec{a}))^2}{N - P},$$

where $x$ is the BPM data, $f(n; \vec{a})$ is the fitting function described in Eq. (2.6) dependent on the turn number and the fitting parameters ($\vec{a}$), the summation is over the turn number index $n$ ranging from 1 to $N$, and $P$ is the length of the fitting parameter vector $\vec{a}$. The SSR/DOF is the average square residual and in the beam position analysis has units of mm$^2$.

Initial guesses for the fitting parameters need to be supplied to *lsqnonlin*. The initial offset guess is obtained by averaging the turn-by-turn RingScan data. This is how the CCR BPM program calculates the CO. Since the fractional tunes are $\sim .2$, it is very important to have a multiple of five turns of BPM data in order to average over an integer number of betatron oscillations. When a non-integer number of cycles are averaged, a weighting affects the averaging and does not average to the offset of the cosine wave. This is taken into consideration for the purposes of calculating the initial offset guess, and only a multiple of five turns of data are averaged, ignoring the last few turns if necessary.

The initial tune guess chosen to be either the tune measured using the CCR BPM
program for a baseline measurement (which is static and the same for all RingScans) or the tune derived from the peak frequency of a fast Fourier transform (FFT) of the turn-by-turn data. An FFT for only 40 turns is not very accurate, but is useful when tunes vary over several sets of RingScan measurements as in the case of a beta function measurement using the quadrupole perturbation method. The CCR BPM program calculates the tune by supplying many guesses and chooses the tune guess that fits the data best using a linear regression method.

Like the tune, there are two choices for the amplitude initial guess. The initial guess for the amplitude parameter is either the distance between the turn-by-turn data point that is furthest from the offset initial guess \( \max |x_n - O_0| \) where \( x_n \) is the turn-by-turn BPM data and \( O_0 \) is the initial offset guess) or the amplitude of the FFT frequency peak. It is necessary to include the first method as an option for the initial amplitude guess in order to avoid obtaining an imaginary initial guess for the phase parameter.

Finally, the initial phase guess is calculated by inverting Eq. (2.6) and solving for \( \phi \). Two data points are needed to determine the phase, one for calculating the \( \cos \phi \) and the other to resolve which of the two \( \phi \)'s that give the same \( \cos \phi \) is the correct phase. Two consecutive turns are needed in the calculation of the initial phase guess, so the last turn, not having a turn afterward, is not applied to solve the inversion of Eq. (2.6). The four possible combinations of amplitude and tune guesses are employed to obtain four possible initial phases for all turns of data except the last. The resulting initial parameter guess is the combination of the four possibilities (of tune and amplitude) and phase for each turn (1 through \( N - 1 \)) that best fits the turn-by-turn BPM data.

The initial guess is inputted in the Matlab fitting function \textit{lsqnonlin}, and the fitted parameters are outputted and saved. These are the quantities of interest: the fitted amplitude, tune, phase, and offset. However, aside from the values of the fitted
parameters, it is also of interest to know the fitting error on the fitting parameters from the cosine wave fit, as well as a measure of the goodness of fit (SSR/DOF), and the BPM single turn measurement error. A maximum likelihood (ML) error analysis[19] is implemented to obtain these quantities plus the covariance and correlation matrices, which describe how the fitting parameters are related. The ML error analysis only works for mean zero random errors. Figure 2.5 plots the average residual for each cosine wave fit after all scans with data acquisition errors have been removed from the reproducibility dataset as explained in Sec. 2.4. The average residual for most scans is less than a nanometer because the residuals tend to cancel out in the averaging. This indicates that the residual distribution is random and mean zero, so the ML error analysis may be applied to this dataset. See Appx B for an introduction to the ML error analysis procedure employed in this thesis.

![Figure 2.5: The average residual of each cosine wave fit for all scans without data acquisition errors. The vertical line separates the horizontal BPMs (left) and the vertical BPMs, right.](image)

For the convenience of analysis, an ORM BPM naming convention is employed. This convention distinguishes the two dimensions of a single BPM, dividing a two-directional BPM into two different BPMs, a horizontal and vertical BPM. The convention then gathers all BPMs of the same direction and numbers them consecutively.
This applies to the RingScan analysis as follows. The 20 BPMs in the PSR (18 real and 2 that do not really exist, but are included in the analysis because data is still collected for these BPMs) are divided into 40 different BPMs. BPMs 1-20 are the horizontal BPMs, and BPMs 21-40 are the vertical BPMs such that BPM 1 and BPM 21 are the horizontal and vertical divisions of SRPM01, BPMs 2 and 22 are the horizontal and vertical parts of SRPM02, and so on. This means that the missing BPMs are indexed as BPMs 10, 15, 30, and 35.

Results from the cosine wave fit to the BPM data, averages, rms measurement spreads, fitting errors, and correlations are reported in Sec. 2.6.

2.4 Errors in Data Acquisition

All scans (40 turns at a BPM) should look like Fig. 2.4, but some scans do not look like a cosine wave and could not possibly describe beam motion at the BPM. There are several types of these data acquisition errors and each will be described in detail in this section. How these errors are identified and removed from the dataset, possible causes for these errors, how these errors would affect the total RingScan beam position measurement if not removed from the dataset, and the rate of occurrence of these errors in this reproducibility measurement will also be discussed. The rates for the data acquisition errors are derived from the number of scans at existing BPMs (2 × 18 × 101 = 3636), and scans from missing BPMs are not considered in the total count.

A technique often employed in the analysis of data acquisition errors is a power ratio comparison. The power ratio is a convenient way to compare data from different BPMs of the same direction because the power ratio, unlike the position, only depends on the calibration coefficients of the horizontal or vertical AFEs, as described in Sec. 2.1. For a particular turn, the power ratios can be calculated by solving the nonlinear
coupled Eq. (2.3) for $R_x$ and $R_y$ using the RingScan position data $x$ and $y$. Again the Matlab fitting function \textit{lsqnonlin} is employed. This conversion, although time consuming, is very accurate with a maximum SSR $6\times10^{-20}$.

Some data acquisition errors are said to be x-y symmetric meaning that if the error occurs in the horizontal, it is also observed in the vertical. This information will help when trying to diagnose the cause of the error. A symmetric error points to something that will affect both horizontal and vertical measurements like a trigger (since the horizontal and vertical data are taken, selected, digitized, and saved in buffers separately but simultaneously). A data acquisition error that is not x-y symmetric suggests an error due to bad signal or extreme measurement error as in the case of outliers in the fitted parameters.

2.4.1 BPM selection errors

BPM selection errors look like cosine waves and fit cosine waves very well. This is because they are cosine waves. These errors were originally only identifiable by comparing several scans from the same BPM in a multiple RingScan measurement. BPM selection errors are x-y symmetric and tend to be outliers in the fitted amplitude and offset parameters. The fitted phases of these outlying scans match the phase of the previous BPM. The tune, however, is unaffected by this error. The matching phase with the previous BPM led to comparing data from the outlying scan with the scan taken immediately before it (at the previous BPM). The positions and power ratios for an outlying scan (BPM 38 scan 62) and the previous scan (BPM 37 scan 62) are shown in Fig. 2.6.

Comparing the positions of the outlying scan with the scan taken immediately before it, Fig. 2.6, show that the positions (aside from a nonlinear scale factor) track each other completely. The nonlinear scale factor is due to the different geometric
2.4 Errors in Data Acquisition

coefficients for each BPM applied by the IOC to compute the position from the power ratio, Eq. (2.3). For the example in Fig. 2.6, BPM 38 (SRPM82y) is a 4” and BPM 37 (SRPM81y) is a vertical diamond type BPM. If the BMs were the same type, their positions would track each other with a constant difference equal to the sum of their calibration offset coefficients.

It is much more useful to compare the power ratios of the outlying scan with the scan taken immediately before it. Such comparison shows that the power ratios are identical with a power ratio comparison SSR/DOF < 10^{-10}. Thus two phenomena may be occurring, data may be taken twice at the previous BPM or the same data is analyzed more than once. Power ratio comparisons of data taken at the same BPM have an SSR/DOF on the order of 10^{-3} to 10^{-5} because of measurement variations due to measurement error and the pulse-to-pulse central momentum variations of the linac. This proves that data is not taken twice at the previous BPM, but that the
IOC must read the same digitized voltages multiple times. Normally the ADC buffer, where the digitized voltages are stored, is triggered to clear between machine cycles.

Data is taken at BPM A, but the ADC does not clear its buffer. Then the BPM program triggers the MUX to switch to the next BPM, B. Data may be taken at the newly MUX selected BPM B; it is unknown but does not matter because if data is taken, it is written to the end of the ADC buffer, which is read by the IOC. However, the IOC only reads the first $N$ turns of BPM data, which are from the first BPM, A. The IOC does not know which BPM produced the data, and analyzes the repeated data from BPM A as if it was from the MUX selected BPM B. Since the power ratio depends only on horizontal or vertical calibration coefficients of the AFE (Eq. (2.2)), the same digitized voltage values will yield the same power ratios. The recorded positions are different because the geometric coefficients applied in Eq. (2.3) are BPM specific. Since BPM selection errors confuse horizontal data with horizontal data and vertical data with vertical data, the fitted tune parameter is unaffected.

Investigations were pursued to see if the power ratio comparison SSR/DOF could be implemented to identify BPM selection errors, how close are the power ratio differences for good scans, and how different could power ratios be and still have a BPM selection error. For each scan there are 40 power ratios, one for each position data point. A SSR/DOF can be calculated when comparing the power ratios of a scan with the scan taken immediately before it, Fig. 2.7. Since there is no fitting, the number of degrees of freedom is equal to the number of turns of RingScan data.

A couple of interesting features are shown in Fig. 2.7. First, note that there is a large gap of several orders of magnitude between scans without BPM selection errors and scans with identical power ratios, BPM selection errors. This proves that this sort of power ratio analysis can be implemented to identify BPM selection errors. (It will be shown later in Sec. 2.6.3 why the fitted phase parameter could not be applied to identify BPM selection errors.) The threshold value is chosen to be $10^{-10}$ where
2.4 Errors in Data Acquisition

![Graph showing power ratios for BPMs](image)

**Figure 2.7:** A comparison of the power ratios of a scan with the preceding scan for all scans at all BPMs. The horizontal line shows the threshold value, $10^{-10}$. The vertical line divides horizontal (left) and vertical (right) BPMs.

All scans with a power ratio comparison SSR/DOF below the threshold are identified as data acquisition errors. Scans identified as possessing BPM selection errors have finite SSR/DOF ($\sim 10^{-15}$) and are not identically zero. This is taken as round off error in the power ratio fitting or machine precision if the IOC. Also note that the power ratio comparison does not yield x-y symmetric SSR/DOF most likely due to measurement and rounding error. Some scans from the missing BPMs (BPMs 10, 15, 30, and 35) have matching power ratios as the previous scan indicating that BPM selection errors occur even for the missing BPMs.

Scans are identified as BPM selection errors if the power ratio comparison SSR/DOF with the previous scan is less than the threshold value of $10^{-10}$ and the number of turns in the previous scan is the same. (Some scans have a different number of turns than $N$ due to early loss of the 201.25 MHz longitudinal frequency structure).

There are a few scans above the BPM selection error threshold with much smaller power ratio comparison SSR/DOF compared to the other scans without BPM selection errors. The power ratio comparison SSR/DOF for these scans resembles more the power ratio comparisons of scans taken at the same BPM. Meaning that data
could have been taken twice in row at the previous BPM and the second time it was analyzed as if it were data from the current BPM, a stuck MUX error. Stuck MUX errors are discussed later in Sec. 2.4.8. A last comment on Fig. 2.7, note the seven scans in the vertical with the largest power ratio comparison SSR/DOF. These scans will later be identified as large offset and plateau errors.

The same data has been observed to be processed up to four times and even across different RingScan measurements, Fig. 2.8. The cause of the ADC missing the trigger to clear its buffer is unknown, but if this error happens once, it is most likely to happen again in that RingScan.

Seventy eight of the 3636 scans taken were removed for the RingScan reproducibility measurement because they possessed a BPM selection error (2.15%), and eight RingScans of the 101 in the reproducibility dataset had a scan with a BPM selection error, 7.95%. This error does not affect the tune, but does yield a fitted phase equal to the previous BPM. If the CO is similar at the previous BPM or if the coefficients of Eq. (2.3) just happen to work out, this error may not influence the amplitude and CO measurement. However, it is most likely that a BPM selection error will yield
faulty amplitude and offset results.

Since the BPM selection error is due to a missed trigger to clear the ADC buffer, the missing BPMs are not immune from this error. Ten BPM selection errors were observed in the missing BPMs. If the missing BPMs were included in the rate of occurrence statistics, 88 BPM selection errors were found in 4040 scans, 2.18%.

2.4.2 Flat line errors

At first glance, flat line errors are obvious errors, and thus easily identified and removed for the data set, but how this error can manifest itself is not completely understood. Flat line errors are x-y symmetric. Figure 2.9 shows a typical flat line error.

Flat line errors have only been observed in SRPM51 and SRPM72, which immediately follow the missing BPMs. Remember, the BPM program still collects data for these missing BPMs. Multiple flat line errors are consecutively occurring flat line errors and are observed in other BPMs, but the first flat line error of each series
always originates in SRPM51 or SRPM72. Multiple flat line errors are discussed later in this subsubsection. The scan structure of Fig. 2.9 is consistent with all flat line errors, six or seven points of random noise with significant amplitude and the rest of the data equal to zero.

The zeros are easy to explain. The IOC will fill the scan with zeros if it receives less digitized turns from the ADC than the user inputted number of turns requested, \( N \). But this begs the question, if the ADC digitizes all turns the beam is in the ring with appreciable 201.25 MHz intensity before the data is sent to the IOC as described in Sec. 2.1, what could cause the ADC to digitize less than 40 turns?

To understand why there are always six or seven turns of random noise, one needs to look at data recorded from the previous, missing BPM. Data from the missing BPMs always consists of one or two points of random noise. The first data point can be attributed to noise from the ADC disarm trigger, which fires \( \sim 1 \text{ ms} \) after \( T_0 \), long after the beam has been extracted from the PSR[20]. If there is a second point, it seems to be a digitization \( \sim 100 \mu \text{s} \) after the disarm trigger. This could be a result of the software clearing the ADC pipeline (where three turns of digitized data are stored before being saved to the buffer to increase the ADC sampling speed) before reading the buffer. The exact mechanics of how the ADC buffer is flushed is unknown.

The power ratios of the first (and/or second) non-zero point from the missing BPM are the same as the power ratios of the first one or two points of the scan with the flat line error. The flat line error has an additional five points of random noise. So, if the missing BPM records one point of non-zero data, the flat line error will have six points, the first with power ratios matching the non-zero data point from the missing BPM.

For missing BPMs, there is no beam present signal to trigger the ADC to digitize voltage from the AFE. This is why there are only one or two points of data, where the ADC is triggered to digitize data by noise from other triggers, like the disarm trigger.
Since there is no voltage output from the AFE (no BPM for the missing BPMs), the ADC digitizes random noise, which could have sizable amplitude. Data from scans with no beam in the ring look the same as scans from missing BPMs because there are no beam present triggers. Because the first point or two of the flat line error has the same power ratios as the data recorded for the missing BPM, this means that the ADC buffer is not clearing properly and the same data is analyzed for multiple BPMs. The addition of five points of random noise to the data from the missing BPM is presumably digitizations triggered by noise from both the ADC arm and disarm triggers and three points from flushing the pipeline.

![Figure 2.10:](Color) The horizontal power ratio in dB for the missing BPM (BPM 15) and 7 consecutive flat line errors of a multiple flat line error starting with BPM 15 (SRPM71x) scan 7 and ending with BPM 2 (SRPM02x) scan 8.

While it did not occur in this RingScan reproducibility measurement, a multiple flat line error was observed in the ORM dataset, Fig. 2.10. This is where flat line errors happen for consecutive BPMs. Each new flat line error possesses matching
power ratios with the missing BPM and all of the previous flat line errors with five additional points of random noise. It should be noted however, in a particular case where seven flat line errors occurred in a row (Fig. 2.10), five new points of data were not always added. Once, six points of random noise was added to the data from the previous flat line error and once only four points. However, five additional points of random noise were added for the other five flat line errors. The multiple flat line error in Fig. 2.10 starts in BPM 16 scan 7 and continues into the next RingScan to BPM 2 scan 8. The RingScan program had to be reengaged to measure another scan. The time between RingScans is generally about 3 s. This indicates the scale on which the digitized voltages can be stored.

The random nature of the recorded BPM data from missing BPMs or flat line errors is reminiscent of the type of scan seen when RingScan data is taken and both outputs from the MUX are unplugged and unterminated, Fig. 2.11. Digitized signals from unterminated cables in air can lead to positions with significant amplitude. This indicates that the digitized data in scans for missing BPMs or with flat line errors are most likely digitizations of random noise from an unterminated source.

![Figure 2.11: Horizontal position data taken while both horizontal outputs from the MUX were unplugged and unterminated.](image-url)
2.4 Errors in Data Acquisition

For a flat line error to occur, data is recorded for the missing BPM, and most importantly, the ADC buffer does not clear. What happens next is not known, but it is believed that the ADC arm trigger misfires, arming the ADC in the middle or between machine cycles. There are no beam present triggers because the 201.25 MHz intensity has decayed or the beam gate is off at this time. The ADC is then triggered to digitize by noise from other triggers such as the ADC arm and disarm triggers. However, this does not explain why five points of data are recorded for real BPMs and only one or two for the missing BPMs.

Flat line errors are identified as scans that have matching power ratios (power ratio comparison SSR/DOF less than the threshold value of $10^{-10}$) and more points of turn-by-turn BPM data compared the previous scan. Flat line errors were found in ten of the 3636 scans (.28%) and in five of the 101 RingScans (4.95%) taken. If left in the data set, a flat line error will result in bad measurements for all of the fitted parameters.

2.4.3 Modifications to the ADC triggering system

During the August-September 2009 maintenance week, E. Bjorklund of AOT-IC implemented a new triggering scheme for the PSR BPM ADCs[21]. The new triggering scheme was implemented in hopes of curing the flat line data acquisition error. It was believed that one possible cause of the flat line error was that the ADC was mistakenly armed by noise from other triggers. Then, the ADC could start digitizing data in the middle of the beam pulse instead of the beginning. Somehow this mis-triggering also affects the clearing of the ADC buffer, but the exact mechanics are unknown.

The ADC triggering system was modified by sending an additional disarm trigger just prior to the arm trigger at the beginning of the beam pulse. This forces a possibly mis-armed ADC to disarm and synchronize with the IOC and EPICS data acquisition
timings. Table 2.5 holds a record of the number of BPM selection and flat line errors found in the data acquisition of RingScan experiments performed prior to the August-September 2009 maintenance week. Notice the totally random nature of these data acquisition errors. Sometimes the combined rate of error for the BPM selection and flat line errors is as high as 4.5% as was found on October 21, 2008, and other times it is as low as .5% as on July 26, 2009.

<table>
<thead>
<tr>
<th>Date</th>
<th>Scans</th>
<th>BPM Selection Error</th>
<th>Flat Line Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>June 26, 2008</td>
<td>86832</td>
<td>229 (2.567%)</td>
<td>316 (.364%)</td>
</tr>
<tr>
<td>Sept. 26, 2008</td>
<td>3456</td>
<td>10 (.289%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Oct. 21, 2008</td>
<td>2808</td>
<td>126 (4.487%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Nov. 11, 2008</td>
<td>44496</td>
<td>290 (.652%)</td>
<td>26 (.058%)</td>
</tr>
<tr>
<td>May 26, 2009</td>
<td>3420</td>
<td>72 (2.105%)</td>
<td>12 (.351%)</td>
</tr>
<tr>
<td>July 26, 2009</td>
<td>25452</td>
<td>119 (.468%)</td>
<td>16 (.065%)</td>
</tr>
<tr>
<td>Aug. 3, 2009</td>
<td>6480</td>
<td>68 (1.049%)</td>
<td>2 (.031%)</td>
</tr>
<tr>
<td>Aug. 20, 2009</td>
<td>7200</td>
<td>350 (4.861%)</td>
<td>48 (.667%)</td>
</tr>
</tbody>
</table>

Table 2.5: A record of the number of BPM selection and flat line errors found in the data acquisition of RingScan experiments between June 26, 2008 and August 20, 2009 before the modification to the ADC triggering system.

Compare the results in Tab. 2.5 with the record of BPM selection and flat line errors observed in the acquisition of RingScan data after the modification to the ADC triggering system, Tab. 2.6. The modification to the ADC triggering system has but completely removed the BPM selection and flat line error from the list of data acquisition errors. It appears that the trigger modification has somehow forced the clearing of the ADC buffer such that BPM selection and flat line errors are very rarely observed in the RingScan data. Only one BPM selection error has been identified
by the analysis scripts since the modification to the ADC triggering. Although the BPM selection and flat line errors are now very rare, the analysis scripts still execute the time consuming calculation to solve for the power ratios from the beam position data for comparison with the previous scan to identify the BPM selection and flat line errors.

<table>
<thead>
<tr>
<th>Date</th>
<th>Scans</th>
<th>BPM Selection Error</th>
<th>Flat Line Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sept. 8, 2009</td>
<td>3600</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Sept. 26, 2009</td>
<td>66276</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Oct. 3, 2009</td>
<td>10080</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Dec. 22, 2009</td>
<td>12240</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>Sept. 25, 2010</td>
<td>87228</td>
<td>2 (.002%)</td>
<td>0 (0%)</td>
</tr>
</tbody>
</table>

**Table 2.6:** A record of the number of BPM selection and flat line errors found in the data acquisition of RingScan experiments between September 8, 2009 and September 25, 2010 after the modification to the ADC triggering system.

### 2.4.4 Missing turn errors

Missing turn errors were one of the first errors identified in the RingScan measurement. After taking the reproducibility measurement, it was of interest to gain an understanding of the quality of the tune measurement using the RingScan program. The as measured tune-tune plot is shown in Fig. 2.12. The plot reveals a main island with several outliers. There are two types of outliers. The non x-y symmetric outliers (the outliers directly above, below, left, and right of the main island) will be shown to be large offset and plateau errors, which are described Secs. 2.4.5 and 2.4.6 respectively. The x-y symmetric outliers (on the diagonal from the main island forming a
line with slope $\sim 1$) are scans with missing turn errors. Notice there are two groups of missing turn errors which form this line of slope $\sim 1$. The group closer to the main island is spread out, and the second group is well clustered and much further away from the main island.

Figure 2.12: Tune-tune plot for all scans at existing BPMs with no data acquisition errors removed.

Figure 2.13 shows an example of a typical scan with a missing turn error. The missed turn is observed by comparing the BPM data with the initial cosine wave fit guess, green. Although the initial cosine guess is not perfect, it does have a constant frequency that in the beginning of the scan matches up well with the BPM data. At turn 24, the cosine guess indicates a maximum in the oscillation, but the BPM data, goes down from the previous turn instead of up like the initial cosine guess. From turn 24 on, the RingScan data is one turn ahead of the initial cosine guess. However, if the BPM data were delayed a turn for turns after 24, the BPM data with the added turn and the initial cosine guess match for the whole scan.

A missing turn error occurs when the ADC digitizes the AFE output voltage as normal, but then for some turn, the beam present trigger does not trigger the ADC to digitize data. The beam passes by the BPM and no data is taken. The following
beam present triggers prompt the ADC to digitize data as normal.

Investigations were made to see which BPMs had missing turn errors and which turns were missed, Figs. 2.14 and 2.15. From Fig. 2.14, it is easy to see that only five out of the 18 existing BPMs have missing turn errors. There is also a correlation between the turn that is missed and the BPM. For instance, BPM 8 (SRPM32) only has missing turns between turn 17 and 25, while BPM 1 (SRPM01) has missing turns at turn 11 and 35 through 39. The RingScan data also suggests that the same turn is missed in both the horizontal and the vertical scans. This is understood in the above explanation that this error is due to a missing beam present trigger which triggers the ADC to digitize a turn of both horizontal and vertical data. This is why in Fig. 2.15 the missing turns always come in pairs. The reason why missing turn errors only manifest in five of the 18 existing BPMs is unknown, but since the mechanism for the ADC not to digitize a single turn of BPM data is a missing beam present trigger (which is sent if the beam signal on the bottom BPM electrode is above some
2.4 Errors in Data Acquisition

Figure 2.14: Shows the BPM and the turn added for every missing turn error. The turn missed is x-y symmetric. The vertical line separates the horizontal (left) and vertical (right) BPMs.

Figure 2.15: A histogram of the turn missed for each missing turn error. Turns are missed in pairs, one for the horizontal and one for the vertical scan.
threshold), one wonders if there is something different about the cables and stripline of the bottom BPM electrode for these BPMs.

In the RingScan reproducibility measurement, 106 turns were missed out of 145440 (.073%), 96 scans of 3636 were removed from the data set because they had missing turn errors (2.64%), and 42 out of 101 RingScans were observed to contain a scan with a missing turn error, 41.58%. If left in the data set, the missing turn error will compromise the tune measurement and most likely the phase through correlation in the fitting. The RingScan amplitude and offset measurement are left unaffected. However, a CCR BPM program style measurement of the CO would suffer because it would no longer be averaging over an integer number of oscillations.

A few remarkable notes about this error: Two scans removed for a BPM selection error were also observed to have a missing turn error. These two scans are counted as BPM selection errors and do not figure into the statistics of the missing turn error. An observant reader would have noticed in the preceding paragraph 106 turns were
2.4 Errors in Data Acquisition

quoted as missing but only 96 scans were removed for this error. This is because 10 scans had double missing turn errors, Fig. 2.16. In Fig. 2.16, the BPM data skips ahead of the initial cosine guess by one turn at turn 15. The BPM data is then one turn early until turn 24 where it jumps ahead by another turn compared to the constant frequency initial cosine guess. If turns are added to the BPM data to make up for the missing turns of data red the BPM data matches the initial cosine guess. While not in the RingScan reproducibility dataset, scans with up to three missing turns have been observed.

Looking back at the tune-tune plot of Fig. 2.12, the structure of the two outlying groups of tune measurements on the diagonal line of slope $\sim 1$ can be explained. The cluster of five points furthest from the main island is comprised of the scans with two missing turns. The scans closer to the island only have one missing turn. The reason why the scans with one missing turn have such spread is that the fitted tune depends on which turn is missed. If a turn in the middle of the scan is missed, the fitting program will try to compensate for the apparent phase shift in the middle of the scan and yield a fitted tune further from the main island. If a turn is missed early or late in the scan, the majority of the scan is at the same phase and the tune fit is less affected, fitting a tune closer to the main island.

A scan is identified as missing turn error and removed from the dataset if both horizontal and vertical scans are outliers in the fitted tune parameter, are not identified as large offset or plateau errors (discussed later in this section), and the SSR/DOF of between the initial cosine wave guess and the RingScan data can be improved by adding the same turn to both horizontal and vertical scans.
2.4.5 Large offset errors

Large offset errors are called such because these scans fit offsets \(\sim 17\) mm with very small amplitude oscillations, Fig. 2.17. This error constitutes some of the non x-y symmetric outliers (directly above, below, left, and right of the main island) in the tune-tune plot of Fig. 2.12, and some of the large power ratio SSR/DOF comparisons in the vertical of Fig. 2.7. Large offset errors are not x-y symmetric, suggesting a signal problem.

![Figure 2.17: (Color) BPM 25 scan 78 is a typical large offset error, many points of small amplitude random noise centered around \(\sim 17\) mm. The dashed red line is the fitted offset for this scan.]

Large offset errors are scans identified as outliers in the fitted tune and offset parameters and with all moving five turn position averages greater than 15 mm. These scans cannot be identified using the SSR for the cosine wave fit because the small variation of the data leads to a small residual for any small amplitude cosine fit. Unlike the large offset error shown in Fig. 2.17, some large offset errors have been observed to possess sinusoidal motion of appreciable amplitude instead of the small amplitude random noise. For this reason, the large offset errors are not identified
2.4 Errors in Data Acquisition

based on small fitted amplitudes. Large offset errors have only been observed in the positive vertical direction.

It is interesting that all large offset errors have offsets of $\sim 17$ mm. It is also interesting that this is the approximate distance from the center of the BPM where the BPM measurement starts to become nonlinear. Real beam positions of 17 mm produce AFE voltages just outside the linear region of the tangent function in Eq. (2.2), meaning that a small change in voltage due to measurement error could lead to a huge change in the power ratio. And when the IOC converts the digitized voltage back to a beam position via Eqs. (2.2) and (2.3), the measured beam position is observed to possess some nonlinear saturation. How the BPMs respond to large positions and saturate is not fully understood. As an example of how the BPM measurement responds to a completely saturated BPM signal, RingScan measurements were made when one of the electrode outputs of the MUX was unplugged and left unterminated, Fig. 2.18. Although the positions do not exactly match in Figs. 2.17 and 2.18, qualitatively these scans appear very similar. The first turn of data is different than the rest and closer to the zero position, and both scans max out at with large offsets and have small amplitude oscillations.

The comparison of the large offset error with data from an unplugged MUX cable leads to the following explanation of this error. When the MUX switches from BPM to BPM, there are actually four switches switching (four MUXs), one for each of the BPM electrodes. All large offset errors were observed in vertical BPMs with a resulting positive offset, suggesting that for these scans the bottom electrode switch did not connect properly, and thus only minimal signal from the bottom BPM electrode is allowed to pass the MUX. To produce the noisy signal, the switch, which is barely connected, must be “mostly unterminated” like the unplugged cable that made Fig. 2.18. Since the MUXs are mechanical switches, one could imagine an error rate of $\sim 1\%$[22], which is about the rate the large offset errors were observed in the
RingScan reproducibility dataset. The minimal amount of signal allowed to pass the MUX must be enough to activate the beam present trigger because the digitization in the other direction is unaffected, yielding scans in the other dimension without a data acquisition error.

Even though good signals may be gathered in the other direction from the large offset error, the resulting calculated position is affected by the large offset error because of the coupling term in Eq. (2.3), which the IOC invokes to calculate the beam position from the power ratios. So to ensure good data, both scans are removed from the dataset.

Only four scans out of the 3636 (.11%) and four RingScans out of the 101 (3.96%) taken for the reproducibility measurement were observed to have large offset errors. Large offset errors will yield bad fits for all of the fitting parameters.
2.4.6 Plateau errors

Plateau errors are like large offset errors, but for only part of the scan, Fig. 2.19. Three plateau errors were observed in the RingScan reproducibility measurement, .08% of the 3636 scans, and in 3 of the 101 RingScans, 2.97%. Each of the plateau errors observed was a little different. There is no typical shape for a plateau error, but each of the plateau errors has a few characteristics in common. Plateau errors, like large offset errors, are not x-y symmetric, are only observed to plateau around a positive 17 mm (the same position as the large offset errors), the jump from the floor to the plateau top only takes one or two turns (360-720ns), and when the scan is not plateauing the scan possesses normal betatron oscillation.

![Figure 2.19](Color) BPM 21 scan 3 with a plateau error. The maximum and minimum moving five turn position average are shown by the dashed red lines.

Observing the similarities between plateau errors and large offset errors, especially in what position the scans plateau, one could envision the same type of explanation for this error. Plateau errors only plateau in the positive direction; this indicates the lower and outside (left) electrodes are not connecting properly or securely when
the MUX is switched. In this case it appears that the quality of contact can vary on the time scale of \( \sim \) 360 ns, or one turn. Although, the actual reported switching time for the mechanical MUX is on the order of a few milliseconds, one can imagine the actual contact of a metal whisker (the switch whose contact end is only a few atoms in width) to either be connected or not. And this connection time scale could be on the order of a few nanoseconds\(^{[22]}\). When the MUX switch is not connected firmly, it is not terminated properly and only allows limited signal to pass producing a saturated position.

Plateau errors along with large offset errors are the non symmetric outliers directly above, below, left, and right of the main island in the tune-tune plot of Fig. 2.12. This is why scans with plateau errors are identified as tune outliers. Plateau errors are also identified as outliers in the fitted offset and if the difference in the minimum and maximum moving five turn position average is greater than 15 mm minus the fitted offset. If not removed from the dataset, plateau errors will contribute bad fits for all of the fitting parameters.

Because large offset and plateau errors are so different compared to the nominal cosine wave scan, their power ratio comparison with the previous scan results is a large SSR/DOF. This can be seen in the series of large SSR/DOF in the vertical BPMs in Fig. 2.7.

### 2.4.7 Drift across the scan errors

Drift across the scan errors are normal sinusoidal-type scans, but the offset of the cosine wave is observed to change linearly across the scan, Fig. 2.20. While the CO can change due to a drifting magnet or momentum change, this measurement was taken with the buncher off (no momentum change), and magnet drifts are on a timescale of minutes and not the \( \sim 14 \) \( \mu \)s across a scan, meaning that the beam
motion described by this scan is unphysical and thus an error.

Figure 2.20: (Color) BPM 20 (SRPM92x) scan 72 has the most significant drift across the scan error in the RingScan reproducibility dataset. The offset of the BPM data (blue circles) drifts $\sim 1$ mm in 40 turns, or 14.4 $\mu$s. The green square trace is the cosine wave fit.

Figure 2.20 shows the most pronounced drift across the scan error in the RingScan reproducibility dataset. The RingScan data starts at the beginning of the scan with an offset of $\sim 2$ mm, but at the end of the scan the offset is $\sim 3$ mm. The cosine wave fit minimizes the least squares by fitting a cosine wave with an offset equal to the offset of the middle of the scan. This fit has negative residuals in the beginning of the scan and positive at the end, but the average of all the residuals is zero. Ultimately one should ask which offset is the correct CO? Is the CO the offset at the beginning of the scan, the middle, or the end? This question has not been answered, but it has sparked investigations into fitting the data differently.

If one scan exhibits this drift across the scan error, perhaps all scans drift to some degree. This motivated fitting the RingScan data to a cosine wave with a linear offset term,

$$x_n = A_1 \cos(2\pi n + \phi) + \text{Offset} + A_2 n,$$  

(2.8)
where $\nu$, $n$, $\phi$, and $O_{\text{offset}}$ are the same as in Eq. (2.6), $A_1$ is the amplitude of Eq. (2.6), and $A_2$ is the slope of this linear offset drift across the scan, [mm/turn]. Equation (2.8) was employed to fit the RingScan reproducibility dataset with the same procedure as described in the Sec. 2.3. The initial $A_2$ guess was zero for all scans. The results of fitting Eq. (2.8) to the RingScan reproducibility dataset and extracting the slope of the offset drift are shown in Fig. 2.21.

From Fig. 2.21 it can clearly be seen that all scans have some kind of drift across the scan. There are a few interesting things to note from Fig. 2.21. The outlying point for BPM 20 is the scan plotted in Fig. 2.20. The drift across the scan is not x-y symmetric. The average drift for the horizontal BPMs is $\sim 0.01$ mm/turn. For a scan of 40 turns, this is a $0.4$ mm drift across the scan, which is about twice the BPM single turn measurement error and almost three times the horizontal CO \textit{rms} measurement spread, detailed in Sec. 2.6. The vertical drift is smaller, averaging $\sim 0.005$ mm/turn. Notice that BPMs 37 and 39 (SRPM81y and SRPM91y) have the largest drifts out of all of the BPMs. These are the vertical diamond type BPMs, which are observed to be noisier than the others, discussed more in the Sec. 2.6. BPM 13 (SRPM61x) is

\textbf{Figure 2.21:} The $A_2$ of Eq. (2.8), the slope of the offset drift across the scan, for all scans in the reproducibility dataset with all data acquisition errors except offset outlier errors removed. The vertical line separates the horizontal (left) and vertical (right) BPMs.
the only BPM that straddles zero. One would expect all of the BPMs to display this type of behavior. The last BPM that is unlike the others is BPM 34 (SRPM62y). It is the only BPM that drifts negatively.

The mechanism that causes these drifts is unknown, but since it seems to have some dependence on BPM, anything from the BPM electrodes to the MUX is suspect. Since most of the BPMs drift positively, the termination on the lower and left (outer) electrode or the cables from these electrodes to the MUX could be faulty. Or could differential charge accumulation from beam losses on the striplines be an explanation of this error? Any explanation of this drift should include how signal from one of the electrodes could vary linearly, even with the AM to PM modulation performed in the AFE, for the time scale of a scan, \( \sim 14 \mu s \).

Drift across the scan errors are not specifically identified and removed from the dataset. The scans with the most significant drifts across the scan are later selected as offset outliers and removed from the dataset. If left in the dataset, drifts across the scan errors increase the spread in the fitted offset distribution while yielding good measurements in the other fitting parameters.

### 2.4.8 Single scan drift errors

A single scan drift error is an outlier in the fitted offset that varies quite a bit from the average offset measured at a particular BPM. Figure 2.22. It is a scan whose fitted offset seems to have drifted significantly, but just for that one scan. In general, single scan drift errors are not x-y symmetric. Figure 2.22 shows two outliers in the fitted offset parameter at BPM 7 (SRPM31x) scans 11 and 18. Scan 11 is \(-5.9\) \( \text{rms} \) standard deviations from the average, and scan 18 differs from the average by 6.8 \( \text{rms} \) standard deviations. These scans obviously do not correctly represent the measured CO at BPM 7 and thus must be measurement error. But what could cause a single
scan to be so different than the rest at that BPM?

![Figure 2.22](image.png)

**Figure 2.22**: (Color) The fitted offsets for all scans at BPM 7 (SRPM31x) in the reproducibility measurement including two single scan drift errors (scans 11 and 18). Red squares represent scans removed from the dataset for possessing data acquisition errors, while the blue circles constitute the remaining offset distribution.

One such single scan drift error was observed in BPM 3 and 23 (SRPM11) scan 52. There is fairly good evidence that these two scans are new data taken at the previous BPM as in the case of a stuck MUX error. The power ratio comparison SSR/DOF with the scans at the previous BPM more resembles a comparison of two scans at the same BPM, Fig. 2.7. The fitted amplitudes of these scans are outliers at their BPM, but they match the amplitudes of the BPM before it. And the offsets for these scans also match the offsets of the previous BPM, making the vertical scan an outlier in the offset by \(-109 \text{ rms}\) standard deviations. These two scans are identified and removed from the dataset as outliers in the fitted parameters.

So if one single scan drift error can be a stuck MUX error, perhaps all of these errors could be explained in the same way. However, it is not as straight forward as this. First off, the complete stuck MUX error, where all four MUXs do not switch, is x-y symmetric in the power ratio comparison (if not necessarily x-y symmetric in amplitude and offset outliers), and there are no other x-y symmetric scans in Fig.
2.4 Errors in Data Acquisition

2.7 that have power ratio comparisons similar to SRPM11 (BPM 3 and 23) scan 52. Also observing fitted amplitudes similar to the previous BPM can happen for every scan taken at a BPM immediately after a missing BPM because the BPMs are in the same type of quadrupole (focusing or defocusing) and thus should have a maximum or minimum in the beta function. And observing the offset to be the same as the previous BPM could happen for all scans at a BPM if the CO at the two BPMs is steered just right or well centered.

All of the single scan drift errors are not complete stuck MUX errors. A more likely but unsubstantiated explanation follows. What would a scan look like if only three out of the four MUXs switched to the next BPM and new data was taken, a partial stuck MUX error? This would yield normal scans in one dimension. In the other direction, the power ratio of Eq. (2.3) would compare signal from electrodes at two different BPMs. The beam in the PSR is \( \sim 72 \) m in length, which is more than enough to cover two consecutive BPMs (\( \sim 5 \) m apart) at the same time. This could lead to very different results in the fitted amplitude, phase, and offset than normal at that BPM. This type of error would almost be impossible to specifically diagnose from the beam position data.

In order to mitigate all complete or partial stuck MUX errors, scans with fitted parameters outside of three \( \text{rms} \) standard deviations of the distribution are identified and removed from the dataset. Of course if left in the dataset, the single scan drift errors would affect the CO measurement and may, by their very nature, influence the amplitude and phase fits as well.

2.4.9 Drift across multiple scans errors

The last of the drift-type errors, drift across multiple scans errors are only observed in BPM 20 (SRPM92x). Figure 2.23 plots the fitted offset of all of the scans at BPM
2.4 Errors in Data Acquisition

20. The first ~50 scans form a narrow distribution as expected, but scans in the latter half of the reproducibility measurement fit erratic offset values. This is the drift across multiple scans error. The CO measurement for BPM 20 was so unstable that it could not be used in the ORM analysis, described in Sec. 4.1. This sort of behavior at a single BPM suggests that there might be a faulty termination on one of the horizontal electrodes. Scans with this type of error are removed as offset outliers, as explained later in this section. If left in the dataset, these scans would contribute to the spread in the CO measurement. Unlike the single scan drifts, drift across multiple scans do not seem to affect the amplitude or phase, so are not believed to be due to a partial stuck MUX error.

Figure 2.23: (Color) The fitted offsets for all scans at BPM 20 (SRPM92x) in the reproducibility measurement. Red squares represent scans removed from the dataset for possessing data acquisition errors, while the blue circles constitute the remaining offset distribution.

2.4.10 Amplitude growth errors

Although not seen in the reproducibility dataset, this error is observed in other datasets. A typical amplitude growth error is plotted in Fig. 2.24. Like the drift
errors, amplitude growth errors are not specifically searched for in the data error analysis scripts. However, if the scans are very bad, they will be caught as fitted parameter outliers. Scans with amplitude growth errors generally fit similar tunes, phases, and offsets when compared to other scans at the same BPM. The disturbing characteristic about amplitude growth errors is that for a measurement of 20 RingScans if one BPM has an amplitude growth error, all scans at that BPM will have an amplitude growth error. Perhaps this is why the fitted parameters of a scan with an amplitude growth error are in good agreement with the other parameter fits at a particular BPM. So aside from fitting the RingScan data with a cosine wave of varying amplitude, it is virtually impossible to identify these errors. If because of an amplitude growth error, the tune is not fit well, then all scans at a BPM may be removed as tune outliers. This leaves a hole in the RingScan data for that measurement.

![Figure 2.24](image)

**Figure 2.24:** (Color) A typical amplitude growth error. The RingScan data is plotted in the blue circles, and the cosine wave fit is the green squares.

The scan in Fig. 2.24 starts with an amplitude of ~1 mm, but by the end of the scan the amplitude has more than doubled to ~2.5 mm, while the offset is unaffected.
2.4 Errors in Data Acquisition

Amplitude growth errors are not x-y symmetric. Amplitude growth errors cannot represent real beam motion because it is very unlikely for an instability to occur over 40 turns in single shot mode in the PSR, especially when amplitude growth errors are not x-y symmetric and are observed to occur consistently for only one BPM. Remember that data is taken at the other 19 BPMs before data is again collected at the BPM in question. This points to a problem in the data acquisition, but somewhere that only affects one dimension (cables, MUX, AFE, ADC) and in a way that increases the amplitude but does not affect the offset.

2.4.11 Fitted parameter outliers

After all of the scans are fit to a cosine wave and the above BPM data acquisition errors are identified and removed, each fitted parameter distribution is checked for outliers. This grooming serves two purposes: one, it identifies many of the BPM data acquisition errors including possible total and partial stuck MUX errors, and two, it cleans up the RingScan measurement resulting in smaller measurement spreads for each fitted parameter. Each fitted parameter distribution has a maximum measurement spread imposed on the measurement distribution which is calculated taking into account some aspects of the RingScan data. Outliers are removed from each fitted parameter distribution until the $\text{rms}$ standard deviation of the fitted parameter distribution is less than the maximum measurement spread. The scans that reduce the $\text{rms}$ spread the most are removed first. Lastly, all remaining outliers outside of three standard deviations of the parameter distribution are identified as outliers. In order not to favor one parameter over another for the purposes of recording the parameter outliers, the outliers for each fitted parameter distribution are first found independently and then all scans with an outlying parameter are removed at once. So it is possible for a single scan to possess outliers in multiple fitted parameters. In general,
fitted parameter outliers are not x-y symmetric.

I Amplitude outliers

There is a fitted amplitude distribution for each BPM. Since the amplitude measurement spread is dominated by the pulse-to-pulse central momentum variations, the fitting errors cannot be employed to calculate the maximum amplitude measurement spread. The effect of the pulse-to-pulse momentum variations on the position measurement can be quantified by the spread in the turn-by-turn BPM data. Since the amplitude describes the height of the cosine wave, it is easy to conclude that the \( \text{rms} \) standard deviation of the amplitude measurement distribution should be less than or equal to the average \( \text{rms} \) position spread for each turn in the turn-by-turn BPM data. The average \( \text{rms} \) position spreads for BPMs with small amplitude betatron oscillation range from 1.5-1.7 mm and 2.8-4.5 mm in BPMs with large amplitude betatron oscillations. Amplitude outliers outside of four or five \( \text{rms} \) standard deviations could indicate a total or partial stuck MUX error, but amplitude outliers just outside of three \( \text{rms} \) standard deviations most likely result from extremes in the pulse-to-pulse central momentum variations changing the injection offset for that scan. Seven (.19\%) scans were removed from the RingScan reproducibility dataset as amplitude outliers. The spread in the amplitude measurement will be larger if these scans are not removed from the dataset.

II Tune outliers

Tune outliers are the scans left over from the hunt for missing turn, large offset, and plateau errors. Since all scans in the same direction measure the same tune, all horizontal scans are treated together as one measurement distribution and likewise for the vertical. Outlying scans are removed one at time from the tune distribution
until the $rms$ standard deviation is less the maximum tune measurement spread. The measurement spread for the fitted tune parameter is dominated by the fitting error and any systematic difference in the average tune measurement at each BPM. So, the maximum tune measurement spread is calculated as the average of the fitting errors on the fitted tune parameter plus the $rms$ spread in the tune average at each BPM. For the reproducibility measurement, the limits on the $rms$ tune spread were $[8.35 \times 10^{-4}, 6.20 \times 10^{-4}]$, while the final $rms$ tune spread over all BPMs of the same direction was $[4.26 \times 10^{-4}, 3.38 \times 10^{-4}]$. The scans removed as tune outliers fit tune values just outside the three standard deviation range. There were no tune outliers identified and removed from the reproducibility dataset. However, if tune outliers are not removed from the dataset, they affect the spread in the tune measurement.

### III Phase outliers

The spread in the measured phase distribution is also dominated by the pulse-to-pulse central momentum deviations. However, there is no convenient way to convert the spread in the position measurement to a spread in the measured phase. Thus, there is no maximum measurement spread for the fitted phase distributions. Outliers are continually removed from the measured phase distribution until there are no longer fitted phase values outside of three $rms$ standard deviations of the measured phase distribution. Each BPM has its own measured phase distribution. Outliers in the fitted phase most likely arise due to extremes in the pulse-to-pulse central momentum deviations changing the injection offset. However, a phase outlier outside of four or five $rms$ standard deviations may be due to a partial stuck MUX error. There were 19 (.52%) scans removed from the reproducibility dataset as phase outliers. All but one of the phase outliers were in horizontal scans, and they were all outliers on the lower side of the phase distributions. This suggests that the horizontal phase is more
variable than the vertical and is most likely due to the smaller injection offset in the horizontal. The near-on-axis injection scheme used for the RingScan reproducibility measurement greatly increases the variability in the phase measurement due to the pulse-to-pulse momentum variations. The spread in the phase measurement increases if these scans are not removed from the dataset.

IV Offset outliers

Offset outliers also include drift across the scan errors, single scan drift errors, and drift across multiple scans. The horizontal offset measurement spread is dominated by the pulse-to-pulse central momentum variations. Since the average fitted offset can be thought of as the average of the average of position for each turn, the $rms$ standard deviation of the fitted offset distribution must be less than the average $rms$ position spread for each turn. Each BPM has its own measured offset distribution. Offset outliers just outside three $rms$ standard deviations of the offset distribution most likely arise from extremes in the pulse-to-pulse momentum variations changing the CO through dispersion. Offset outliers outside four or five $rms$ standard deviations, such as single scan drift errors, indicate total or partial stuck MUX errors. There were 30 (.82%) scans identified and removed as offset outliers, five of which were scans with drift across multiple scans errors from BPM 20. If these scans are not removed, they will cause a larger spread in the CO measurement.

V Scans with multiple fitted parameter outliers

Since outliers are identified for each fitted parameter distribution independently and before any of the outlying scans are removed from the dataset, it is possible for a scan to be tagged as possessing an outlier in more than one fitted parameter distribution. While there is no difference in the characteristics of these fitted parameter outliers
2.4 Errors in Data Acquisition

compared to the outliers of the same type described above, scans with multiple fitted parameter outliers are listed separately as to not prefer outliers of one parameter over another. Scans with multiple fitted parameter outliers are also better candidates for total and partial stuck MUX errors.

There are four combinations of multiple parameter outliers: amplitude and phase; amplitude and offset; phase and offset; and amplitude, phase, and offset. The fitted tune parameter is not considered in the combinations of multiple parameter outliers because it is believed that total and partial stuck MUX errors do not fit a tune outlier. Also combinations of tune outliers are considered during the identification of large offset and plateau errors. There were no scans identified as amplitude and offset outliers, but one (.03%) scan was removed from the reproducibility dataset for each of the other outlier combinations: amplitude and phase; phase and offset; and amplitude, phase, and offset.

VI $\sigma_{BPM}$ outliers

Along with the other fitted parameters, the single turn BPM error ($\sigma_{BPM}$) is an output of the ML error analysis of the cosine wave fit to each scan. The single turn BPM measurement error is the $\text{rms}$ standard deviation of the residual distribution. Grooming the $\sigma_{BPM}$ distribution serves as a second filter for missing turn errors. Some scans with missing turn errors may have luckily yielded a good tune fit, however the missing turn error will always affect the single turn BPM measurement error. So all scans identified as $\sigma_{BPM}$ outliers are first checked for missing turn errors. The single turn measurement error is positive definite, so only outliers on the large side of the $\sigma_{BPM}$ distribution are identified such that well fit scans (scans on the lower $\sigma_{BPM}$ side of the distribution) are not removed from the dataset.

One might suspect that the error estimate on $\sigma_{BPM}$ from the ML error analy-
2.4 Errors in Data Acquisition

sis could be applied to the $\sigma_{BPM}$ distributions as a maximum measurement spread. However, there is another source of spread in the $\sigma_{BPM}$ distributions due to the drift across the scan error, so the $\sigma_{BPM}$ parameter is treated like the fitted phase parameter, and outliers are removed until no more remain outside of three $rms$ standard deviations. Each BPM has its own $\sigma_{BPM}$ distribution. There were 32 (.88%) scans in the reproducibility dataset removed as $\sigma_{BPM}$ outliers.

Most of the $\sigma_{BPM}$ outliers are in vertical scans. Five scans at BPM 40 (SRPM91y) and 12 scans at BPM 28 (SRPM32y) were identified as $\sigma_{BPM}$ outliers. A $\sigma_{BPM}$ outlier could be the result of a single turn of bad BPM data. But both BPMs 28 and 40 are also observed to possess missed turn error. It will be shown later in Sec. 2.5 that BPMs observed to yield missing turn errors have large variability in the residual during the range of where the BPM misses turns.

2.4.12 Data take out

After all of the scans are fit with a cosine wave, scans with data acquisition errors are identified as described in this section and removed. Most of the errors are x-y symmetric and thus both scans (one in each direction, whose data was taken at the same time) at that BPM are removed. Both scans are also removed for scans with data acquisition errors and fitted parameter outliers that are not x-y symmetric. Obviously, in this case, the scan with the error should be removed. The scan in the other direction is removed for data quality purposes. Although the data acquisition for this scan may have been unaffected by the error in the scan of the other direction, when the IOC calculates the position for the good scan (Eq. (2.3)) it couples the power ratios from the other dimension causing the ICO calculated positions for the scan without a data acquisition error to be bad. So when there is an error, scans in both directions are removed.
2.4 Errors in Data Acquisition

After removing all of the scans with data acquisition errors as described above, one should check how many scans remain for each BPM and determine if there are enough scans remaining to discuss statistically the reproducibility of the RingScan measurement. Figure 2.25 is a histogram of the number of scans remaining at each BPM. Note that at least one scan has been removed from each BPM. The BPM with the most scans removed is SRPM22 (BPMs 8 and 28). Missing turn errors and $\sigma_{BPM}$ outliers account for most of the removed scans for this BPM. However, 76 scans still remain for SRPM22, enough for statistics.

Another representation of the distribution of scans removed because of data acquisition errors is shown in Fig. 2.26. The randomness of these errors becomes apparent. The only visible correlations are vertically the missed turn errors and horizontally the BPM selection errors. There was at least one scan removed for every BPM. Totally, 372 scans out of 3636 were removed from the dataset for possessing a data acquisition error or for being identified as a fitted parameter outlier, 10.23%. At least one scan was removed from 83 of the 101 RingScans, 82.18%, so only 18 RingScans were error.
2.4 Errors in Data Acquisition

![Diagram of Scan and BPM for all scans removed from the reproducibility dataset as data acquisition errors or fitting parameter outliers. Shapes represent scans with errors, and x’s are scans in the other direction of removed scans that were also removed. The solid vertical black line separates the horizontal BPMs (left) and the vertical BPMs, right.](image)

**Figure 2.26:** (Color) Scan and BPM for all scans removed from the reproducibility dataset as data acquisition errors or fitting parameter outliers. Shapes represent scans with errors, and x’s are scans in the other direction of removed scans that were also removed. The solid vertical black line separates the horizontal BPMs (left) and the vertical BPMs, right.

free. These numbers do not build an experimenter’s confidence in the data acquisition of the BPM measurement system for the PSR. However, as clearly shown in Fig. 2.25, if one takes enough RingScans, knows of the data acquisition errors, and is able to remove bad data from the dataset, there will be enough scans remaining to obtain quality beam position measurement results.
2.5 Residuals Along the Scan

When fitting data to a function one must inquire several times at different stages in the analysis if the fitting function is the proper function to be invoking. A study of the residuals can help answer this question and provide confidence in the fitting scheme. It will also reveal some aspects of the fitting or data collection that have not come to light in the previous analysis.

Each scan in the RingScan reproducibility dataset has 40 points of turn-by-turn BPM data. This data is fit to a cosine wave, Eq. (2.6). A comparison of the fit to the measured data point results in 40 residuals, one for each turn. The residual of the $n^{th}$ turn is defined as

$$\text{Residual}_n = x_n - (A \cos(2\pi n + \phi) + O_{\text{set}}).$$  \hspace{1cm} (2.9)

Here all of the variables are the same as in Eq. (2.6). Since at least 76 scans remain at each BPM after scans with data acquisition errors are taken out, it is of interest to study the residual distribution for each turn separately at each BPM. This is different than the SSR/DOF parameter employed as the goodness of fit quality factor, where the square of all residuals for a scan are summed. The residual across the scan analysis will give information related to the goodness of data collection and fitting as a function of turn.

Three statistical quantities of the residual distributions are discussed: the average residual (Sec. 2.5.1), the $rms$ standard deviation of the residual distribution (Sec. 2.5.2), and the maximum residual, Sec. 2.5.3.

2.5.1 Average residual

The first quantity of interest is simply the average residual for each turn at each BPM. Figure 2.27 plots the average of the residual distribution for each turn at BPM
2.5 Residuals Along the Scan

5 (SRPM21x). The first thing to notice is the constant slope upward across the plot. This is an artifact of the fitting scheme of Eq. (2.6) fitting BPM data that has a constant offset drift across the scan. As explained in Sec. 2.4.7, in order to minimize the SSR/DOF, the minimization routine will chose the fitted offset as the average offset (the offset in the middle of the scan) such that if the offset drift is positive, the fit will have more positive values in the beginning of the scan compared to the BPM data and more negative values at the end. This is represented in the slope across the average residual. Note that the average residuals are approximately symmetric about turn 20 where the average residual is 0, meaning this is the turn where the fitting routine has chosen the fitted offset to be.

![Figure 2.27:](image)

In the case of BPM 5, it appears the average residual drift across the scan is $\sim 0.4$ mm. This corresponds to the same result derived through fitting the turn-by-turn BPM data to a cosine wave with a linear offset term (Eq. (2.8)), Fig. 2.21. The variation in the average residual at the end of the scan is could to be due to the loss of the 201.25 MHz intensity, but since this variation is not observed in the vertical it is more likely due to energy loss from collisions with the foil each turn.
BPM 5 (SRPM21x) is in a focusing quadrupole, so the betatron oscillation amplitude is large at this BPM. Turns with more extreme positions can be seen in Fig. 2.27 by the sizable variation and oscillatory behavior of the average residual. For example in Fig. 2.27, turn six is a maximum in the betatron oscillation and turn eight is a minimum. This behavior should be compared to the average residual for a BPM in a defocusing quadrupole, Fig. 2.28.

The oscillations in the average residual are still present in Fig. 2.28, indicating a position dependence in the residual, but the variation is much smaller in the case of BPM 4 (SRPM12x), which is in a defocusing quadrupole. This result shows explicitly that the residual actually does have amplitude (or position) dependence, which is not definitive in the SSR/DOF analysis since the CO was well centered and the beam was injected near-on-axis for the reproducibility experiment.

Some of the average residuals, particularly those at the end of the scans, are larger than the calculated $\sigma_{BPM}$, the single turn measurement error. This is acceptable because $\sigma_{BPM}$ is the $rms$ standard deviation of the residual distribution, where the distribution may be approximated by the average residual at each turn. It then
2.5 Residuals Along the Scan

becomes apparent that the main cause to the spread of residuals and source of the larger than expected calculated $\sigma_{BPM}$ is the constant offset drift across the scan, which is much greater than the oscillations in the average residual due to the position dependence.

The average residuals displayed in Fig. 2.27 are of horizontal data. The vertical average residual holds an interesting surprise. Figure 2.29 plots the average residual at BPM 23 (SRPM11y), which is in a horizontal focusing quadrupole, so the vertical betatron oscillation amplitude is small. Note the drift of the average residual across the scan and the variation in average residual with position in Fig. 2.29 as previously observed in the horizontal data, Figs. 2.27 and 2.28. However, these effects are dwarfed because the point that dominates the scale of the y-axis is the first turn.

The first turn has an average residual of greater than .6 mm! How can this be when the single turn BPM measurement error ($\sigma_{BPM}$) calculated with the ML error analysis for BPM 23 is .12 mm? The answer is obviously that the other 39 turns of data fit well yielding a smaller $\sigma_{BPM}$, but this leads to the suspicion that the first turn of vertical data may be bad. The large first turn average residual is observed for
all vertical BPMs but is not seen in the horizontal. In additional, the \( \textit{rms} \) spread in the first turn residual distribution for the vertical BPMs is about \( \sim 0.1 \text{ mm} \) and is not enough to explain the deviation from the trend in the other turns.

Remember that two consecutive turns of data are combined to calculate the initial phase guess for the cosine wave fit, as described in Sec. 2.3. It has been observed that the first turn is never chosen to calculate the initial phase guess for any of the 1818 vertical scans, while it is used a good portion of the time in the horizontal. This is further evidence indicating that the first turn of vertical data does not belong to the cosine wave motion of the beam and is bad data. This first turn issue will surface again in the studies of the injection offset, Sec. 2.7.

The vertical first turn residual is almost always more negative than the other turns indicating that the measured position is always less than it should be (more down). Also, recall that the first turn of data in the large offset errors (Sec. 2.4.5), which was only observed in the vertical, was always much different than the rest of the scan and more negative. Is signal from the top BPM electrode not at full strength for the first turn of data acquisition? Like in Sec. 2.4, this leads to questions about how the first turn of data could be poor in only one direction. This line of thought guides one to inquiries on the rise time of the 201.25 MHz filters in the AFEs and if single shot beam intensity is enough signal to make a BPM measurement. These questions have not been pursued and this issue has not been resolved.

### 2.5.2 Residual spread

Another aspect of the turn-by-turn residual analysis that is of interest is the \( \textit{rms} \) spread in the residual distribution of each turn at each BPM. This value expresses the variability in the residual. Figure 2.30 is a good representation for the \( \textit{rms} \) spread in the residuals for all horizontal and vertical BPMs. There looks to be some settling
2.5 Residuals Along the Scan

Time in the beginning of each scan where the measurement is a little more variable than in turns 10-25, which seems to have a fairly constant and minimum spread in the residual distribution. The growth in the \( \text{rms} \) measurement spread at the end of the scan is attributed to the energy loss due to beam collisions with the foil each turn.

![Figure 2.30](image.png)

Figure 2.30: The \( \text{rms} \) spread of the residual distribution for each turn at BPM 5 (SRPM21x) after scans are removed from the dataset for data acquisition errors.

Again the first turn is singled out as being different from the other turns. Noting that in Fig. 2.30 the first turn \( \text{rms} \) spread in the residual distribution is one of the largest, one might also worry that the quality of the horizontal first turn. However for BPM 5, the first turn average is within two standard deviations of where it is expected to be. Contrast this with the four standard deviation difference in the first turn average residual at BPM 23. Also, as noted above, some horizontal scans use the first turn for the initial phase guess in the cosine wave fit, meaning that the first turn in the horizontal is part of the sinusoidal motion. The larger first turn \( \text{rms} \) spread in the horizontal could be a result of the large measurement spread in the fitted phase parameter, Sec. 2.6.3.

There are some peculiar outcomes in examining the standard deviation of the residual distributions on a turn-to-turn basis. One particular result can be observed
2.5 Residuals Along the Scan

in both the horizontal and vertical for BPMs that exhibit missing turn errors. Since it is believed that missing turn errors are caused by missing beam present triggers (see the Sec. 2.4.4 for more details) and the beam present trigger is prompted by 201.25 MHz intensity signal off the bottom BPM electrode, it is of interest to look at the vertical data. The \( \text{rms} \) spread of the residual distribution of each turn at BPM 26 (a BPM with missing turn errors) is plotted in Fig. 2.31.

![BPM 26, rms spread, Ord1, Reproducibility 06/27/2008](image)

**Figure 2.31:** The \( \text{rms} \) spread of the residual distribution for each turn at BPM 26 (SRPM22y), a BPM with missing turn errors, after scans are removed from the dataset for data acquisition errors.

Figure 2.31 has the same general layout as Fig. 2.30, however the structure is greatly distorted by the large values in the \( \text{rms} \) spread for turns 15-30. The large \( \text{rms} \) spread in the residual distribution for turns in the middle of the scan is not observed in most BPMs. It is only found in BPMs with missing turn errors. Turns 15-30 are also the range of turns where BPM 26 missed a turn in the data acquisition, Fig. 2.14. It is remarkable that these two should coincide. Note that all figures in this discussion of residuals along the scan represent data with all data acquisition errors removed. It is interesting that a possible footprint of the missing turn error still exists. What would cause the residual to be twice as variable for 15 turns in the middle of the data acquisition for BPMs with missing turn errors, especially when turns 15-30 are shown
to have the smallest spread for BPMs without missing turn errors?

2.5.3 Maximum residual

The last characteristic of this analysis of the residual distribution is the maximum residual for every turn at each BPM. The maximum residual interestingly enough tracks the *rms* standard deviation of the residual distributions for each turn. This is understandable because the *rms* standard deviation is sensitive to the extremes of the distribution.

Figure 2.32 compares the maximum residual for each turn at BPM 5 (SRPM21x) for all scans without data acquisition errors. Contrast all of the plots for BPM 5, Figs. 2.27, 2.30, and 2.32. Comparing the maximum residual of BPM 5 (Fig. 2.32) with the *rms* spread of the residual distribution at each turn (Fig. 2.30) one observes their qualitative similarity in shape. The only difference is that the first turn does not have a large maximum residual even though the first turn spread is largest. This indicates a constant or bimodal, non-Gaussian distribution and is more evidence that the first turn of data in the horizontal is not
bad. The maximum residual plot for BPM 5 (Fig. 2.32) is not compellingly related to the average residual (Fig. 2.27) but one can see a possible connection in the larger than expected average residuals at the end of the scan with the larger maximum residuals in the same location.

The maximum residual does correlate with the average and $rms$ spread of the residual distribution in the case of bad first turn data in the vertical, Fig. 2.33. Relate Fig. 2.33 with the average residuals at BPM 23 (Fig. 2.29) and note that the first turn has an average residual of $-0.6$ mm, an $rms$ spread of $0.13$ mm (not plotted), and a maximum residual of $1$ mm. The $rms$ spreads for the other turns at BPM 23 range between $0.05$ and $0.08$ mm.

![Figure 2.33: The absolute value of the maximum residual for each turn at BPM 23 (SRPM11y) for all scans without data acquisition errors.](image)

A large maximum residual also corresponds with the residual spread for BPMs with missing turn errors, Fig. 2.34. Comparison of the maximum residual for BPM 26 (Fig. 2.34) with the $rms$ spread in the residual (Fig. 2.31) shows that the spread and the maximum of the residual track each other. The turns with large spread also have large maximum residuals. Although not as large as the first turn in Fig. 2.33, the maximums in the large spread regions of Fig. 2.34 are about twice as big as the
2.6 RingScan Reproducibility Results

After the RingScan data is fit to a cosine wave and all data acquisition errors have been identified and removed, the resulting good scans are left describing the reproducibility of the RingScan beam position measurement. The uncertainty of the fitted parameters from the fit itself is calculated by performing a ML error analysis for each scan. This analysis yields the fitting error of each of the fitting parameters, the covariance and correlation matrices relating the fitting parameters, and the single turn measurement error for that scan, see Appx. B.

There are three quantities of interest for each parameter distribution: the average, the distribution $rms$ standard deviation or measurement spread, and the error on the average. The $rms$ spread in the measurement distribution quantifies the variability or reproducibility of the RingScan measurement, where as the error on the average quantifies the precision of the average RingScan measurement and is dependent on

![Figure 2.34](image.png) The absolute value of the maximum residual for each turn at BPM 26 (SRPM22y) for all scans without data acquisition errors.
the number of scans in the distribution, $N$,

$$
\sigma_{\text{ave}} = \frac{\sigma_{\text{meas}}}{\sqrt{N-1}},
$$

(2.10)

where $\sigma_{\text{ave}}$ is the error on the average and $\sigma_{\text{meas}}$ is the rms distribution spread. A minimum limit due to the intrinsic BPM resolution is imposed on the error on the average of the fitted amplitude and offset distributions. When $\sigma_{\text{ave}}$ is less than the intrinsic BPM resolution, the random errors (pulse-to-pulse momentum variations, BPM measurement errors, etc.) have averaged to zero and the discrete digitization is the main source of uncertainty in the fitted amplitude and offset measurements. So the random error on the average fitted amplitude and offset distributions will never be quoted less than the intrinsic BPM resolution. The intrinsic BPM resolution for each BPM type was displayed in Tab. 2.4.

2.6.1 Fitted amplitude

One benefit of injecting the beam near-on-axis for the reproducibility measurement is that the resulting amplitude measurements are very good. Although the RingScan reproducibility measurement at production injection offsets was not studied in detail, it was observed that the amplitude measurement spread is larger for the larger injection offsets. This is believed to be due to BPM saturation and uncalibrated nonlinear measurement effects.

As observed in Fig. 2.35, the amplitude oscillates from large to small making it very obvious if a particular BPM is in a focusing or defocusing quadrupole. The vertical amplitude sizes (large or small) are opposite of the horizontal. (If the horizontal amplitude is large then the vertical amplitude is small and the other way around). Remember that the amplitude is related to the beta function and the action by Eq. (1.40). Equation (1.40) is the basis of calculating the injection offset using the amplitude method described in Sec. 2.7.2.
2.6 RingScan Reproducibility Results

Figure 2.35: (Color) The fitted amplitude (blue circles) for every scan without a data acquisition error and the average amplitude with one $rms$ standard deviation of the fitted amplitude distribution at each BPM, red squares. The vertical line separates horizontal (left) and vertical (right) BPMs.

Figure 2.36 plots the fitting error on all of the fitted amplitudes for each scan not removed from the dataset due to data acquisition errors. The average fitting error is very small for all BPMs, $\sim0.03$ mm. This describes the quality of the RingScan data. The small fitting error on the amplitude states that the amplitude part of the fit is well constrained. In Fig. 2.36, the $rms$ spread in the fitted amplitude distribution varies for large and small amplitudes of the betatron oscillation. For the horizontal, the $rms$ standard deviation of the fitted amplitude distribution for the large betatron oscillations is $\sim0.3$ mm (15% of the fitted amplitude), and for the small oscillations it is $\sim0.1$ mm, 13% of the fitted amplitude. In the vertical, the $rms$ standard deviation of the fitted amplitude distribution is $\sim0.12$ mm (5%) for the large betatron oscillations and $\sim0.06$ mm (5% of the fitted amplitude) for the small oscillations.

If there was no other influence on the measurement for all RingScans, the fitting error would be the only source of uncertainty, and it would be expected that the $rms$ spread in the measurement distribution be the same as the fitting error. However, it is clearly observed in Fig. 2.36 that the amplitude measurement spread does not
match the amplitude fitting error. Thus, there must be something that is the same across one scan but changes from scan-to-scan.

The common effect to blame for the increase in the measurement spreads in the PSR is the so-called pulse-to-pulse momentum variation. This is where the central momentum of the beam varies from pulse-to-pulse. Pulse-to-pulse momentum variations are discussed in detail in the Sec. 2.8. These momentum variations will change the CO through dispersion for each scan. A dispersion measurement, verifies the existence of a small vertical dispersion function, so the CO will also change slightly in the vertical, Sec. 3.3. Moreover, both the horizontal and vertical COs will also change at the foil (the injection point) changing the injection offset and the action on a scan-by-scan basis. The pulse-to-pulse momentum variations could also change the steering in the Ring In (RI) line and thus the injection offset. But it is believed that this last effect is small and the major contribution to the varying injection offset is the change in the CO at the foil.
Changes in the injection offset will vary the action and initial phase on a scan-by-scan basis. Equation (1.40) may be applied to relate the change in the action and the spread in the fitted amplitude via the beta function. In the horizontal, the beta functions are \(\sim 5\) times larger for BPMs in the focusing quadrupoles where the scans possess big amplitudes, compared to the smaller amplitude scans from BPMs in defocusing quadrupoles, while in the vertical the beta functions in the vertically focusing quadrupoles are only \(\sim 4\) time larger, see Sec. 3.4 for results of the beta function measurement. Applying the differences in the beta functions in Eq. (1.40) explains the amplitude dependence observed in the amplitude \(rms\) measurement spread, as shown in Fig. 2.36. The horizontal amplitude measurement spread is additionally affected by the change in the horizontal CO at the point of injection due to the pulse-to-pulse momentum variations. This is an additive effect not proportional to the beta functions and is why the horizontal amplitude measurement spread is larger than the vertical.

The number of scans required to precision limit the error on the average amplitude distribution may be calculated by solving Eq. (2.10) for \(N\) and substituting the intrinsic BPM resolutions from Tab. 2.4 for \(\sigma_{ave}\). The number of scans required to obtain a precision limited error on the average amplitude at each BPM is 404.

### 2.6.2 Fitted tune

The fitted tune measurement is applied to calculate the beta functions in the quadrupole perturbation method of Sec. 3.4 and all methods of the injection offset presented later in Sec. 2.7 also employ the measured tune. The measured tune is also a good first test for any model of the PSR. Since the tune is the frequency of the cosine wave fit, the more points of turn-by-turn BPM data in a scan, the better the fit. In the RingScan reproducibility dataset, 40 turns of data (8 betatron oscillations) were collected in
RingScan Reproducibility Results

Each scan yielding a very good tune measurement. Since the tune is multiplied by $2\pi$ in Eq. (2.6), only the fractional tune can be fit. The integer tune number will only contribute to another betatron oscillation between the turns and will not be observed in the discrete time measurement of the BPM, $\cos 2\pi = \cos 2\pi i$ where $i = 1, 2, 3, \ldots$.

Figure 2.37 plots the fitted fractional tune for all scans and the weighted average with one $\text{rms}$ standard deviation for each BPM. Notice, not surprisingly, that all horizontal BPMs (1-20) and all vertical BPMs (21-40) yield the same tune result. There are no outlying BPMs that give tune results much different than the rest. The tune measured by the CCR BPM program was [.1914, .1981].

Since all of the BPMs in one direction measure the same tune, fitted tunes from all scans in one direction constitute the total tune measurement distribution. The final tune measurement is actually the average of all scans in that direction and not just at one BPM as in Fig. 2.37. Figures 2.38 and 2.39 show the histograms.
Figure 2.38: Histogram of all horizontal fitted tunes with average $1.91429 \pm 1.0 \times 10^{-5}$ and rms standard deviation $4.3 \times 10^{-4}$.

Figure 2.39: Histogram of all vertical fitted tunes with average $1.97937 \pm 8 \times 10^{-6}$ and rms standard deviation $3.3 \times 10^{-4}$.
of the total horizontal and vertical fitted fractional tune distributions. The $rms$ standard deviations of these profiles, $4.3 \times 10^{-4}$ (0.22% of the fitted fractional tune) in the horizontal and $3.4 \times 10^{-4}$ (0.17%) in the vertical, are about 10 times smaller than the expected error on the CCR BPM measurement. The tune averages and the errors on the averages are $0.191429 \pm 1.0 \times 10^{-5}$ in the horizontal and $0.197937 \pm 8.4 \times 10^{-6}$ for the vertical. The larger measurement spread in the horizontal fitted tune distribution is believed to be due to the pulse-to-pulse momentum variations and a more negative chromaticity in the horizontal.

The error on the tune average is very small yielding a very precise tune measurement. This is of course because so many scans contributed to the tune average (Eq. (2.10)), but this result is encouraging for the beta function measurement. Since the tune is a frequency measurement, the error on the average is not limited by the digitization of the ADC as in the case of the amplitude. Compare the RingScan and the CCR BPM program tune measurements: RingScan $[0.191429 \pm 1.0 \times 10^{-5}, 0.197937 \pm 8.2 \times 10^{-6}]$ and CCR $[0.1914, 0.1981]$. The CCR BPM program agrees well with the RingScan result, but the RingScan result does possess higher precision.

The resulting tune-tune plot after all scans with data acquisition errors are taken out of the dataset is shown in Fig. 2.40. Compare the tune-tune plot of Fig. 2.40 with the tune-tune plot before the scans with data acquisition errors were removed, Fig. 2.12. After the scans with data acquisition errors are removed from the dataset, the RingScan measurement yields a well defined tune-tune island.

Figure 2.41 is a graph of the fitting error on the fitted tune parameter from the cosine wave fit and the $rms$ standard deviation of the measured tune distribution at each BPM. The fitting error has a slight BPM dependence. Actually, it is more like an amplitude dependence. The BPMs in focusing quadrupoles (where the amplitude of the betatron oscillation is larger) have a smaller fitting error in the fitted tune parameter. The tune fitting error is less for most vertical BPMs compared to the
horizontal. However, there are larger fitting errors in BPMs 37 and 39 (SRPM81y and 91y), the vertical diamond type BPMs. The amplitude dependence of the tune fitting error suggests that a better tune measurement could be made at larger injection offsets. Also note how the \(\text{rms}\) measurement spread at each BPM is comparable with the fitting error. This suggests that there is no outside effect smearing out the measurement distribution. Since the pulse-to-pulse momentum variations do not seem to have an effect on the fitted fractional tune parameter, one can say that the chromatic effects of such momentum variations must be small. This is why the fitting errors on the tune parameter may be applied to define the limit of the tune measurement spread when searching for outliers in the measured tune distribution.

2.6.3 Fitted phase

The phase is like the fitted fractional tune parameter in that multiple values of the phase can yield the same value in the cosine function. Here \(\cos \phi = \cos(\phi + 2\pi i)\) where \(i = 1, 2, 3, \ldots\). Thus, the fitted phases for each scan can be arranged in any manner. The data analysis scripts assemble the fitted phase parameter in ascending
order from SRPM02 (BPM 2 and 22, SRPM02), the first BPM after the foil, Fig. 2.42. The choice of selecting SRPM02 as the smallest phase BPM is for convenience in calculating the injection offset and for modeling purposes. Figure 2.42 plots the fitted phases for each scan and the average with three \( \text{rms} \) standard deviation error bars for the measured phase distribution at each BPM. Notice there is a large measurement spread in the fitted phase parameter for all BPMs.

Three standard deviations from the average of one BPM distribution overlap with three standard deviations of the neighboring BPM distributions. Phases at one BPM could be very similar to the phases at the previous BPM. Actually, it has been observed for some RingScans that a scan at a BPM fits a phase that is less than the phase fitted for the previous scan, suggesting a negative phase advance between the two BPMs, which is of course not physical and must be a result of the measurement uncertainty in the phase parameter due to the pulse-to-pulse momentum variations. Since the phases measured at adjacent BPMs overlap, a phase comparison could not be employed to identify the BPM selection data acquisition errors in Sec. 2.4.1.
Figure 2.42: (Color) The fitted phase for each scan without data acquisition errors (blue circles) and the average with three $\text{rms}$ standard deviations for the fitted phase distribution at each BPM, red squares. The vertical line separates the horizontal BPMs (left) and the vertical BPMs, right.

Figure 2.43 plots the fitting error for the phase parameter for each scan. Like in the case of the tune parameter, there seems to be an amplitude dependence to how well the phase is constrained in the fit. BPMs in focusing quadrupoles (where the betatron oscillations have larger amplitudes) fit the phase better by a factor of $\sim 2$ compared to when the amplitude is smaller in the defocusing quadrupoles. A reason for the phase fitting error to be dependent on the amplitude is consideration that the single turn BPM measurement error would have more effect on the fitted phase for smaller amplitude oscillations because the ratio of single turn measurement error to amplitude is larger. It is observed that for production injection offsets the $\text{rms}$ measurement spread is reduced by a factor of 5-10. Note that the fitting error on the phase parameter is much smaller than the $\text{rms}$ standard deviation of the fitted phase distribution at each BPM, which is about .22 and .2 radians in the horizontal and vertical respectively, 19.95% and 28.97% of the average phase advance between BPMs. This is further evidence of the effect of pulse-to-pulse momentum variations affecting the distribution of several pulses but not the fitting of an individual scan.
2.6 RingScan Reproducibility Results

The fitted phase is very sensitive to changes in the injection offset where the initial phase is determined by the position and angle of the injected beam. Again the vertical diamond type BPMs, BPMs 37 and 38 (SRPM81y and 92y), have the largest measurement spread of the vertical BPMs. Note that these BPMs also possess the largest intrinsic BPM resolutions.

![Figure 2.43:](Image)

Figure 2.43: (Color) The fitting error on the fitted phase parameter for all scans without data acquisition errors (blue circles) and the \textit{rms} spread in the fitted phase distribution at each BPM, red squares. The vertical line separates the horizontal BPMs (left) and the vertical BPMs, right.

Although the measurement distribution spread is large, the error on the average is just tens of milliradians for each BPM. The difference in phase in between two BPMs is the same as the betatron phase advance and could be applied to further constrain the model of PSR.

2.6.4 Fitted offset

The fitted offset is the same as the CO. Many offset (CO) measurements will be required in the measurement of the ORM, Sec. 4.1. Figure 2.44 plots the results of the CO measurement for the RingScan reproducibility dataset. The fitted offset is plotted...
at each BPM for all scans without data acquisition errors and accompanied by the average and three rms standard deviation error bars of the fitted offset distribution at each BPM. The fitted offset points in the vertical can barely be seen. This is because the spread in the vertical offset distribution is \( \sim 0.02 \) mm and much too small to be seen on the scale of the y-axis of Fig. 2.44. In contrast to the small vertical offset measurement spread, the horizontal offset measurement spread is five times larger (\( \sim 0.1 \) mm) and can be seen clearly in Fig. 2.44. Once again the explanation for the horizontal and vertical asymmetry in the measurement spread is due to the pulse-to-pulse momentum variations, where the CO change is directly proportional to a change in the momentum. It is clear from Fig. 2.44 that the CO for the RingScan reproducibility measurement was not as centered as it could have been.

As with the other parameters, the difference between the fitting error on the offset and the rms measurement spread of the offset distribution at a BPM gives a clue pointing to the pulse-to-pulse momentum variations influencing the horizontal offset measurement spread. In Fig. 2.45, the fitting error on the offset for a particular scan is

![Figure 2.44:](Image) The fitted offset (CO) for all scans without data acquisition errors (blue circles) and the average with three rms standard deviations for the fitted offset distribution at each BPM, red squares. The vertical line separates the horizontal BPMs (left) and the vertical BPMs, right.
2.6 RingScan Reproducibility Results

fairly constant across all BPMs, and the fitting errors is slightly larger than the \( \sim 0.02 \) mm intrinsic BPM resolution from digitization of the ADC. Since the vertical offset measurement spread matches the fitting errors which are about the intrinsic resolution of the BPMs, it appears that the precision of the vertical CO measurement is limited by the intrinsic BPM resolution. The horizontal measurement spread is \( \sim 5 \) times larger than the fitting error. Because the vertical dispersion function is small, the pulse-to-pulse momentum variations do not influence the vertical CO measurement like they do the vertical amplitude and phase via changes in the injection offset.

Interestingly, in Fig. 2.46 the horizontal measurement spread tracks the measured dispersion function. Although there is an arbitrary vertical scale in Fig. 2.46, the correlation between the measured dispersion function and the \( \text{rms} \) measurement spread fitted offset distribution at each BPM is clearly observed. This correlation is unmistakably broken at BPM 20 (SRPM92x) providing more proof for the suspect CO measurement at that BPM. (Since the CO is needed to measure the dispersion function, one should also question the validity of the dispersion function at BPM 20).
2.6 RingScan Reproducibility Results

calculated correlation between the CO measurement spread and the measured dispersion function is .91 when all horizontal BPMs are considered and .97 when BPM 20 is not included. If the pulse-to-pulse momentum variations are truly the cause of the larger horizontal CO measurement spread, it makes sense that momentum changes would have more effect on the CO where the dispersion function is largest. This is because from Eq. (1.13) the CO change at a BPM is equal to the relative momentum change multiplied by the dispersion function at that BPM, $\Delta x_{CO} = D\delta$. A calculation of the magnitude of the pulse-to-pulse momentum variations based on this discussion will be pursued in Sec. 2.8.

**Figure 2.46:** (Color) The $rms$ measurement spread of the fitted offset distribution at each horizontal BPM (blue circles) and the measured horizontal dispersion function (green squares). The vertical line separates the horizontal BPMs (left) and the vertical BPMs, right.

The calculated error on the average of the offset distribution at each BPM is less than $\sim .02$ mm. Like for the amplitude parameter, this is a statistical figure for the best case when the systematic effects from the discrete nature of the digitization in the ADC cancel. The precision of the average CO measurement for this number of scans is limited by the intrinsic resolution of the BPMs due to digitization, $\sim .02$ mm. A result of this exactitude was not expected with the PSR BPMs, but one should be reminded that many scans with data acquisition errors that affect the CO measurement were removed from the dataset and are not represented in the figures in
2.6 RingScan Reproducibility Results

Figure 2.47: The measured CO distribution for BPM 14 (SRPM62x) for all scans without data acquisition errors.

Figure 2.48: The measured CO distribution for BPM 34 (SRPM62y) for all scans without data acquisition errors.
2.6 RingScan Reproducibility Results

this section. Distributions of the measured offset parameter are shown in Figs. 2.22, 2.23, 2.47, and 2.48.

The number of scans required to precision limit the average of the offset measurement may be calculated in a similar manner as for the amplitude fitted parameter. Seventeen scans are required to precision limit the horizontal offset measurement, but only 4 scans are needed to precision limit the vertical measurement.

2.6.5 Single turn BPM measurement error, $\sigma_{BPM}$

When the ML error analysis is performed for the cosine fit of each scan not only are the fitting errors on the fitting parameters calculated, but the measurement error for each scan is also evaluated. In the ML error analysis, the difference between the data and the fit (residual defined by Eq. (2.9)) is assumed to be random, zero average, and Gaussian, as shown in Fig. 2.5. The single turn BPM measurement error is the rms standard deviation of the Gaussian residual distribution,

$$
\sigma_{BPM} = \sqrt{\frac{\sum_{n}(x_n - f(n; \vec{a}))^2}{N}},
$$

where $n$ is the turn number index, $x_n$ is the turn-by-turn BPM position data, $f(\vec{a}; n)$ is the fitting function depending on turn number and the fitting parameters $\vec{a}$ (for this exercise $f(\vec{a}; n)$ is described in Eq. (2.6)), and $N$ is the total number of turns. In application to the RingScan data, $\sigma_{BPM}$ is the single turn BPM measurement error. Every turn of data taken at one BPM should have the same single turn measurement error. Figure 2.49 plots the single turn BPM measurement error for all scans. All BPMs have about the same single turn measurement error, between .1 and .2 mm. Previously, the quality of the BPM measurement was assumed to be poor, and this results is much lower than expected. But many data acquisition errors were removed from the dataset to obtain this result. The largest single turn measurement errors are
in the vertical diamond type BPMs, BPMs 37 and 39 (SRPM81y and 91y). BPMs 17 and 19, the horizontal diamond type BPMs, also have a larger $\sigma_{BPM}$.

The fitted single turn BPM measurement error is about ten times larger than the intrinsic resolution of the BPMs. $\sigma_{BPM}$ not only takes into account the discrete digitization of the ADC but also noise from cables, reflections, and connections. The linear offset drift across each scan of RingScan data observed in Fig. 2.21 and in Sec. 2.5 is the leading contributor to the size of $\sigma_{BPM}$. Interestingly, the uncertainty of the $\sigma_{BPM}$ parameter from the ML error analysis more resembles the intrinsic resolution of the BPMs, which in most cases matches the $rms$ spread of the $\sigma_{BPM}$ distribution at each BPM, Fig. 2.50.

The $rms$ spread in the $\sigma_{BPM}$ distribution for each BPM is plotted in Fig. 2.50 along with the uncertainty of the single turn BPM measurement error due to the cosine wave fit. The error on the fitted $\sigma_{BPM}$ value is $\sim$10% and fairly consistent across all BPMs. However, there are four BPMs that stick out, BPMs 1, 13, 26, and 28 (SRPM01x, SRPM61x, SRPM22y and SRPM32y), because the $rms$ spread
in the $\sigma_{BPM}$ distributions at these BPMs is greater than the uncertainty on the calculated $\sigma_{BPM}$ value. These BPMs have long tails on the upper side of the fitted $\sigma_{BPM}$ distribution, Fig. 2.49, increasing the rms spread at these BPMs. Missing turn errors have been observed in three of these BPMs, and it was shown in the discussion in Sec. 2.5 that BPMs with missing turn errors have larger spread in their residuals.

2.6.6 Sum of squares of residuals

The sum of squares of residuals per degree of freedom (SSR/DOF) is the quantity describing the goodness of the cosine wave fit to a scan of BPM data. The residual is the difference between a data point and the fit, Eq. (2.9). The SSR/DOF describes the square of the average residual for each data point in the fit. The SSR/DOF is very closely related to the single turn BPM measurement error ($\sigma_{BPM}$),

$$SSR/DOF = \frac{\sum_n (x_n - f(n; \vec{a}))^2}{N - P}.$$  

(2.7)
All of the variables in Eq. (2.7) are the same as those in Eq. (2.11), and $P$ is the number of fitting parameters, four for the basic cosine wave fit of Eq. (2.6). The SSR/DOF is naturally larger for scans with errors. Plateau errors have the largest SSR/DOF followed by the missing turn errors, and the large offset errors have slightly larger SSR/DOF’s than scans without data acquisition errors because their amplitudes are so small. However, the SSR/DOF is not invoked to find or identify data acquisition errors. This is because the SSR/DOF is not robust; it depends on the fitting scheme employed and on the amplitude of betatron oscillation.

Figure 2.51: (Color) The SSR/DOF for the cosine wave fit of each scan without data acquisition errors (blue circles), and the average with one $rms$ standard deviation of the SSR/DOF distribution at each BPM, red squares. The vertical line separates the horizontal (left) and vertical (right) BPMs.

Figure 2.51 plots the SSR/DOF for all scans without data acquisition errors. All scans without data acquisition errors have a SSR/DOF less than .1 mm$^2$. This means that all scans are fit well. The residuals, and thus the SSR/DOF, are sensitive to the amplitude of the betatron oscillations, so sometimes the SSR/DOF is better presented normalized to the fitted amplitude for that scan or BPM. However, the amplitudes are small in the RingScan reproducibility dataset, and the fits are good, so this is not done in Fig. 2.51.
2.6.7 Fitting correlations

One last aspect of the fitting is the correlation between the fitting parameters. The correlation describes how one fitting parameter’s fit effects the other fitting parameters. The correlation matrix for each cosine wave fit can be easily found by manipulating the error matrix (covariance matrix) calculated as part of the ML error analysis. The average correlation for every fitted parameter for all scans at each BPM is plotted in Fig. 2.52. All correlations are less than ±.1 except the tune-phase correlation, which is −.87. So aside from the tune and the phase, the fitting parameters are not correlated. This lends confidence to the robustness of the fitting scheme. The tune and the phase however are very anti-correlated. This can be easily understood because the first derivatives of Eq. (2.6) with respect to the tune and the phase only differ by a constant, 2πn. Another example of the close relation between tune and phase is in the missing turn error where the phase is displaced by missing a turn and the resulting tune fit produces a faulty measurement.

2.7 Studies of the Injection Offset Measurement

It has been the general feeling over the past few years that the beam size in the ring is larger than that inferred from the CCR BPM injection offset measurement program. This notion helps to explain the recent increase in losses and lowering of the injection offset in the PSR during production. The RingScan reproducibility dataset can be applied to calculate and analyze the variability of the injection offset measurement.

The injection offset is an operational parameter that relates how the beam is injected into the PSR in relation to the CO at the point of injection, the foil. The injection offset is reported as a coordinate in phase space, [position, angle], and defines the action of the beam in the ring, J. The position and angle of the injection offset
determine where on the cosine wave of the betatron motion the beam starts when it enters the PSR, thus defining the initial phase.

In these studies, three different methods of calculating the injection offset will be compared with each other and with the CCR BPM program result, Sec. 2.7.1. The three different methods are called here the amplitude method (Sec. 2.7.2), scan method (Sec. 2.7.3), and turn method (Sec. 2.7.4). All three methods are in theory mathematically equivalent, yielding similar results, and as will be discussed, they each have their own advantages and disadvantages.
2.7 Studies of the Injection Offset Measurement

2.7.1 CCR BPM program measurement of the injection offset

The CCR BPM program performs what is called a first-turn analysis to calculate the injection offset. The program reads the first turn of BPM data at every BPM. The first turn data is employed to calculate the position and angle at the foil (the injection offset) by applying a linear regression fit. The injection offset is extracted from the first turn data fit to,

\[ x_{i,n} = x_0 \sqrt{\frac{\beta_i}{\beta_0}} (\sin \Theta + \alpha_0 \cos \Theta) + x'_0 \sqrt{\frac{\beta_i}{\beta_0}} \sin \Theta - x_{CO}, \quad (2.12) \]

where \( \Theta = 2\pi \nu (n - 1) + \mu_{0\rightarrow i} \).

where the index \( i \) indicates values at the \( i^{th} \) BPM and the index \( \theta \) indicates values at the foil, the index \( n \) is the turn number running from 1 to the number of BPMs, \( x \) is the position data (first turn data at each BPM or location at the foil), \( x' \) is the phase space angle, \( \beta \) and \( \alpha \) are the beta function and its derivative with respect to the longitudinal coordinate respectively, \( \mu \) is the positive betatron phase advance from the foil to the BPM, and \( x_{CO} \) is the closed orbit at each BPM. \( n \) is set to 1 for the first turn calculation applied by the CCR BPM program. Equation (2.12) is derived from propagating the injection offset (betatron coordinates at the foil) to the location of the BPMs by applying the generic transfer matrix from Eq. (1.36). Since the initial phase space coordinates in Eq. (1.36) were betatron coordinates, the final coordinates of the linear map are also betatron coordinates. Thus, the CO is subtracted in Eq. (2.12) in order to obtain the betatron coordinates with respect to the CO at the BPMs.

The CCR BPM program simplifies Eq. (2.12) by assuming the coefficients of \( x_0 \) and \( x'_0 \) are constants, Eq. (2.13). The coefficient vectors \( \vec{A} \) and \( \vec{B} \) are read from a text file, \textit{bpmfsttrn.dat}, when the calculation is performed. The text file \textit{bpmfsttrn.dat} was
last updated in 1998. An improved operations tool would read the magnet currents, run a model, and produce a new set of coefficients $\vec{A}$ and $\vec{B}$ every time the injection offset is measured. $C$ is the saved CO at each BPM. Thus the CO must be set and saved before the injection offset can be calculated.

$$x_i = A_i x_0 + B_i x'_0 - C_i$$

(2.13)

The CCR BPM program calculated an injection offset for the RingScan reproducibility measurement of $[-0.72 \pm 0.088 \text{ mm}, \ 0.31 \pm 0.03 \text{ mrad}]$ in the horizontal and in the vertical $[1.99 \pm 0.168 \text{ mm}, \ 0.3 \pm 0.027 \text{ mrad}]$. The CCR BPM program operates on a different dataset than the RingScan reproducibility dataset and does not remove scans with data acquisition errors from the dataset as is done in the analysis of RingScan data. In Sec. 2.7.6, the RingScan reproducibility data without data acquisition errors removed is employed to measure the injection offset like the CCR BPM program.

### 2.7.2 Amplitude method

The first method of calculating the injection offset with the RingScan data is the amplitude method. In the amplitude method, only one scan is needed to calculate the injection offset. The amplitude method applies the fitted amplitude and phase of each scan and the average tune measurement for all scans in one dimension to calculate the injection offset[23],

$$x_0 = A_i \sqrt{\frac{\beta_0}{\beta_i}} \cos \Theta \quad \text{and} \quad x'_0 = -\frac{A_i}{\sqrt{\beta_0 \beta_i}} (\alpha_0 \cos \Theta + \sin \Theta),$$

(2.14)

where \[ \Theta = 2\pi \nu + \phi_i - \mu_{0 \rightarrow i}, \]

$x_0$ and $x'_0$ are the position and angle of the injection offset, $A_i$ and $\phi_i$ are the fitted amplitude and phase for the scan calculating the injection offset, $\beta$ and $\alpha$ are the
model beta functions and its derivative with respect to the longitudinal, the index $\theta$ represents quantities at the foil and index $i$ indicates quantities at the $i^{th}$ BPM, $\nu$ is the average fractional tune for all scans without data acquisition errors taken in one dimension, and $\mu$ is the positive betatron phase advance from the foil to the BPM.

Equation (2.14) is derived from the solution to the betatron EOM, Eq. (1.38). Thus, the fitted phase in the $\Theta$-equation of Eq. (2.14) must yield from a fitted cosine function such as that in Eq. (2.6). This is why the RingScan data is fit to a cosine wave instead of a sine wave.

Since the amplitude method provides a direct calculation of the injection offset by applying the fitted parameters whose uncertainties are already known from the ML error analysis of the cosine wave fit to the turn-by-turn RingScan data, the uncertainties on the calculated $x_0$ and $x'_0$ can be easily obtained by propagating the fitting errors of $A$ and $\phi$ via their covariance matrix. Since so many scans are included in the calculation of the average tune, the random error on the average tune is very small and is not considered in the propagation of uncertainty. It is also assumed for this calculation that any systematic error on the tune measurement is small. Figures 2.53 and 2.54 compare the results of the injection offset measurement by the amplitude method with the turn and scan methods and the CCR BPM program measurement.

A peculiar characteristic appears in Figs. 2.53 and 2.54. The individual uncertainty of the injection offset calculation by the amplitude method for each scan is much smaller than the overall distribution $rms$ spread. If the calculation and error analysis were done properly, this would indicate that the actual injection offset was moving in phase space during the measurement. The exact cause of the distribution spread has not been proven, but it is believed to be due to the pulse-to-pulse momentum variations. As discussed earlier in Sec. 2.6.4 and later in Sec. 2.8, it has been proven that the pulse-to-pulse momentum variations change the CO everywhere in the ring including at the point of injection. Since the injection offset is not an
absolute position but a position relative to the CO, changes in the CO at the foil will also change the injection offset. It has also been proposed that the injection steering in the RI line could vary with beam momentum.

It is unlikely that the spread in the distribution is a result of applying model parameters produced by the baseline model. Any error in the model parameters would be systematic since the same model parameters are applied to every measurement of the injection offset via the amplitude method. The effect of systematic errors in the model parameters will shift the entire injection offset distribution not spread it out. However, it is believed that any systematic error introduced by the model parameters is small because it will be shown in Sec. 3.4 that the baseline model beta functions agree with the measured betatron amplitude functions.

The resulting spots in phase space from the amplitude method injection offset
As with all injection offset calculations, model parameters are applied. This is undesirable because the baseline model is known not to predict the measured vertical tune. This is a source of systematic error in the injection offset calculation. The amplitude method also depends on fitting the RingScan data to a cosine wave to extract the amplitude and phase, making this method dependent on what fitting scheme is applied to the turn-by-turn BPM data: Eqs. (2.6) and (2.8) or any other cosine type variation. The amplitude method does have the advantage that it employs a direct calculation of the injection offset, not a fit, and it requires only one scan calculation are quite small. In the horizontal the distribution has an average of \([-0.683 \pm 3.50 \times 10^{-3} \text{ mm}, \ 368 \pm 1.57 \times 10^{-3} \text{ mrad}\)] with an \textit{rms} spread of \([0.141 \text{ mm}, \ 6.34 \times 10^{-2} \text{ mrad}]\). The vertical distribution has an average of \([2.111 \pm 3.51 \times 10^{-3} \text{ mm}, \ 333 \pm 7.70 \times 10^{-4} \text{ mrad}]\) with an \textit{rms} spread of \([0.141 \text{ mm}, \ 3.11 \times 10^{-2} \text{ mrad}]\).
to calculate the injection offset, so measurement errors from different BPMs and momentum variations for measurements over multiple pulses do not contribute to the random error in the injection offset calculation.

2.7.3 Scan method

The scan method applies all of the position data in one scan to calculate the injection offset. Although this method still relies on model parameters, it does not depend on fitting the turn-by-turn BPM data to a cosine wave. And since the method requires only one scan to make the injection offset calculation, the scan method is not affected by momentum variations between scans or measurement errors from different BPMs.

The scan method invokes the same equation as the CCR BPM program Eq. (2.12). For the scan method, \( i \) is fixed because the scan only occurs at one BPM, and the turn index \( n \) is ran to form a system of linear equations for \( x_0 \) and \( x'_0 \). The first turn is chosen to have an \( n \) index of 1, so \( n \) can run from 1 to the number of turns of BPM data taken in a scan, \( N \). For the RingScan reproducibility dataset, \( n = 1, 2, 3, \ldots 40 \).

In order to minimize systematic errors due to uncertainties in the coefficients of the linear fit, the average fractional tune for all scans without data acquisition errors is set as \( \nu \). However, since the injection offset is a relative position, the fitted offset for each scan is applied as \( x_{CO} \). It is believed that the fit for the injection offset in the scan method is still independent of the fitting scheme since the fitted offset can easily be estimated as the average of the turn-by-turn BPM data.

Equation (2.12) yields a system of 40 equations relating the turn-by-turn BPM data for a scan and \( x_0 \) and \( x'_0 \). Linear regression is invoked to solve for the injection offset (see Appx. A.1 for an introduction to linear regression), and a ML error analysis is performed to calculate the fitting errors. The errors in the model parameters are systematic errors in the fit for the injection offset and are assumed to be small.
Likewise, errors in the average tune and fitted offset are also systematic errors in the fit and assumed to be small. Figures 2.55 and 2.56 plot the results of the injection offset calculation using the scan method.

**Figure 2.55:** (Color) The horizontal injection offset with one uncertainty calculated via the scan method (blue) for each scan in the RingScan reproducibility dataset after data acquisition errors have been removed, and the average with one $rms$ standard deviation for the amplitude (green), scan (red), and first turn methods, black. The CCR BPM program measurement with its reported error is plotted in magenta.

The same peculiar characteristic observed in the amplitude method also appears in the results of the scan method. The individual calculation errors are much smaller than the overall distribution spread. Some possible causes for this are discussed in Sec. 2.7.2.

The scan method does employ a fit to the BPM data, so it is of interest to quantify the quality of such a fit, Fig. 2.57. Since the linear regression fitting has an exact algebraic solution, the SSR/DOF of the fit best describes the quality of the data applied in the fitting of Eq. (2.12). Interestingly enough, Fig. 2.57 is almost a reproduction of the SSR/DOF derived from the cosine wave fit to the turn-by-turn
BPM data, Fig. 2.51. This is not surprising since the cosine wave fit to turn-by-turn data at a BPM (Eq. (2.6)) and the fitting scheme for the injection offset calculation (Eq. (2.12)) are both derived from Eq. (1.38).

The ML error analysis of the scan method fit also yields the single turn measurement error sigma, Fig. 2.58. The sigma from this analysis is expected to match the single turn measurement error found in the error analysis of the cosine wave fit because the SSR/DOF’s for each fit are very similar, Fig. 2.49. Comparing Figs. 2.58 and 2.49, the two results agree fairly well lending an independent measurement of the single turn BPM measurement error. Remember that the single turn BPM measurement error is dominated by the constant offset drift across the scan, which also affects the injection offset fits.

The scan method calculates an injection offset of $[-0.680 \pm 3.49 \times 10^{-3} \text{ mm}, 0.367 \pm$
2.7 Studies of the Injection Offset Measurement

Figure 2.57: The SSR/DOF from the fit to calculate the injection offset via the scan method for all scans without data acquisition errors.

Figure 2.58: (Color) The sigma from a ML error analysis of the fit employed to calculate the injection offset via the scan method for all scans without data acquisition errors (blue) and the average with one rms standard deviation for each BPM, red.
2.7 Studies of the Injection Offset Measurement

1.61 \times 10^{-3} \text{ mradian} \] with an \textit{rms} measurement spread of \([.141 \text{ mm}, .065 \text{ mradian}]\) in the horizontal and in the vertical an injection offset of \([2.10 \pm 3.53 \times 10^{-3} \text{ mm}, .333 \pm 7.74 \times 10^{-4} \text{ mradian}]\) with an \textit{rms} measurement spread of \([.142 \text{ mm}, .031 \text{ mradian}]\).

2.7.4 Turn method

The first-turn method that the CCR BPM program applies is a special case of the turn method. The turn method also invokes Eq. (2.12) to fit BPM data. There is nothing special about the first turn (any turn may be invoked) except the tune term goes to zero in the \(\Theta\)-equation of Eq. (2.12). In the turn method, the turn number index \((n)\) is held constant and the BPM index \((i)\) is ran to form a system of 18 equations relating the \(n^{\text{th}}\) turn of BPM data at all BPMs and the injection offset, \(x_0\) and \(x'_0\). Like in the scan method, the injection offset is solved by linear regression, and the errors of the fit are calculated by a ML error analysis.

The turn method does not depend on the fitting scheme invoked to fit the turn-by-turn BPM data. It does however have the disadvantage of requiring data taken over 20 different pulses at 18 different BPMs. While this method does yield the largest individual fitting error (as observed in Figs. 2.59 and 2.60), there is an advantage to applying data from 18 different scans. In the case of the CCR BPM program where the BPM data acquisition errors are not taken out of the dataset, if there is a bad scan, the effects will be minimized by the other 17 good scans. This is in contrast to employing just one scan for the measurement of the injection offset; if that one scan is bad, the whole measurement is bad. The discussion of the injection offset calculation results without bad data taken out will be pursued in Sec. 2.7.6.

Figures 2.59 and 2.60 shows the injection offset results from the turn method analysis. All turns for all RingScans are plotted in Figs. 2.59 and 2.60, yielding the
most calculations of the injection offset for any method, 4040 (number of turns $\times$ RingScans = 40 $\times$ 101). The amplitude and scan method only produce 1818 calculations of the injection offset, $BPMs \times RingScans = 18 \times 101$.

In the turn method, the single injection offset calculation errors are more representative of the $rms$ spread of the whole injection offset distribution. This is an indication that the process that is smearing out the injection offset distributions in the amplitude and scan methods (the pulse-to-pulse momentum variations) is taken into account in the fitting error result of the turn method. The turn method includes momentum variation affects because data from 18 different pulses are applied in the injection offset calculation. Note, the correlation in the vertical injection offset, Fig. 2.60. The cause of this is unknown, but the trend is independent of the turn index.

The turn method calculates the horizontal injection offset as $[-0.693 \pm 1.42 \times 10^{-3}$

Figure 2.59: (Color) The horizontal injection offset with one uncertainty calculated via the turn method (blue) for each scan in the RingScan reproducibility dataset after data acquisition errors have been removed, and the average with one $rms$ standard deviation for the amplitude (green), scan (red), and first turn methods, black. The CCR BPM program measurement with its reported error is plotted in magenta.
Figure 2.60: (Color) The vertical injection offset with one uncertainty calculated via the turn method (blue) for each scan in the RingScan reproducibility dataset after data acquisition errors have been removed, and the average with one \textit{rms} standard deviation for the amplitude (green), scan (red), and first turn methods, black. The CCR BPM program measurement with its reported error is plotted in magenta.

mm, $0.373 \pm 5.93 \times 10^{-4}$ mrad\n\nwith an \textit{rms} measurement spread of $[9.03 \times 10^{-2}$ mm, $3.77 \times 10^{-2}$ mrad\n\nand an injection offset in the vertical of $[2.10 \pm 1.79 \times 10^{-3}$ mm, $0.333 \pm 3.00 \times 10^{-4}$ mrad\n\nwith an \textit{rms} measurement spread of $[0.114$ mm, $1.91 \times 10^{-2}$ mrad\n\nThe ML error analysis for the turn method fit yields the measurement error ($\sigma$) for different turn numbers, Figs. 2.61 and 2.61. Compare the measurement error from the turn method with the single measurement errors from the scan method fit (Figs. ?? and ??) and the single turn measurement error from the cosine wave fit, Fig. 2.49. The measurement error is largest for the turn method fit. This is because the turn method analyzes data from 18 different pulses with slightly different COs and includes error contributions from the pulse-to-pulse momentum variations.

Note in Fig. 2.62, the vertical first turn $\sigma$ is about twice as large as the other
2.7 Studies of the Injection Offset Measurement

Figure 2.61: (Color) The horizontal measurement error ($\sigma$) derived from the ML error analysis of the turn method fit for all turns and RingScans (blue) without data acquisition errors and the average for each turn with a one $rms$ standard deviation, red.

Figure 2.62: (Color) The vertical measurement error ($\sigma$) derived from the ML error analysis of the turn method fit for all turns and RingScans (blue) without data acquisition errors and the average for each turn with a one $rms$ standard deviation, red.
turns in the vertical. This is further evidence that the first turn in the vertical may be incorrect (see the analysis in Sec. 2.5). Observing no difference in the horizontal first turn measurement error (Fig. 2.61) lends confidence in the earlier statement that only the vertical first turn is bad. However, it is observed that the later turns in the horizontal have a slightly larger $\sigma$. This is believed to be a result of the loss of the 201.25 MHz intensity at later turns. It is unknown why the loss of the 201.25 MHz does not manifest in the vertical. Note that the loss of the 201.25 MHz was also not observed in the vertical in Sec. 2.5.

With the verification that the first turn of vertical data may be bad, one may worry about the quality of the first turn analysis employed in the CCR BPM program and in the turn method to calculate the injection offset. However, when the injection offsets calculated by the first turn are compare with injection offsets calculated by the other turns in the turn method, all results are within one $rms$ standard deviation of each other. The first turn method also agrees with the results from the amplitude and scan methods, and the CCR measurement. So although the first turn of vertical data seems to be faulty, it does not affect the first turn injection offset fit in the turn method. However, this could be because the beam was injected near-on-axis for the RingScan reproducibility measurement. The first turn injection offset method should be scrutinized under the larger production offset conditions.

2.7.5 Comparison of the injection offset calculations

Table 2.7 summarizes the results of the injection offset calculations for all methods discussed earlier in this section and the CCR BPM program measurement. As can be seen in the plots of the injection offsets for the different methods (Figs. 2.53, 2.54, 2.55, 2.56, 2.59, and 2.60), all methods yield about the same result. This is naturally to be expected since all three methods of calculating the injection offset are derived from the
partial propagation equation (Eq. (1.38)), and thus are algebraically identical. The amplitude and scan methods are most similar of the three methods because they both analyze only one scan of data to calculate the injection offset. The turn method and the CCR BPM program use data from all BPMs taken over several different pulses. The smaller distribution spreads of the measurements for the turn method are due to the sheer number of calculated points of the injection offset, 4040, compared to the 1818 calculations of the injection offset in the scan and amplitude methods. To better understand the injection offset measurement made during operations, this study of the injection offsets should be repeated with RingScan data at production injection offsets.

2.7.6 Injection offset calculations with bad data included

All of the plots and results reported in this discussion on calculations of the injection offset have involved data with all of the BPM data acquisition errors removed, as discussed in Sec. 2.4. Since the general feeling is that the beam size inferred from the injection offset calculation performed by the CCR BPM program is smaller, it is of interest to investigate what effects the presence of data acquisition errors have on the injection offset calculation.

Applying the first turn of the turn method, as is done by the CCR BPM program, the injection offset can be calculated for each RingScan in the RingScan reproducibility dataset. All scans will be included in the calculation, even scans with data acquisition errors. This will yield 101 injection offset measurements.

Figures 2.63 and 2.64 plot what an operator might see if they were to make 101 measurements of the injection offset with the CCR BPM program. This time the average CO at each BPM is applied as $x_{CO}$ in Eq. (2.12) to simulate the saved CO needed in the BPM injection offset program. In both figures, there is a well
2.7 Studies of the Injection Offset Measurement

<table>
<thead>
<tr>
<th>Comparison of Injection Offset Calculations</th>
<th>CCR</th>
<th>Amplitude</th>
<th>Scan</th>
<th>Turn</th>
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<td><strong>Horizontal Position [mm]</strong></td>
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<td></td>
<td></td>
<td></td>
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<td>-.6830</td>
<td>-.68034</td>
<td>-.693</td>
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<td>.1411</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>Error</td>
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<td>.1.57×10⁻³</td>
<td>.1.61×10⁻³</td>
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<td>.3.13×10⁻²</td>
<td>.1.91×10⁻²</td>
</tr>
</tbody>
</table>

**Table 2.7:** Restates the results of the injection offset calculation for all methods. The numbers listed under “Position” and “Angle” are the average of those quantities, “Error” is the error on the average calculated by Eq. (2.10), and “Spread” is the \( \text{rms} \) standard deviation of the measurement distribution. All turns and RingScans without data acquisition errors are applied in calculations with the turn method.
2.7 Studies of the Injection Offset Measurement

Figure 2.63: The horizontal injection offset with one uncertainty calculated by applying the first turn method to all data in the RingScan reproducibility dataset including scans with data acquisition errors.

defined island that is the real injection offset. Also accompanying the islands are 10 outlying results (best observed in the vertical plot of the injection offset) that yield bad calculations of the injection offset. These outliers have the largest error bars in Figs. 2.63 and 2.64. Analyzing data from the RingScan reproducibility dataset with the first turn method for calculating the injection offset without removing the bad BPM data to simulate the CCR BPM program measurement, it is shown that the CCR BPM program would calculate the wrong injection offset 9.9% of the time. This is a huge percentage as the injection offset is normally only calculated once with the CCR BPM program, and there is no way of knowing if the result is correct without taking multiple measurements.

Also note that in both the vertical and horizontal injection offset calculations, it is possible to yield results with the incorrect sign in the position and the angle. The maximum difference in position between an outlying measurement and the island is $\sim 3$ mm in the vertical. It would be interesting to test if the magnitude of the errors scales with larger injection offsets.
Understanding which of the BPM acquisition errors could affect the injection offset calculations performed in the CCR BPM program is a little tricky because the turn method has a tendency to smear out individual BPM errors. Surely, any error that affects the first turn in relation to the average CO is a candidate: flat line, BPM selection, large offset, plateau, drifts across the scan, single turn drifts, and possibly offset outlier errors. The next thing necessary is for several of these errors to occur at different BPMs in one RingScan. It has been observed that BPM selection errors happen many times in one RingScan, and eight RingScans were observed to possess this error in the reproducibility dataset. Seven of the outliers shown in Figs. 2.63 and 2.64 are calculations from RingScans with at least four scans possessing BPM selection errors. Although a lucky combination of a few of the errors mentioned above occurring in the same RingScan could result in an injection offset outlier, the other three outliers have no errors that affect the first turn. This would indicate an extreme change in the pulse-to-pulse momentum variations over the 20 pulses in the RingScan. These three outliers are actually contained in the injection offset islands of Figs. 2.59
2.8 Calculation of the Magnitude of the Pulse-to-Pulse Momentum Variations

The pulse-to-pulse momentum variations were the default explanation in Sec. 2.6 for the measurement spreads in the fitted parameter distributions (amplitude, phase, and offset). These variations have been known to exist but a calculation of the magnitude of the pulse-to-pulse momentum variations has never been properly documented. The fitted CO rms measurement spread from the RingScan reproducibility dataset and the measured dispersion function will be applied in this derivation. The dispersion function measurement is described in detail in Sec. 3.3.

Assume that the error in the CO measurement of the RingScan reproducibility dataset only has two contributing factors, the BPM measurement error and the pulse-to-pulse momentum variation,

\[ \epsilon_{CO} = \epsilon_{BPM} + D\epsilon_{\delta}, \]  

(2.15)

where \( \epsilon_{CO} \) is the total error on the CO measurement, \( D \) is the dispersion function, \( \epsilon_{\delta} \) is the error in the momentum due to the pulse-to-pulse variations, and \( \epsilon_{BPM} \) is the CO measurement error of the BPM measurement. It is obvious that the BPM measurement error contributes to the CO measurement spread, and recall that it was shown that the CO measurement spread was very correlated with the dispersion function, Fig. 2.46. Since the absolute individual error values are not known from the RingScan reproducibility dataset, Eq. (2.15) is written in terms of standard deviations,

\[ \sigma^2_{CO} = \sigma^2_{BPM} + 2D\langle \epsilon_{BPM} , \epsilon_{\delta} \rangle + D^2\sigma^2_{\delta}, \]  

(2.16)
2.8 Calculation of the Magnitude of the Pulse-to-Pulse Momentum Variations

where $\sigma$ is the $rms$ standard deviation and $\langle \epsilon_{BPM}, \epsilon_4 \rangle$ is the covariance between the errors due to the momentum variation and the BPM measurement error which is expected to be small. Since the CO $rms$ spread and the dispersion function are known, one can fit for the standard deviation of the momentum variations and the BPM measurement error, Eq. (2.17). The BPM measurement error is expected to equal the $rms$ measurement spread observed in the vertical CO measurement, $\sim 0.02$ mm.

$$\sigma^2_{BPM} = a + 2bD + cD^2$$  \hspace{1cm} (2.17)

The details of the dispersion function measurement in the PSR are reported in Sec. 3.3, but in brief, the change in the CO was measured by averaging the results of 10 RingScans. The change in the revolution frequency was measured with SRWC41 by a TOF delay measurement in the same procedure as correcting the energy during the setup of the PSR, Sec. 2.2. The change in time was converted to a change in momentum by the momentum compaction factor method. Three different momentums settings where employed for the dispersion measurement: plus, minus, and a baseline momentum (the same as the momentum for the RingScan reproducibility measurement). The three data points were fit to a line, and the dispersion function (the slope of the line) was extracted.

Several variations in the fitting scheme of Eq. (2.17) were performed to explore the properties of the pulse-to-pulse momentum variations and CO $rms$ spread data. The variations include, fitting dispersion and $rms$ CO measurement spread data from all BPMs, fitting only data from the vertical BPMs, fitting all horizontal BPMs except BPM 20, fitting data from all BPMs expect BPM 20, and all of these variations with the added constraint that $b$ of Eq. (2.17) equals zero.

It was found that the vertical CO also changed during the dispersion function measurement indicating a vertical dispersion function, so the vertical CO is included
2.8 Calculation of the Magnitude of the Pulse-to-Pulse Momentum Variations

in some of the variations of the fit for the magnitude of the pulse-to-pulse momentum variations. As it turns out, the vertical data points are needed to obtain the expected BPM measurement error of $\sim 0.02$ mm. It was found that if only the vertical data was fit to Eq. (2.17), the fitted $\text{rms}$ spread in the momentum variation was imaginary. This is understandable as the dispersion function is very small in the vertical and not able to constrain the momentum variation fit.

When Eq. (2.17) is fit with only the horizontal BPMs, the fit crosses the $\delta = 0$ axis at a negative value, yielding an imaginary $\sigma_{BPM}$. So the horizontal data also is unable to constrain the momentum variation fit and the fit does not converge to the observed BPM measurement error.

When all BPMs were fit, it was found that data from BPM 20 (SRPM92x) was an outlier. This is not surprising considering the previous discussion about this BPM possessing a much larger CO measurement spread compared to the other BPMs, Figs. 2.23 and 2.45. Since the vertical CO measurement at SRPM92 (BPM 40) is reasonable, only BPM 20 was removed from the data for the best fit.

Equation (2.17) was also fit, as one could easily assume, with the covariance between the BPM error and the momentum error equal to zero, $\langle \epsilon_\delta, \epsilon_{BPM} \rangle = 0$ or $b = 0$. Fitting Eq. (2.17) for all BPMs with the constrained that $b = 0$ yields a BPM error larger than that observed in vertical CO measurement spread, $0.02$ mm. Thus the pulse-to-pulse momentum variation spread calculated in the fitting variation that uses all BPMs except BPM 20 and includes the covariance term in the fitting will be quoted as the result, $\sigma_\delta = 4.56 \times 10^{-5}$.

Table 2.8 summarizes the results of fitting Eq. (2.17) to the square of the CO measurement spread via a linear regression method. There are several things to note. First one notes that the worst fits, the fits with the largest SSR/DOF, include BPM 20. Also note that the best fits involve only the vertical BPMs. This is because all of the vertical BPM data points are close together and easy to fit, see Fig. 2.65.
### Summary of Fitting Eq. (2.17) to CO Measurement Spread

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_{BPM}$ [mm]</th>
<th>$\langle \epsilon_{BPM}, \epsilon_{\delta} \rangle$ [mm]</th>
<th>$\sigma_{\delta}$</th>
<th>SSR/DOF [mm$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>All BPMs</td>
<td>1.94×10$^{-2}$</td>
<td>-2.8×10$^{-7}$</td>
<td>5.60×10$^{-5}$</td>
<td>1.24×10$^{-5}$</td>
</tr>
<tr>
<td>All BPMs, no BPM 20</td>
<td>1.83×10$^{-2}$</td>
<td>-1.1×10$^{-6}$</td>
<td>4.56×10$^{-5}$</td>
<td>1.53×10$^{-6}$</td>
</tr>
<tr>
<td>Horizontal BPMs, no BPM 20</td>
<td>NaN</td>
<td>-2.4×10$^{-6}$</td>
<td>3.64×10$^{-5}$</td>
<td>3.28×10$^{-6}$</td>
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<td>Vertical BPMs</td>
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<td>-8.2×10$^{-7}$</td>
<td>NaN</td>
<td>1.53×10$^{-7}$</td>
</tr>
<tr>
<td>All BPMs $b = 0$</td>
<td>2.22×10$^{-2}$</td>
<td>0</td>
<td>5.90×10$^{-5}$</td>
<td>1.45×10$^{-5}$</td>
</tr>
<tr>
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<td>0</td>
<td>5.34×10$^{-5}$</td>
<td>1.75×10$^{-6}$</td>
</tr>
<tr>
<td>Horizontal BPMs, no BPM20, $b = 0$</td>
<td>4.43×10$^{-2}$</td>
<td>0</td>
<td>5.18×10$^{-5}$</td>
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<td>Vertical BPMs, $b = 0$</td>
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<td>0</td>
<td>7.04×10$^{-5}$</td>
<td>1.42×10$^{-7}$</td>
</tr>
</tbody>
</table>

**Table 2.8:** Summarizes the results of fitting the square of the CO measurement spread to the measured dispersion function, Eq. (2.17). NaN indicates an imaginary value, and zero indicates that the parameter was not fit for in that variation.
Also observed in Tab. 2.8 is confirmation that the covariance of the BPM measurement error and the momentum error is small, $\langle \epsilon_3, \epsilon_{BPM} \rangle \approx 0$. Even so, including the covariance significantly changes the results of the BPM measurement error and the pulse-to-pulse momentum variation spread. The pulse-to-pulse momentum variation spread can couple into the BPM measurement error by changing the CO and pushing the extremes of the betatron oscillation towards the nonlinear part of the BPM response. The pulse-to-pulse momentum variations also change the CO at the injection point changing the injection offset and the amplitude of betatron oscillation, which can also couple into the BPM measurement. From the residuals along the scan analysis in Sec. 2.5, the residuals of the cosine wave fit are known to be dependent on position which is determined by the momentum. But since the CO reproducibility measurement was performed at near-on-axis injection with small betatron oscillations, these effects are small as seen in the almost zero fitting of the covariance term.

**Figure 2.65:** (Color) The square of the fitted offset rms measurement spread from the RingScan reproducibility dataset (blue circles) and the fit of Eq. (2.17) (red) including the covariance term and data from all BPMs except BPM20 (SRPM92x). The outlying blue circle is data from BPM 20, graphed here, but not included in the fit.

Figure 2.65 shows the fitting result from including the covariance term and data
2.9 Summary

from all BPMs except BPM 20. BPM 20 is shown in Fig. 2.65, but it is not included in the fit. The data point for BPM 20 is the outlying blue circle with dispersion function $\sim -2.2 \text{ m}$. In Fig. 2.65, the vertical BPMs comprise the group of points centered about zero on the x-axis. One might notice that the dispersion function is negative for all of the horizontal BPMs, this is because in the coordinate system of the PSR inward is positive. An increase in particle momentum will push the CO outward (negatively), thus the horizontal dispersion function is negative.

The pulse-to-pulse momentum variation of the central momentum of the beam pulse varies between pulses with an rms standard deviation spread of .0045%. This is about 10 times less than the momentum spread in a single pulse[6]. Although the pulse-to-pulse momentum variations are small, they contribute most of the horizontal CO measurement spread.

2.9 Summary

I have studied the beam position measurement in a statistically rigorous manner. I discovered 9 distinctly different data acquisition error. I developed a suite of analysis scripts to identify and take out scans possessing data acquisition errors from the dataset. I have theorized possible methods for the data acquisition errors to manifest in the collection of the BPM data. System experts have been able to apply my hypotheses and have reconfigured their systems to mitigate the BPM selection and flat line errors.

I analyzed the residuals of the cosine wave fitting to the turn-by-turn as a function of turn number. I found that the dominating component in the single turn BPM error was a constant offset drift across the scan. I also found evidence that the first turn of vertical data may be compromised because the average residual for this turn is so large.
I reported the measurement results of the fitting parameters. Most important are the tune and offset results. The RingScan reproducibility measurement yields tune distributions with \( \text{rms} \) measurement spreads on the order of \( 10^{-4} \). This result is believed to be about 10 times as precise as the current tune measurement employed during operations. The offset measurement was also found to be better than thought. The horizontal CO measurement spread was \( \sim 0.1 \) mm, while the vertical CO measurement spread was \( \sim 0.02 \) mm, which seems to be limited by the intrinsic resolution of the BPM.

I showed that the CO measurement spread was correlated with the measured dispersion function. I took advantage of this discover in a calculation of the magnitude of the pulse-to-pulse momentum variations. I found that magnitude of the pulse-to-pulse momentum variations was \( 4.5 \times 10^{-5} \), which is about 10% of the momentum spread in the injected beam. Although small, this pulse-to-pulse is the major contributor to the measurement spread of amplitude, phase, and offset measurement spreads.

This study of the reproducibility of the RingScan beam position measurement has shown that if I account for the BPM data acquisition errors and remove scans with these errors from the dataset, the resulting RingScan measurement is very good. The results from the RingScan data are more accurate and have less \( \text{rms} \) measurement spread than the methods currently applied during operations. Only one set of RingScan data is needed to calculate the CO, fractional tune, and the injection offset, where three different sets of data must be collected to measure these quantities with the CCR BPM program. The RingScan data can also be saved for off-line analysis.

It is clear that I possess a measurement device in the RingScan BPM program that will be essential in collecting data for experiments aimed to improve the model of the PSR. I am also confident that the suite of RingScan analysis scripts developed to analysis the turn-by-turn BPM data is able to identify and take out all scans with data acquisition errors. Thus, it is only necessary that I acquire the proper number

...
of RingScans for each measurement setting.

Now that I understand the basic beam position measurement in the PSR, I may extend the position measurement to the supporting and model improvement experiments discussed in Chaps. 3 and 4 respectively.
Chapter 3

Supporting Experiments

In this chapter, I discuss the supporting measurements. The supporting measurements have two functions; they verify experimental procedures, and they describe how the measurements are performed. The first experiment is a beam-based measurement of the PSR dipole hysteresis, Sec. 3.1. This measurement shows that small current changes in the PSR benders as those applied in the orbit response matrix measurement and the center the CO method for measurement of the beam momentum do not suffer from hysteresis in the PSR dipoles. The measurement of the dispersion function is discussed in Sec. 3.3. Here, I compare two competing methods in the beam momentum measurement. I apply both methods for measurement of the dispersion function. I also introduce a technique for fast momentum measurements with a dispersion function calibrated LDPM03 (Sec. 3.3.1 III) for use in the septum fringe field characterization experiment, Sec. 4.2. The last supporting measurement is the betatron amplitude measurement, Sec. 3.4. I employ the classic quadrupole perturbation method to measure the beta functions.

The remaining sections in the chapter (Secs. 3.5 and 3.6) report additional calculations that I performed on the beta function measurement dataset. The first of
3.1 A Beam-Based Hysteresis Measurement in the Los Alamos Proton Storage Ring

these (Sec. 3.5) is a check of the relative BPM gains by comparing the measured beta function at a BPM with the amplitude of betatron oscillation at that BPM. The second digression (Sec. 3.6) reports the results of a beam-based alignment analysis.

Understanding the betatron amplitude and dispersion function measurements is important because these measurements will be compared with the baseline model and any possibly improved model in order to establish an improved model of the PSR.

3.1 A Beam-Based Hysteresis Measurement in the Los Alamos Proton Storage Ring

The beam-based hysteresis measurement makes use of the beam to determine whether the hysteresis of the corrector magnets to be employed in the ORM measurement will affect the reproducibility of the baseline CO. To measure the ORM, the beam is bumped with a corrector magnet and the resulting CO measured by beam position monitors (BPMs) is compared with the CO before the bump. The difference in the CO per unit bump strength is a column in the ORM[24, 25]. To cancel systematic errors in the BPM measurement, the corrector gain, and drifts in the magnet power supply, both plus and minus bumps are applied to the beam for each corrector. To check whether the corrector magnet hysteresis will affect the reproducibility of the baseline CO in the ORM measurement, a baseline CO will be compared with a second CO (called the hysteresis CO) measured after the magnet current has been changed to the ORM bump settings (both plus and minus) and returned to the baseline current. If the two COs are the same to within the CO measurement error of the BPMs, then the hysteresis of the corrector magnets will not affect the reproducibility of the baseline CO in the ORM measurement.

There are nine vertical corrector magnets in the PSR which typically operate
between $\pm 10$ A. These magnets are mostly coil and have little iron to cause hysteresis. The bend angle in the vertical correctors is also about two orders of magnitude smaller compared to the horizontal benders. So, hysteresis would appear much earlier in the horizontal bending magnets. Thus, the vertical corrector magnets are not considered in this study.

While there are no horizontal correctors in the PSR, each horizontal bending magnet is individually shunted and may be employed as a horizontal corrector in addition to operating as a horizontal bending magnet. There are two types of horizontal bending magnets. SRBM11 and 12 are C-magnets operating at 1050 A and 770 A respectively, and there are nine original PSR (here called common) $36^\circ$ dipole magnets operating at 1060 A. A twelfth horizontal bending magnet, RIBM09, is the injection merging magnet. RIBM09 determines the horizontal injection offset into the PSR and thus will not be applied to bump the beam during the ORM measurement. Due to experimental time limitations only two horizontal bending magnets were tested for hysteresis. The most saturated C-magnet (SRBM11) and one of the common PSR benders (SRBM21) were chosen.

The beam-based hysteresis measurement was performed during the June 27, 2008 accelerator development. The PSR was set up in the same manner as the RingScan reproducibility measurement, which was discussed in Sec. 2.2. At the beginning of the day, the beam was injected near-on-axis ($[-0.72 \text{ mm}, 0.31 \text{ mrad}]$ in the horizontal and $[1.99 \text{ mm}, 0.3 \text{ mrad}]$ in the vertical), however, the beam-based hysteresis measurement took place at the end of a 12 hour shift and the injection offset had slipped to ($\sim[-0.65 \text{ mm}, 0.57 \text{ mrad}]$ in the horizontal and $\sim[1.5 \text{ mm}, -0.03 \text{ mrad}]$ in the vertical). This slippage in the injection offset was due to a mis-reset vertical corrector (SRVM81) from the ORM measurement, and led to more spread in the vertical beam position data because of the small vertical injection angle, as discussed in Sec. 3.4.
After the PSR was set up, ten RingScans were collected to measure the baseline CO. The magnet current was changed to the positive ORM bump setting and allowed to rest. The magnet current was then adjusted to the negative ORM bump setting and allowed to settle. Lastly the magnet current was returned to the baseline current set point and ten more RingScans were taken to measure the hysteresis CO after the magnet current changes. The current changes for SRBM11 were ±4 A, and the current changes for SRBM21 were ±2 A. Both current changes applied approximately the same strength kick to the beam, 1 mrad. One should also note that the SRBM21 shunt current was the knob employed, so the actual current change in the magnet is opposite that described above.

### 3.1.1 Results

After the RingScan data was fit to a cosine wave and all of the BPM data acquisition errors were removed from the RingScan data as explained in Sec. 2.4, the average CO at each BPM from the baseline CO was subtracted from the average hysteresis CO. The errors on the average COs were calculated by Eq. (2.10). So the calculation error on the difference between the baseline and hysteresis COs is calculated in the usual way for uncorrelated errors by adding the squares of the individual measurement error of the average COs:

$$\sigma_{\Delta CO} = \sqrt{\sigma_{\text{ave,baseline}}^2 + \sigma_{\text{ave,hysteresis}}^2}.$$  \hspace{1cm} (3.1)

Figure 3.1 shows matching baseline and hysteresis CO measurements for the current change in SRBM11. This is of course if BPM 20 (SRPM92x) is ignored. For now suppose that the BPM 20 measurement is good and must arise from the dipole kick due to the hysteresis of SRBM11. However, a dipole kick should affect the measurements of all BPMs around the ring, Eq. (1.61). Since all of the other BPMs measure zero change in the CO, it must be concluded that the SRPM92x measurement is
faulty. The performance of SRPM92x during this experiment will be discussed in Sec. 3.1.2.

Note that the difference in the CO at BPM 19 (SRPM91x) is also more than three standard deviations from zeros. This is one of the diamond-type BPMs and is known to possess more spread in the fitted parameters than the other BPMs. The CO measurements for both the baseline and hysteresis measurements at BPM 19 overlap a little, but there are not enough scans to fill both measurement distributions.

Ignoring data from BPM 19 (SRPM91x) and BPM 20 (SRPM92x), Fig. 3.1 shows the difference in the hysteresis and baseline COs is zero within one uncertainty for most BPMs and zero within two uncertainties for all BPMs. In Fig. 3.1, BPMs 1 and 7 (SRPM01x and 31x) are shown to have slightly larger error bars than the other horizontal BPMs. Both of these BPMs are in horizontally focusing quadrupoles where the dispersion function is large. These slightly larger errors are derived directly from the measured CO variability which for this measurement is largest in these BPMs. The variability in the CO measurement at these BPMs is believed to be due to the
3.1 A Beam-Based Hysteresis Measurement in the Los Alamos Proton Storage Ring

pulse-to-pulse momentum variations of the linac.

![Graph showing hysteresis measurement](image)

Figure 3.2: (Color) The difference between the hysteresis CO measured after the current change in SRBM21 and the baseline CO with three uncertainties, blue. The horizontal red line is zero. The vertical black line separates horizontal (left) and vertical (right) BPMs.

The results of the hysteresis measurement for the common PSR bending dipole (SRBM21) are shown in Fig. 3.2. Again the hysteresis CO and the baseline CO are shown to be zero within one \textit{rms} uncertainty for most BPMs and zero within two \textit{rms} uncertainties for all BPMs. BPM 20 (SRPM92x) sticks out showing an error bar at least twice as large as the other BPMs.

BPMs 37 and 39 (SRPM81y and 91y) have much larger error bars compared to the other vertical BPMs. These are the vertical diamond-type BPMs and were shown to have a more variable CO measurement in Sec. 2.6.4. BPM 40 (SRPM92y) suffers from bad statistics, but only has a slightly larger error bar compared to the other vertical BPMs. Most of the scans for BPM 40 were thrown out because they corresponded to scans taken at the same time as the offset outliers identified in BPM 20, SRPM92x. Perhaps more than ten RingScans are needed to average out the pulse-to-pulse momentum variations and fill the measurement distribution to obtain a quality CO measurement.
Both hysteresis measurements from SRBM11 and 21 yield null results for the hysteresis of these magnets affecting the reproducibility of the CO. If one assumes that the other eight common PSR bending magnets behave similarly to SRBM21, this shows that the hysteresis of the horizontal benders to be employed as corrector magnets in the ORM measurement will not affect the reproducibility of the baseline CO. This means that the magnet power supplies will not have to be cycled for each ORM bump saving several hours of experiment time.

### 3.1.2 Performance of SRPM92x

The beam-based hysteresis measurement was actually the experiment that first showed the inconsistency of the CO measurement for BPM 20 (SRPM92x) and led to a more in depth analysis of the RingScan reproducibility data in Sec. 2.4.9. Figure 3.1 shows that the CO measurement for BPM 20 drifted by more than 2 mm in the three minutes it took to make the hysteresis measurement. Checking the number of scans remaining in the data set after the BPM data acquisition errors were removed, it was found that only 15 of the 40 scans taken at SRPM92x during the hysteresis measurement of both SRBM11 and SRBM21 were not removed from the dataset for possessing data acquisition errors, 37.5%. The number of scans remaining at BPM 20 (SRPM92x) for each CO measurement is shown in Fig. 3.3. It is obvious that none of these CO measurements possess enough scans for sufficient statistics.

Each fitted offset measured at SRPM92x for the SRBM11 hysteresis measurement is shown in Fig. 3.4. For the baseline case, scans 1, 5, 6, 8, and 9 were removed. Scans 5 and 8 have a missing turn error, scans 1 and 9 were found to be amplitude outliers in the vertical, and scan 6 was a fitted phase outlier in the vertical. The vertical fitting parameter outliers were most likely from the increased measurement spread due to the small vertical injection angle. All of the baseline fitted offsets, including those
3.1 A Beam-Based Hysteresis Measurement in the Los Alamos Proton Storage Ring

**Figure 3.3:** (Color) A bar graph showing the number of remaining scans at BPM 20 (SRPM92x) after scans with errors were removed from the dataset, blue. The red line shows the number of scans taken for each measurement, ten.

removed for data acquisition errors save scan 1, yield a consistent offset measurement.

Compare the baseline to the erratic fitted offset measurement of the hysteresis CO measurement for SRBM11, where most scans were removed as data acquisition errors. Scan 2 was removed because it had a missing turn error, and scans 8 and 9 were found to be large offset errors. The rest (scans 1, 4, and 10) were removed as outliers in the fitted offset. Ignoring the scans with largest fitted offsets (scans 4, 8, and 9), there is still large variability in the fitted offset, \( \sim 2 \) mm full range. Scans 3, 5, 6, and 7 remain because they are the closest to each other in the fitted offset. The RingScan analysis scripts removed scans 1 and 10 to minimize the offset measurement spread. Even so, it is impossible to believe this CO measurement. Also note that none of the fitted offsets for the hysteresis CO measurement come close to the more consistent baseline measurements. Something has caused all of the offset measurements for BPM20 to drift positively.

It is not known what causes the inconsistency of the CO measurement for BPM20 (SRPM92x) or how the BPM can yield consistent measurements for only a short time. The consistent to variable behavior of BPM 20 was also observed in analysis.
3.1 A Beam-Based Hysteresis Measurement in the Los Alamos Proton Storage Ring

Figure 3.4: (Color) The fitted offsets measured at BPM20 (SRPM92x) for the entire magnet hysteresis measurement of SRBM11. The blue circles are data from the baseline measurement, and the green squares are data from the hysteresis CO measurement. The shapes not filled in with color are scans removed from the dataset because they possess data acquisition errors.

of the RingScan reproducibility dataset, Fig. 2.23 of Sec. 2.4.9. One notes that the inconsistent fitted offset measurements are always positive. Could this indicate a faulty termination on one of the horizontal BPM stripline electrodes, or does this point to a switching contact problem in one of the MUX switches which select BPM 20 for measurement? Further investigation needs to occur to understand and fix this BPM. However, it is unlikely that this will be done before the proposed ORM measurement, so data from BPM 20 should not be included in the ORM analysis.

3.1.3 Summary

It was found that the CO was reproducible with small changes in the most saturated C-magnet (SRBM11) and a common PSR bender, SRBM21. I believe that these results can be propagated to the entire set of PSR dipoles. Thus, if the current changes remain small, hysteresis of the horizontal dipoles will not affect the ORM
3.2 Quadrupole Hysteresis Measurement

Preliminary results from the ORM analysis indicated that the strength of the model defocusing quadrupoles was \(\sim 5\%\) larger than the real value. This 5\% difference corresponds with a 10 A discrepancy, which is hard to believe. Nonetheless, the control channel read backs, power supply displays, and quadrupole hysteresis was checked for this 5\% anomaly.

It was found that the control channel read backs could differ \(\sim 5\%\) compared to the magnet power supply gages. So, the discrepancy is not in the controls system.

During production, the operators are continuously tuning the accelerator to minimize losses. It is possible that the defocusing quadrupoles may have walked away on the hysteresis curve. All PSR magnets are meant to run on the upper hysteresis curve because this is where the magnet measurements were made.

A ring entry was made on October 16, 2008 during the maintenance week to test for the affect of quadrupole hysteresis. The magnetic field at the pole-tip of SRQU41 was measured with a hall probe. SRQU41 was chosen because it has a large bore field clamp and allowed the hall probe to rest on the pole-tip. The pole-tip magnetic field was measured at the defocusing quadrupole production current set point at both
3.2 Quadrupole Hysteresis Measurement

<table>
<thead>
<tr>
<th>Hysteresis Measurements for SRQU41</th>
<th>Magnetic Field [T]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Current Set Point [A]</strong></td>
<td><strong>Defocusing Set Point</strong></td>
</tr>
<tr>
<td>274.9*</td>
<td>0.20045</td>
</tr>
<tr>
<td>600.00</td>
<td>0.43984</td>
</tr>
<tr>
<td>274.9*</td>
<td>0.20152</td>
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<tr>
<td><strong>Focusing Set Point</strong></td>
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</tr>
<tr>
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<tr>
<td>0</td>
<td>0.00265</td>
</tr>
<tr>
<td>455.4*</td>
<td>0.33413</td>
</tr>
</tbody>
</table>

**Table 3.1:** Results of the quadrupole hysteresis measurement. The *-ed currents indicate production current set points.

The results presented in Tab. 3.1 indicate a .53% difference in quadrupole strength between the top and bottom hysteresis curves at the defocusing quadrupole production current set point. This effect is an order of magnitude smaller than the 5% difference between model and real defocusing quadrupole strengths suggested by the preliminary ORM result.

The hysteresis of the quadrupole yields of difference of .39% in the quadrupole strength between the top and bottom hysteresis curves at the focusing quadrupole production current set point. The fractional hysteresis effect at the focusing quadrupole current set point is less than that at the defocusing current set point because the
focusing quadrupole strength is stronger.

The preliminary ORM analysis results of $\sim 5\%$ lower strength in the defocusing quadrupoles was not observed in the power supply current output, read back or set point in the controls system, or in the hysteresis of the quadrupole magnets.

### 3.3 Dispersion Measurement

Many measurements of the dispersion function were made during the 2008 and 2009 run cycles. These include the developments of May 24, June 27, November 14, 2008 and July 26, September 25, and December 22, 2009. Data and analysis presented in this section will be limited to that of the December 22, 2009 development, where two momentum measurement procedures were consecutively executed at the same PSR settings for direct comparison of the $\alpha_c$ and the center methods.

#### 3.3.1 Two methods to measure the momentum

In order to measure the fractional change in momentum ($\delta$), a time of flight (TOF) measurement is made. The procedure for this TOF measurement is the same as that applied in correcting the beam energy discussed in Sec. 2.2. The time delay between the $N^{th}$ revolution and the $N^{th}$ trigger of the “moving” 2.8 MHz imposed design revolution frequency is eyeballed with cursors on an oscilloscope in the central control room (CCR) yielding an estimated measurement error of 5 ns.

Presented and compared in this subsection are two methods to convert the TOF measurement to a $\delta$ here called the $\alpha_c$ and center methods.
I Momentum compaction factor method

The momentum compaction factor method applies a model momentum compaction factor ($\alpha_c$) and the on-momentum Lorentz factor $\gamma_0$ to calculate the fractional momentum deviation from the TOF measurement. The $\alpha_c$ describes the fractional change in circumference per fractional momentum deviation and is defined in Eq. (1.72). The $\alpha_c$ method makes use of the relationship between the fractional period change and the $\delta$ derived in Eq. (1.76) in Sec. 1.1.10 and is repeated here for convenience,

$$\delta = \left( \alpha_c - \frac{1}{\gamma_0} \right)^{-1} \frac{\Delta T}{T_0}. \quad (3.2)$$

The synchronous period ($T_0$) is defined by the imposed “moving” 2.8 MHz revolution frequency of the PSR. The synchronous circumference ($C_0$) is derived from alignment data taken during the 2005-2006 long maintenance period and is 90.301 m. Together $T_0$ and $C_0$ define the synchronous Lorentz factor, $\gamma_0$. The $\alpha_c$ depends on the PSR model. However, the $\alpha_c$ does not vary significantly with model parameters and an imperfect model will contribute only a small systematic error to the $\delta$ calculation. An estimate of the magnitude of this systematic error due to employing the model $\alpha_c$ is discussed later in Sec. 3.3.3 II. The random error on the calculated $\delta$ propagated from the error of the TOF measurement ($\sigma_T$) is,

$$\sigma_\delta^2 = \left[ \left( \alpha_c - \frac{1}{\gamma_0} \right)^{-1} \frac{\sigma_T}{T_0} \right]^2. \quad (3.3)$$

II Center method

The center method takes advantage of $\frac{\Delta C}{C_0}$ in Eq. (1.73) going to zero when both the on and off-momentum COs are the same, which happens when both COs are centered. When $\frac{\Delta C}{C_0} = 0$, the fractional momentum deviation may be easily calculated from the
TOF measurement:

\[
\delta = -\frac{\gamma_0^2 \Delta T}{T_0}, \quad \text{and} \quad \sigma_{\delta}^2 = \left(-\frac{\gamma_0^2 \sigma_T}{T_0}\right)^2. \tag{3.4}
\]

However operationally, when there is a change in the beam momentum, following Eq. (1.67), the CO will also change. So in order to achieve the \(\frac{\Delta C}{C_0} = 0\) condition, all of the bending magnets are adjusted to compensate for the change in the CO due to the momentum change. Thus, the on and off-momentum COs are the same and Eq. (3.4) may be applied to calculate \(\delta\) from the TOF measurement.

Since centering the CO with the bending magnets changes the lattice and thus the dispersion function, the CO data for the dispersion fit must be taken at the uncentered CO. However, the TOF measurement for this method is performed for the centered CO. The centering of the CO must be undone before measuring the CO and TOF at the next momentum setting. Centering the CO and undoing the center requires precision and is very time consuming. There are also many sources of systematic errors in this operation: the baseline measurement being on-momentum, each centered CO matching the baseline CO, and CO distortions changing the path length of the circumference due to dipole kicks. The sources and the magnitude of these systematic errors will be discussed in Sec. 3.3.3 III.

III Calibrating LDPM03 to measure \(\delta\)

For experiments with many measurements of the fractional momentum deviation or where there are changes to the PSR lattice and thus \(\alpha_c\) (if small), a faster method to measure \(\delta\) independent of the ring optics is desired.

LDPM03 is a BPM located at the high dispersion region of the Line D transport from the linac to the PSR. Thus, according to Eq. (1.67), the beam position will change the most at LDPM03 per unit change in the beam momentum yielding the best resolution for position to \(\delta\) conversions. If the on-momentum position and the
3.3 Dispersion Measurement

dispersion function where known for LDPM03, the conversion from the measured off-momentum position to $\delta$ is straight forward. Applying one of the methods described above in this subsection, the dispersion function and the on-momentum position at LDPM03 may be calculated, thus calibrating LDPM03 for measurements of $\delta$.

Once LDPM03 is calibrated, dispersion function measurements in the PSR are simple. A relationship between the measured off-momentum position and dispersion function at LDPM03 and any or all BPMs in the PSR can be derived. Starting from Eq. (1.67),

$$x_{LD} = a_{LD} + D_{LD} \delta$$
$$x_{PSR} = a_{PSR} + D_{PSR} \delta,$$

(3.5)

where $x$ is the measured off-momentum CO or position, $a$ and $D$ are the on-momentum CO or position including the steering term in Eq. (1.67) and dispersion function respectively known at LDPM03 from calibration and fit at the PSR BPM, and indices $LD$ and $PSR$ indicate values at LDPM03 and a PSR BPM respectively. Using the $LD$ equation to solve for $\delta$ and substitute into the $PSR$ equation yields

$$x_{PSR} = a_{PSR} - \frac{D_{PSR}}{D_{LD}} a_{LD} + \frac{D_{PSR}}{D_{LD}} x_{LD}.$$  

(3.6)

Thus the measured CO at the PSR BPM ($x_{PSR}$) may be written as a function of the measured position at LDPM03 ($x_{LD}$) instead of the fractional momentum deviation, $\delta$.

There are only two unknowns in Eq. (3.6), the on-momentum CO ($a_{PSR}$) and dispersion function ($D_{PSR}$) at the PSR BPM. When measurements are made at two or more values of $\delta$, Eq. (3.6) can solve or fit $a_{PSR}$ and $D_{PSR}$ as described in Appxs A.2, A.3, and A.4.
3.3 Dispersion Measurement

3.3.2 Measurement setup and data analysis

The PSR was setup for single shot, near-on-axis injection as described in Sec. 2.2. The foil was moved all of the way into the beam pipe so that the changing the momentum would not cause the newly injected beam to miss the foil.

First a RingScan reproducibility measurement of 100 RingScans was acquired as a means to check the performance of the data acquisition programs, the BPMs, and the measured injection offset from the CCR BPM program. The RingScan BPM program was able to collect 40 turns of turn-by-turn position data.

Next data for the $\alpha_c$ method was gathered. Data was collected at four different $\delta$'s corresponding to average CO changes of 0 (or the baseline case), +2 mm, +4 mm, and −1 mm. The phases of module (Mod) 47 and 48 were varied to obtain the momentum change. Unfortunately the phase set points of both Mod 47 and 48 must have been close to the crest of the rf wave because higher energies (negative average CO changes) were not easily achieved. The rf amplitude could have mitigated this issue, but that was not done for this experiment. For each momentum setting 50 position measurements at LDPM03, 20 RingScans for CO data in the PSR, and a TOF delay measurement after 1100 turns were acquired. The Save Raw Data BPM program was executed to collect the data at LDPM03.

Lastly data was collected for the center method. The center method was chosen to be measured last because centering the CO changes the lattice which might not be reset properly. The same momentum settings corresponding to the average CO changes that were applied in the $\alpha_c$ method were also dialed in for the center method. For each momentum setting 50 position measurements at LDPM03 and 20 RingScans for the dispersion measurement were acquired. The baseline set point for SRMP999 (the orbit control knob) was recorded. Then, the CO was centered by turning the orbit control knob. The average CO at each BPM was used as an on-line measure to
match the baseline CO. At the centered CO, 20 more RingScans were taken to verify
that the CO was actually centered and matched the baseline CO. The quality of the
CO centering is discussed in Sec. 3.3.3 III. A TOF delay measurement was also taken
after 1100 turns at the centered CO. The centering of the CO was undone by dialing
SRMP999 back to its baseline value.

The RingScan data was fit to a cosine wave and the data acquisition errors were
identified and removed from the dataset as described in Sec. 2.4. The average CO
extracted from the cosine wave fit and the error on the average (defined in Eq. (2.10))
at each BPM is applied in Eq. (1.67) and fit with the fitting scheme described in Eq.
(A.18) for fitting the dispersion function.

The time delay measured from the TOF measurement is converted to a $\delta$ via
either the $\alpha_c$ or center methods. Since the TOF measurement is made only once per
momentum setting, an estimated measurement error of 5 ns was estimated for the
TOF measurement.

Since LDPM03 is in the transport line, where the beam only makes a single pass,
the position data from LDPM03 is not fit to a cosine wave, just averaged. The error
on the average position is calculated by Eq. (2.10).

A fitting scheme which takes into account both dependent and independent vari-
able uncertainties should be applied to fit for the dispersion function and on-momentum
position because both the fractional momentum derivation and the beam CO (or po-
sition at LDPM03) are measured quantities with associated measurement spreads.
Such a fitting scheme is derived in Appx. A.2 ((A.18)) and is reproduced here for
convenience,

$$\chi^2 = \sum_i \left[ \left( \frac{x_i - \eta_k}{\xi_i} \right)^2 + \left( \frac{y_i - f(\eta_k; \vec{a})}{\epsilon_i} \right)^2 \right]. \quad (3.7)$$

The dispersion function and the on-momentum position may be found by fitting
the average CO data for a BPM in the PSR or the average position data at LDPM03
3.3 Dispersion Measurement

to the \( \delta \)'s from the TOF measurement calculated by the \( \alpha_c \) or center methods via Eq. (1.67). First rewrite Eq. (1.67) in the form of the fitting function in Eq. (A.20),

\[
x_i = a + b \delta_i,
\]

(3.8)

where the fitting parameters are identified from Eq. (3.5), \( a \) is the on-momentum CO (or position in the case of LDPM03) including the affects of steering on the on-momentum particle and \( b \) is the dispersion function. Note that Eq. (3.8) is not the fitting function. The fitting function however mirrors Eq. (3.8) replacing the independent and dependent variables with \( \eta_i \) and \( f(\eta_i; \vec{a}) \) respectively, \( f(\eta_i; \vec{a}) = a + b \eta_i \). The errors (\( \xi \) and \( \epsilon \)) in the \( \chi^2 \) equation, Eq. (A.18), for this fit are the estimated measurement error on the TOF measurement (5 ns) and the error on the average CO or position respectively. The form of the fitting function represented in Eq. (3.8) is written with the dependent variable as a function of the independent variable, \( x_i(\delta_i) \).

3.3.3 Dispersion measurement results

Comparisons of the \( \delta \) measurements, fitted dispersion function, and on-momentum position from the \( \alpha_c \) and center methods, an analysis of the reproducibility of the centered CO in the center method, a comparison of the measured dispersion function with the model, as well as estimates of the systematic errors for both methods are reported in this section.

Unfortunately, the quality of the \(-1\) mm \( \Delta\text{CO} \) momentum setting data was so poor that it could not be included in the dispersion function measurement. This momentum change moved the horizontal injection offset to \( \sim[.5 \text{ mm}, .15 \text{ mrad}] \), yielding amplitudes of horizontal betatron oscillation comparable to the single turn measurement error of the BPMs. Injecting so near-on-axis in the horizontal led to large initial phase variations due to the pulse-to-pulse momentum variations and was
observed as large measurement spreads in the amplitude fitting parameter at each horizontal BPM. Because the amplitude of the horizontal betatron motion was small (1.5 mm maximum and .5 mm minimum), the fitted fractional tune measurement also had large spread ($3 \times 10^{-3}$) which is $\sim$10 times larger than the measurement spread observed in Sec. 2.6.2.

The number of turns of good data in each scan for the $-1$ mm $\Delta$CO momentum setting was reduced from 40 turns to 20-30 turns, indicating that the 201.25 MHz longitudinal frequency structure washed out much more quickly at this momentum setting. It could be that in an effort to squeeze more energy out of Mod 47 and 48 by changing the rf phases, the set point of one or both of these Mods could have been set on the other side of the crest, the debunching side of the wave form.

One problem with performing all of the analysis off-line after the development session is that bad machine settings such as this one are not discovered in time to correct them. However, good BPM data was collected for the other three momentum settings, so Eq. (1.67) may still be employed to fit for the dispersion function with $\delta$’s calculated using both the $\alpha_c$ and center methods for comparison.

I Comparison of the two momentum measurement methods

The two methods presented in Sec. 3.3.1 are procedures to measure the beam momentum. After $\delta$ is calculated, the process to obtain the dispersion function is the same for both methods, namely fitting the ring BPM data to a cosine wave in order to extract the CO and applying the fitting scheme presented in Eq. (3.7), which takes into account the dependent and independent variable measurement spreads. The $\delta$ measurements with errors obtained by the $\alpha_c$ and center methods are compared in Tab. 3.2.

The measured time delay at each momentum setting is greater for the center
### 3.3 Dispersion Measurement

#### Table 3.2: Results of the $\alpha_c$ and center methods’ beam momentum measurement.

<table>
<thead>
<tr>
<th>Method</th>
<th>$\Delta CO$ (mm)</th>
<th>$\Delta t$ (ns)</th>
<th>$\sigma_{\Delta t}$ (ns)</th>
<th>$\delta$ ($\times 10^{-3}$)</th>
<th>$\sigma_\delta$ ($\times 10^{-5}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_c$</td>
<td>0</td>
<td>-1.6</td>
<td>5</td>
<td>.022</td>
<td>6.940</td>
</tr>
<tr>
<td></td>
<td>+2</td>
<td>54.4</td>
<td>5</td>
<td>-.755</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+2*</td>
<td>80.4*</td>
<td>5</td>
<td>-1.116*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+4</td>
<td>156.8</td>
<td></td>
<td>-2.1764</td>
<td></td>
</tr>
<tr>
<td>Center</td>
<td>0</td>
<td>-1.6</td>
<td>5</td>
<td>.014</td>
<td>4.316</td>
</tr>
<tr>
<td></td>
<td>+2</td>
<td>129.6</td>
<td>5</td>
<td>-1.119</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+4</td>
<td>248</td>
<td></td>
<td>-2.141</td>
<td></td>
</tr>
</tbody>
</table>

The average $\Delta CO$ is a label for the momentum setting, $\Delta t$ is the measured time delay after 1100 turns, $\sigma_{\Delta t}$ is the estimated error of the TOF measurement, and $\delta$ and $\sigma_\delta$ are the calculated fractional momentum deviation with uncertainty. Negative time delays indicate a fast arriving beam and positive time delays a slow bunch. The * indicates a modified value.

This method is faster than the $\alpha_c$ method. This is expected because the orbits for beams with less than the design momentum (as in momentum settings with positive average $\Delta CO$s) are tighter. Thus, the circumference of the beam orbit is longer for the centered CO than the uncentered CO. Note that the error on the $\delta$ measurement is larger for the $\alpha_c$ method. This is because there is an additional random error associated with multiplying $\alpha_c$ by $\sigma_T / \tau_0$ in Eq. (3.3) that does not appear in the center the CO method of Eq. (3.4). The error on $\delta$ for the center method is equal to the magnitude of the pulse-to-pulse momentum variations reported in Sec. 2.8, so it is unlikely to achieve an experimental error of less than 5 ns on the TOF measurement.

The $\delta$ from both methods for the baseline and $+4 \Delta CO$ momentum settings
Figure 3.5: (Color) An example of fitting the dispersion function at BPM 1 with the $\alpha_c$ method. The extracted COs from the BPM data and $\delta$’s calculated by the TOF measurement with three standard deviations are shown in the blue circles. The black x’s are the initial $\{\eta_i, f(\eta_i)\}$ guesses and the red squares are the fit to the unmodified data (blue). Modifying the +2 mm $\Delta$CO time delay (green circle) yields a modified fit, cyan triangles.

agree to within the measurement error. However, the calculated $\delta$’s for the +2 $\Delta$CO momentum setting disagree. It is believed that the time delay for the $\alpha_c$ method was manually mis-recorded and is shifted early by 26 ns. Observing the $\alpha_c$ method data in Fig. 3.5, it is obvious that the +2 mm $\Delta$CO data point (the middle measurement) is not on a line between the baseline and the +4 mm $\Delta$CO data points but seems shifted early, to the right. If the mis-recorded time delay is taken into account, the corrected +2 mm $\Delta$CO data point lies on a line between the outer two data points. The $\chi^2$/DOF for the unmodified fit is 13.877, while the modified fit has a much smaller $\chi^2$/DOF = 0.107. Correcting the mis-recorded data point does not seem to have an effect on the dispersion function fit or its fitting error. The *-ed row of Tab. 3.2 reports the calculated $\delta$ with a new time delay that incorporates this apparent
26 ns shift. It is believed that the correction to the $\alpha_c +2$ mm $\Delta$CO time delay was justified because the corresponding data point in the center method lies on the line between the baseline and $+4$ mm $\Delta$CO measurements.

The fractional momentum deviations measured by the $\alpha_c$ and center methods agree within one rms standard deviation. While the differences in the measured $\delta$'s can be attributed to measurement error, many sources of systematic errors are known to exist. The magnitudes of these systematic errors are estimated in Sec. 3.3.3 II and III.

II Systematic errors in the $\alpha_c$ method

The $\alpha_c$ method invokes the model $\alpha_c$ to convert from a TOF delay to $\delta$, Eq. (3.2). The model of course does not completely describe the real machine, so the variation in the model $\alpha_c$ due to changes in the model parameters is of interest. Figure 3.6 plots the fractional change in the model $\alpha_c$ as a function of fractional changes in model parameters. Since the $\alpha_c$ is a lattice parameter defined by Eq. (1.72), the model parameters chosen to vary in this study affect the dispersion function or the bending radius. The dipole current directly changes the bending radius via the magnetic rigidity, the quadrupole strengths affect the dispersion function, and the PSR design energy will vary both the dispersion function and the bending radius.

The model $\alpha_c$ is most dependent on the PSR design energy and the focusing quadrupole strength. The $\alpha_c$ is less dependent on the defocusing quadrupole strength because the defocusing quadrupoles are operated with about 200 A less than the focusing quadrupoles, so a 1% change in the defocusing quadrupole strength is about half of the 1% change in the focusing quadrupole strength.

Notice that the $\alpha_c$ only changes a few percent for a percent change in the model parameters. However, some of these model parameters are well constrained. For
3.3 Dispersion Measurement

Figure 3.6: (Color) The fractional change in the model $\alpha_c$ for changes in model parameters: dipole magnet currents (blue circles), focusing quadrupole magnet strength (green squares), defocusing quadrupole strength (red right-pointing triangles), and the PSR design energy, black left-pointing triangles.

instance, the design energy is defined by the imposed revolution frequency (the moving 2.8 MHz) and the machine circumference, which is documented by the alignment data. Because the design energy and thus the $B\rho$ are well defined, the model dipole currents cannot be very different from reality. This leaves the quadrupole strengths or more likely the quadrupole current to gradient length fits applied in the model as a possible discrepancy between the real machine and the model.

Suppose an extreme case where the model focusing quadrupole strength differs from the real focusing quadrupole strength by 1%. Figure 3.6 suggests that this would lead to at most a 3% change in $\alpha_c$. The model $\alpha_c$ is .12038, so 3% of $\alpha_c$ is very small, only $3.6114 \times 10^{-3}$. Estimates of the maximum magnitude of the systematic error on the $\delta$ measurement from the $\alpha_c$ method for each momentum setting is shown in Tab. 3.3.

As stated in Sec. 3.3.3 I, the $\alpha_c$ varies little with changing model parameters. However, the systematic error does scale with the time delay measurement. In an
3.3 Dispersion Measurement

### Table 3.3: The estimated systematic error on the $\delta$ measurement due to an incorrect model momentum compaction factor in the $\alpha_c$ method. The systematic errors are derived assuming a 1% error in the focusing quadrupole strength, which translates to a 3% error in the model $\alpha_c$. 

| Estimated Systematic Error Due to the Model $\alpha_c$ |  |
|---|---|---|
| Momentum Setting | Baseline [×10⁻⁷] | ΔCO [×10⁻⁵] | ΔCO [×10⁻⁵] |
| $\sigma_\delta$ | 3.99 | -3.23 | -6.19 |

extreme case where the model focusing quadrupole strength is assumed to have a 1% discrepancy compared to the real focusing quadrupole strength, the systematic error on the $\delta$ measurement for these momentum settings is less than the random measurement error on the $\delta$ calculation.

### III Systematic errors in the center method

There are many sources of systematic errors in the complicated procedure of the center method. Three sources of systematic errors are discussed here. The first source of systematic error arises if the baseline momentum setting is not exactly the design momentum. The second and third systematic errors comes about if the base assumption for this procedure is not satisfied, that is if $\frac{\Delta C}{C_0} \neq 0$. If this is the case, Eq. (3.2) is the correct calculation instead of Eq. (3.4). There are two ways for $\Delta C$ not to equal zero; one, the centered CO may not have been completely centered to zero yielding an average CO difference, and two, centering the CO may not be done uniformly with all bending dipoles introducing dipole kicks which change the circumference.

First, note in Tab. 3.2 that the baseline momentum setting has a slight time delay
and non-zero $\delta$. This indicates that the energy of the PSR was not exactly set to the design energy and will contribute a systematic error to the center method for calculating $\delta$. The magnitude of the systematic error is on the order of the difference between the baseline $\delta$ measurements of both methods ($\sim 8 \times 10^{-6}$) and is much less than the measurement spread. With that said, the measured baseline momentum deviation from the design momentum is is less than the magnitude of the pulse-to-pulse momentum variations calculated in Sec. 2.8, so the energy cannot reliably be closer to the design energy.

Another source of systematic error in the center method for measuring the beam momentum is a centered CO that is not exactly centered. Since the results of the center method agree with the $\alpha_c$ method, this error must be small. The reproducibility of centering the CO to the baseline CO is shown in Fig. 3.7.

The average CO can approximate the difference in the orbit circumference and the design orbit circumference, thus yielding an estimate of the systematic error due to the centered CO not being exactly center. According to Fig. 3.7, all centered COs were
well centered. The average horizontal and vertical centered CO for each momentum setting is presented in Tab. 3.4. The average CO is calculated by averaging the CO in one dimension at every BPM. BPM 20 (SRPM92x) is not included in the horizontal averaging because it is known to give questionable CO data, Chap. 2 and Sec. 3.1.1. One might worry that the average CO is affected by not including BPM 20 or the two missing BPMs, and it does, but the average CO is a first order approximation and the affects of sampling periodic locations along the circumference is higher order.

<table>
<thead>
<tr>
<th>Average Centered CO</th>
<th>Horizontal [mm]</th>
<th>Vertical [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>$-2.4652 \times 10^{-2}$</td>
<td>0.2784</td>
</tr>
<tr>
<td>$+2$ mm ΔCO</td>
<td>$4.3243 \times 10^{-2}$</td>
<td>0.2185</td>
</tr>
<tr>
<td>$+4$ mm ΔCO</td>
<td>$-7.6866 \times 10^{-2}$</td>
<td>0.1679</td>
</tr>
</tbody>
</table>

Table 3.4: The average horizontal and vertical centered CO for each momentum setting. The average horizontal centered CO calculation does not include BPM 20, SRPM92x.

The non-zero component of $\frac{\Delta C}{C_0}$ due to not exactly centering the CO can be estimated by the average centered COs. The horizontal centered CO averages are really a difference in the average orbit radius and the design orbit radius. Thus, the difference in circumference is the average horizontal centered CO multiplied by $2\pi$. From Eq. (3.2), the systematic error on the $\delta$ measurement is then the non-zero $\frac{\Delta C}{C_0}$ multiplied by the design gamma square, $\gamma_0^2$. The estimated systematic error magnitudes on the $\delta$ measurement for each momentum setting due to not exactly centering the center CO in the center method are reported in Tab. 3.5.

Note that in Fig. 3.7, the difference in the centered horizontal COs between the baseline and the $+2$ and $+4$ mm ΔCO momentum settings is oscillatory. This pattern is reminiscent of the CO distortion due to a dipole kick. In the center method, the orbit control (SRMP999) is knob to center the CO for each momentum setting. The orbit control knob controls three different magnet power supplies in order to adjust
3.3 Dispersion Measurement

Estimated Systematic Error Due to Inexact Centered CO

<table>
<thead>
<tr>
<th></th>
<th>ΔC [mm]</th>
<th>ΔC / C₀ [×10⁻⁶]</th>
<th>σδ [×10⁻⁵]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>−.1550</td>
<td>−1.717</td>
<td>−.5847</td>
</tr>
<tr>
<td>+2 mm ΔCO</td>
<td>.2727</td>
<td>3.011</td>
<td>1.026</td>
</tr>
<tr>
<td>+4 mm ΔCO</td>
<td>−.4830</td>
<td>−5.352</td>
<td>−1.823</td>
</tr>
</tbody>
</table>

Table 3.5: The estimated error on the δ measurement due to not exactly centering the CO for each momentum setting in the center method. ΔC is 2π the average horizontal centered CO, C₀ is the design circumference of the PSR, and σδ is the estimated magnitude of the systematic error on the δ measurement using the center method. σδ is calculated by multiplying by the fractional circumference deviation by the design gamma square, ΔC / C₀ γ₀².

the orbit radius: BEMP01 (Mean Joe Green) controls the currents to the common 36° benders including SRBM01 and the power supply for each of the two C-magnets, SRBM11 and 12. The change in the set point of SRMP999 is modified by a calibration factor to control the change in each power supply. The calibration factors are 1.0 for BEMP01, 1.521 for the SRBM11 power supply, and 1.050 for SRBM12[17]. The source of these calibration factors is not documented, but it is supposed that these factors are the ratios of the slopes of ∮Bdl as a function of current for the different magnets.

Since SRMP999 does not control the merging magnet (RIBM09), it was believed that the CO distortion in Fig. 3.7 was due to a dipole kick from RIBM09. However, analysis shows that other dipole errors than that of RIBM09 are needed to create the CO distortion observed in Fig. 3.7. Other possible sources of dipole kicks come from SRBM11 and 12 if the calibration factors applied by SRMP999 to control the two C-magnets power supplies are incorrect and from SRBM01, since it is shunted by about 100 A to bend 32.8° instead of the common 36°, the ∮Bdl is less in SRBM01 than
the other common PSR benders, which is not taken into account in the calibration factors of the orbit control knob.

A dipole field error will not only cause CO distortion, but it will also change the circumference of the CO. The change in the circumference due to a dipole kick was introduced in Eq. (1.65).

The dipole error at RIBM09 will be investigated for an estimate of the change to the circumference due to a dipole field error. The measured $\delta$’s for each momentum setting may be applied in order to calculate the dipole kick angle due to RIBM09 because it is not one of the devices controlled by SRPM999. The difference in the $\oint Bdl$ at RIBM09 for PSR design momentum and the +4 mm $\Delta$CO momentum setting corresponds to a dipole kick of 0.14197 mradian.

Combining the measured dispersion function at SRPM01 and 02 and model transfer matrices, the dispersion function at RIBM09 can be obtained. The dispersion function in the center of RIBM09 was found to be $-2.1029$ m. Thus, the change in the circumference due to SRPM999 no controlling the current in RIBM09 is $-0.29855$ mm. This propagates to a systematic error on the $\delta$ measurement because the dipole field error changes the length of the circumference with magnitude of $1.1302 \times 10^{-5}$. Although it was not done during this experiment, this systematic error could be mitigated by engaging the correct CO CCR program. This would mean that the centered CO could not be undone by the SRMP999 knob and that the magnet currents would have to be individually reset.

As a final source of possible systematic error, one might worry that while undoing the center CO to reestablish the baseline lattice, hysteresis effects in the magnets may not yield the same magnetic field for the same current. The magnet power supplies were changed by a maximum of 2.78 A for the +4 mm $\Delta$CO momentum setting. It was shown in Sec. 3.1.1 that the magnet hysteresis effects does not affect the CO with current changes of $\pm 4$ A, so it is believed that hysteresis does not affect uncentering
of the CO.

IV Fitted parameters

The dispersion function is the slope of the line fit to the change in the CO as a function of $\delta$. Since the $\delta$'s calculated with both the $\alpha_c$ and center methods are almost identical, one would expect that the fitted dispersion function from the momentum measurements of each method would also be similar, Figs. 3.8 and 3.9. The $\alpha_c$ method derived dispersion function plotted in Figs. 3.8 and 3.9 is fit to the unmodified data. Note in Fig. 3.8, the horizontal dispersion function is negative. This is because in the coordinate system of the PSR diagnostics, positive points inward. An increase in momentum will push the beam orbit outward and in the negative direction. Also notice a very small vertical dispersion function. The coupling to produce the non-zero vertical dispersion function is due to small magnet rolls and misalignments. The $\alpha_c$ method has a larger fitting error on the dispersion function than the center method. The difference in the fitting error on the dispersion function is a consequence of the different $\sigma_\delta$'s for each methods. The baseline model prediction agrees with the measured dispersion function. The model prediction for the dispersion function at each BPM is within three fitting errors of either method.

The difference of the fitted dispersion functions is plotted in Fig. 3.9. It is interesting that the resulting dispersion functions are so similar even when one of the time delays in the $\alpha_c$ method was mis-recorded. The largest difference in the dispersion functions is at BPM 20. The inconsistency of the CO measurement at BPM 20 has been well documented in Chap. 2 and Sec. 3.1.1.

The y-intercept ($a$) is also a fitting parameter in the fit to a line. The $a$ fitting parameter is the fitted on-momentum CO, or the CO for the synchronous particle, and is plotted in Fig. 3.10. Figure 3.10 shows a difference in the fitted on-momentum
CO between the unmodified $\alpha_c$ method data and the center method. However, the deviation does not exist when comparing the on-momentum CO from the modified $\alpha_c$ method data and the center method. The effect of including the mis-recorded +2 mm ΔCO TOF delay shifts the fit line (red in Fig. 3.5) more positive in position changing the y-intercept accordingly. This is observed in Fig. 3.10 where all of the on-momentum CO fits in the horizontal with the unmodified $\alpha_c$ method data are more positive. The effect is minimized in the vertical because the dispersion function is so small. The vertical on-momentum COs all agree to within a few fitting errors.

Figure 3.10 also shows that the on-momentum CO was well centered within ±1 mm in the horizontal. The CO was not as well centered in the vertical with almost a +3 mm CO at BPM 39, SRPM91y.

Since the fitting scheme also fits a value for the $\delta$’s ($\eta_i$ in the fitting scheme derivation of Appx A.2), it is of interest to see if the fitted independent variable values differ from the measured $\delta$’s. Figure 3.11 plots the three fitted $\eta_i$’s at each BPM for...
both methods and compares them to the $\delta$’s measured with the center method. First note that fit with the mis-recorded data point shifts all of the $\eta$’s positively. Even so, the resulting $\eta$’s are within three fitting errors of the measured $\delta$’s. Of course correcting the mis-recorded time delay yields the same results for both methods. The $\eta$ fits are less constrained in the vertical because the vertical dispersion function is small.

The $\chi^2$/DOF is the goodness of fit quality factor for the fitting scheme employed to fit for the on-momentum CO and the dispersion function. Since data from three different momentum settings is fit, there are six measured data points: three CO values and three $\delta$’s. The fitting scheme fits five parameters: three $\eta$’s, the on-momentum position, and the dispersion function. So, there is only one degree of freedom for these fits. A comparison of the $\chi^2$’s resulting from each fit for both methods is plotted in Fig. 3.12. As alluded to earlier, correcting the mis-recorded TOF measurement in the $\alpha_c$ method greatly reduces the $\chi^2$/DOF from above ten to below 1 which is comparable to the $\chi^2$/DOF from the center method fits. The $\chi^2$’s
in the vertical are larger. This is because the dispersion function is much smaller in
the vertical, and the fit is less well constrained. The vertical CO measurement errors
are also $\sim 5$ times smaller in the vertical than in the horizontal demanding a closer
fit to the data for the same $\chi^2$/DOF.

As discussed briefly above, the center method yields smaller fitting errors for
the fitted dispersion function and on-momentum CO fitting parameters than the $\alpha_c$
method. However, gaining a smaller random error in the center TOF measurements
has introduced several small systematic errors as discussed in Sec. 3.3.3 III. The
systematic errors are called small because the measured $\delta$’s from both methods agree.
The magnitudes of some of the systematic errors have been estimated in this analysis,
and all are less than the random measurement error of the $\delta$ measurement.

The $\alpha_c$ method yields an average random error on the horizontal dispersion func-
tion of $\pm 4.52\%$, while the center method yields an average random error of $\pm 2.93\%$
on the horizontal dispersion function.
3.3 Dispersion Measurement

![Graph showing fitted fractional momentum deviations](image)

**Figure 3.11**: (Color) The fitted fractional momentum deviations $\eta_i$ with a fitting error from the fits to the unmodified $\alpha_c$ method data (blue circles), the modified $\alpha_c$ method data (red triangles), and the center method data, green squares. The solid black lines are the measured $\delta$'s by the center the CO method, while the dotted lines mark three standard deviations. The vertical line separates the horizontal (left) and vertical (right) BPMs.

Because of the precise control needed to center and uncenter the CO for each momentum setting, the center method is very time consuming and prone to introducing several systematic errors during the experiment. The $\alpha_c$ method is much simpler and faster, and its only systematic error is that of the model $\alpha_c$, which is slowly varying in model parameter space, Fig. 3.6. Thus, it is suggested that the $\alpha_c$ method be employed for future measurements of the beam momentum in the PSR.

V Calibration of LDPM03

Calibration of LDPM03 allows for faster measurements of the fractional momentum deviation because position data at LDPM03 can be acquired quickly. The random error on the measured $\delta$ is smaller with this method because the pulse-to-pulse momentum variations can be averaged out by collecting many position measurements at
3.3 Dispersion Measurement

LDPM03. Both the $\alpha_c$ and the center methods were applied to calibrate the dispersion function and on-momentum position at LDPM03. Fifty position measurements at LDPM03 were made at each momentum setting for both methods. The average position and the error on the average position calculated by Eq. (2.10) were applied in the fit for the dispersion and on-momentum position at LDPM03. Because the PSR was operated in single shot accumulation mode for this measurement, data was collected from the first minipulse from 50 different macropulses. Table 3.6 displays the results of calibrating LDPM03 comparing both the $\alpha_c$ and center methods.

The on-momentum position at LDPM03 is $-9$ mm, which is far from the center of the pipe. This value is controlled with the horizontal steering and bending magnets in the transport of Line D North. Even with including the bad data point, the on-momentum position fit from the $\alpha_c$ method agrees with the center method to within two fitting errors. The position at LDPM03 for the $+4$ ∆CO momentum setting was about 1 mm, so 1 cm of the 3" pipe at LDPM03 is used during this experiment.

**Figure 3.12:** (Color) The $\chi^2$/DOF from the fits to the unmodified $\alpha_c$ method data (blue circles), the modified $\alpha_c$ method data (red triangles), and the center method data, green squares. The vertical line separates the horizontal BPMs (lft) and the vertical BPMs, right.
3.3 Dispersion Measurement

<table>
<thead>
<tr>
<th>Method</th>
<th>$\alpha_c$</th>
<th>$\alpha_c^*$</th>
<th>Center</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$ [mm]</td>
<td>-8.4339</td>
<td>-9.0163</td>
<td>-8.9471</td>
</tr>
<tr>
<td>$\sigma_a$ [mm]</td>
<td>.2778</td>
<td>.2975</td>
<td>.1848</td>
</tr>
<tr>
<td>$D$ [m]</td>
<td>-4.6628</td>
<td>-4.6775</td>
<td>-4.5554</td>
</tr>
<tr>
<td>$\sigma_D$ [m]</td>
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<td>.2120</td>
<td>.1324</td>
</tr>
<tr>
<td>$\chi^2$/DOF</td>
<td>6.1188</td>
<td>2.9950</td>
<td>2.7662</td>
</tr>
</tbody>
</table>

Table 3.6: The results from calibrating the dispersion function and on-momentum position at LDPM03 with the $\alpha_c$ and center methods. $a$ and $\sigma_a$ are the on-momentum position and its fitting error. $D$ and $\sigma_D$ are the dispersion function and its fitting error. Lastly, $\chi^2$/DOF is the goodness of fit quality factor. The -*-ed method uses a corrected data point.

Both methods yield agreeing fitted dispersion functions at LDPM03. The fitted dispersion at LDPM03 is $-4.6$ m. This value agrees with the dispersion function at LDPM03 from a TRANSPORT model of Line D. The percentage errors on the fitted dispersion function at LDPM03 are similar to those in the ring, 4.52% for the $\alpha_c$ method and 2.91% for the center method. The $\chi^2$'s for the fits at LDPM03 are larger than for the fit for BPMs in the PSR. This could be due to second order dispersion effects at LDPM03 or systematic errors due to beam steering by changing the rf phase of Mod 47 and 48, but is most likely because the position measurement spread at LDPM03 is smaller than at the PSR BPMs.

Once calibrated, LDPM03 may be employed to measure the beam momentum. Any random errors due to the fitting of the calibration parameters $a_{LD}$ and $D_{LD}$ then become systematic errors in the following calculations where LDPM03 measures $\delta$.

3.3.4 Dispersion measurement summary

I compared the $\alpha_c$ and center the CO method for measurements of the beam momentum and found that the $\alpha_c$ method is quicker, possesses less opportunity for
systematic error, and is just as accurate as the center method. I engaged in an intensive systematic error analysis for both methods. The \( \alpha_c \) method possesses only one source of systematic error in the use of the model momentum compaction factor. I found that this systematic error was small because the model \( \alpha_c \) did not vary greatly with model parameters. The center method however had many sources of systematic error. Each step of the complicated procedure in the center the CO method could possibly introduce systematic error. All of the systematic errors in the center method were found to be small for the December 22, 2009 dispersion measurement, but could be very large if this method was sloppily executed. I also introduced and tested a hybrid method of measuring the beam momentum via the beam position at LDPM03.

I reported the results of the dispersion function measurement. I employed the fitting scheme derived in Appx. A.2 to take into account the measurement spreads in both the CO and fractional momentum derivation. The other fitted parameters including the on-momentum CO and the fitted fractional momentum deviations were also discussed. The fitted fractional momentum deviations agreed well with the measured \( \delta \)'s. Since it was shown that the baseline model predicted the dispersion function fairly well, it is necessary for any improved model to predict the dispersion function just as well.

3.4 Beta Function Measurement

The betatron amplitude function measurement is a very important measurement. It will be applied as the second test to any possibly improved model.

In this section, I first introduce the quadrupole perturbation method, Sec. 3.4.1. The results from both the betatron tune measurements and the beta function measurements are discussed in Sec. 3.4.3. A stringent error analysis is applied to the measurement of the beta functions to ascertain the measurement error.
3.4 Beta Function Measurement

3.4.1 Quadrupole perturbation method

The quadrupole perturbation method measures the betatron amplitude function by applying the results of the derivation for the betatron tune shift due to a quadrupole field error derived in Sec. 1.1.7 I, Eq. (1.49). In the quadrupole perturbation method, a known quadrupole error is introduced into the ring’s focusing lattice and the resulting tune is measured. The quadrupole error is generally introduced by changing the current for a single quadrupole so that the strength of the quadrupole error is equal to the change in the quadrupole strength, \( \oint kds = \Delta (KL) \), where \( L \) is the length of the perturbed quadrupole. Solving Eq. (1.49) for the beta function at the location of the quadrupole field error, one can measure the average betatron amplitude function at the perturbed quadrupole,

\[
\langle \beta \rangle = \frac{4\pi \Delta \nu}{\Delta (KL)},
\]

(3.9)

where \( \Delta \nu \) is the change in the betatron tune.

According to Eqs. (1.49) and (3.9) a stronger quadrupole perturbation will yield a larger \( \Delta \nu \), a better measurement of the slope of \( \nu(KL) \), and thus \( \langle \beta \rangle \). However since the small angle approximation was evoked in the derivation of Eq. (1.49), one needs to be careful as to not introduce such a large quadrupole field error that the change in the one revolution betatron phase advance departs from the \( \Delta \Phi \ll 1 \) assumption. If \( \Delta \Phi \) is not small because the quadrupole field error is no longer a perturbation, additional trigonometric terms should appear in Eqs. (1.49) and (3.9) and will be observed in the nonlinearity in the tune as a function of the normalized gradient length. Thus, the linearity of \( \nu \) as a function of \( KL \) is scrutinized in Sec. 3.4.3 II to provide confidence in the results of the measured betatron amplitude functions via the quadrupole perturbation method.
3.4.2 Measurement Setup and Data Analysis

During the June 27, 2008 accelerator development, measurements of the betatron amplitude functions were performed with the quadrupole perturbation method derived in Sec. 3.4.1. For this beta function measurement, the PSR was set up for single shot accumulation and near-on-axis injection as described in Sec. 2.2.

At the beginning of the day, the beam was injected near-on-axis ([-0.72 mm, 0.31 mrad] in the horizontal and [1.99 mm, 0.3 mrad] in the vertical) to avoid scraping and BPM measurement saturation. However, the beta function measurement took place during the last four hours of a 12 hour shift and the injection offset had slipped to (∼[-0.65 mm, 0.57 mrad] in the horizontal and ∼[1.5 mm, −0.03 mrad] in the vertical). This slippage in the injection offset will lead to more spread in the vertical beam position data as discussed later in this subsection.

To measure the betatron amplitude function with the quadrupole perturbation method, a quadrupole field perturbation must be introduced into the focusing lattice and the resulting betatron tune is measured. The quadrupole magnet power supplies (BEMP02 and BEMP03) cannot introduce the quadrupole error because all of the horizontal focusing quadrupoles are powered by BEMP02 and all of the horizontal defocusing quadrupoles are powered by BEMP03. So, a change in a quadrupole power supply current will introduce a quadrupole error in all quadrupoles of that family, focusing or defocusing.

For this experiment, the quadrupole perturbation was introduced to a single quadrupole by a portable shunt. The amount of current that the portable shunt can draw from the selected quadrupole depends on the voltage across the terminals of the quadrupole (which in turn depends on the current supplied to the quadrupole) and the resistance of the cables from the terminals to the shunt, which among other things depends on the cable length. Due to these affects, each quadrupole had a
different current maximum that could be shunted. The portable shunt was able to
draw a maximum of \( \sim 25 \) A of current from the focusing quadrupoles and \( \sim 20 \) A from
the defocusing quadrupoles.

Each quadrupole was shunted in turn and the betatron tune was measured via
beam position data collected at the 18 BPMs around the PSR by the RingScan BPM
program. Four different shunt values were applied to each quadrupole to increase the
accuracy of the \( \nu(KL) \) slope measurement. The quadrupoles were shunted with 0 A
for a baseline measurement, maximum (25 A and 20 A for the focusing and defocusing
quadrupoles respectively), and two values in between, generally 17 and 9 A in the
focusing quadrupoles and 12 and 6 A for the defocusing quadrupoles.

Twenty RingScans were taken for each of the four shunt values at every quadrupole.
The RingScan data consisted of 40 points of turn-by-turn data at each BPM in the
PSR. The turn-by-turn BPM data is fit to a cosine wave and the tune is extracted.
Since all horizontal BPMs measure the horizontal betatron tune and likewise in the
vertical, the measured tune distributions consist of all of the fitted tunes from scans
at BPMs in one dimension. So, there are actually 360 (20 scans \( \times \) 18 BPMs) mea-
surements of the tune for each quadrupole shunt value.

The RingScan data is fit to a cosine wave and the data acquisition errors are
identified and removed from the dataset as described in Chap. 2. The average and
the error on the average (\( \sigma_{\text{ave}} \)) of the betatron tune distributions extracted from the
cosine wave fits are needed to obtain the slope of \( \nu(KL) \). The error on the average of
the tune distribution is calculated by Eq. (2.10).

Fourth order polynomial fits of the current versus gradient length excitation curves
from the second quadrupole mapping session in 1987 are applied to convert the
shunted quadrupole current to a normalized gradient length, \( KL[10, 11] \). The fourth
order fits yield reported \textit{rms} deviations between the fit and the data, defined as
\begin{equation}
\sqrt{\frac{\sum (x_{\text{meas}} - x_{\text{fit}})^2}{N - 1}},
\end{equation}
on the order of \( \times 10^{-4} \). However the precision of the fits is limited mostly by the reproducibility of the measurements, which was found to be .1\%. Reference [10] also notes that the absolute accuracy on the current to gradient length measurements is a few \( 10^{-3} \text{ m}^{-1} \). The uncertainty in the fourth order polynomial fit is a systematic error with random values. Since the same conversion is applied for each shunt value at a quadrupole throughout the entire measurement, the random errors of the current to gradient length measurement become systematic errors in the betatron amplitude function measurement. These systematic errors are addressed in Sec. 3.4.3 II.

Each quadrupole has a slightly different excitation curve, which is taken into account in the model. The model reads the quadrupole magnet power supply current, subtracts the shunt current, and applies the fourth order fit to obtain the \( KL \) for the shunt value.

The four shunt values applied to a quadrupole yield four data points in \((KL, \nu)\) space. The \( \frac{\Delta \nu}{\Delta KL} \) of Eq. (3.9) may be found by fitting a line to the tune and normalized gradient length data. The tune parameter has random uncertainties, which is taken into account in the fit for the betatron amplitude function. The systematic error in the normalized gradient length calculation is not taken into account for the beta function fitting but will play a major role in the systematic error analysis of the betatron amplitude function measurement. Linear regression with dependent variable uncertainties, which is derived in Appx. A.1.2, may be employed to fit for the slope of \( \nu(\Delta KL) \).

The fitted slope \( (b) \) is multiplied by \( 4\pi \) to calculate the average betatron amplitude function in the shunted quadrupole, Eq. (3.9). Since the slope is solved explicitly in Eq. (A.9) by linear regression, the error on the fitted slope is calculated by propagating
the error on the average tune distribution as in Eqs. (A.15) and (A.17). The fitting error for $b$ is propagated to the beta function as normal by

$$
s^2_{\langle \beta \rangle} = (4\pi s_b)^2,
$$

where $s_b$ is the fitting error on the slope and $s_{\langle \beta \rangle}$ is the calculated error on the average betatron amplitude function in the shunted quadrupole.

Recall that the vertical injection offset had slipped from [1.99 mm, .3 mradians] at the beginning of the day to $\sim$[1.5 mm, −0.03 mradian] for the beta function measurement 8 hours later at the end of the development shift. A mis-reset in the current by $−2.6$ A of one of the vertical corrector magnets (SRVM81) during the prior ORM measurement is believed to be the cause of the change in the vertical injection offset.

Notice the very small angle of the injection offset in the vertical. This indicates that the beam was injected at the crest of the betatron oscillation. Unfortunately, the crest is where the derivative of the betatron oscillation with respect to the longitudinal changes most rapidly. This means that any small variation in the beam injection (mostly due to the pulse-to-pulse momentum variations) will greatly change the angle of the injection offset, which translates into larger than normal variations in the action. According to Eq. (1.40), the variability of the action will be observed as a large measurement spread in the fitted amplitude from the cosine wave fit to the RingScan data.

Interestingly, injecting the beam at the crest of the betatron oscillation improves the measurement spread in the fitted phase parameter. This is because the first derivation of the betatron oscillation with respect to the longitudinal is zero at the crest so the phase changes very little as a function of injection angle.

As a means to identify partial and total stuck MUX errors, the RingScan analysis scripts groom each fitted parameter distribution and removes all outliers in each dis-
tribute as described in Sec. 2.4. Because of the variability in the vertical amplitude due to the small angle of the vertical injection offset, many scans are removed from the dataset as fitted amplitude outliers in the vertical. This is because the \(\text{rms}\) standard deviation of the ungroomed vertical amplitude distributions exceeds the limit for the maximum measurement spread imposed on the amplitude distribution by the analysis scripts. The maximum measurement spread limit for the fitted amplitude distribution is calculated as the average of the \(\text{rms}\) spread in position at each turn for all scans at the BPM. To achieve this maximum measurement spread in the fitted amplitude distribution, the analysis scripts removed about a quarter (3-5) of the scans from each vertical BPM as amplitude outliers. This leaves more than 15 scans at each BPM and is enough for statistics even for distributions of fitted parameters at individual BPMs like the fitted amplitude, phase, offset, and \(\sigma_{\text{BPM}}\). Even with 3-5 scans removed at each BPM, the measured tune distribution contains \(~270\) (15 scans \(\times\) 18 BPMs) measurements of the tune and a precise betatron tune measurement can still be obtained.

In this analysis, the quadrupoles are numbered consecutively 1 through 20 as they appear in the PSR: SRQF01, SRQU01, SRQF11, \ldots, SRQU91. Thus all horizontal focusing quadrupoles are odd numbered and all even number quadrupoles are horizontal defocusing quadrupoles.

### 3.4.3 Beta function measurement results

This subsection is divided into two subsubsections. The first describes the quality and results of the betatron tune measurement. This measurement is very important because it is the measured quantity needed to obtain the betatron amplitude functions. The second subsubsection details the beta function calculation quality and discusses the results of a thorough error analysis of the betatron amplitude measurement.
I Tune measurement

The measurement of the betatron amplitude function with the quadrupole perturbation method relies upon a quality betatron tune measurement. It was shown in Sec. 2.6.2 that the RingScan beam position data could yield precise measurements of the tune that agreed with the results obtained by the central control room (CCR) BPM program. The RingScan analysis can produce a more precise tune measurement because the turn-by-turn BPM data is fit to a cosine wave and the tune is extracted from the fit, where as the CCR BPM program supplies many guesses for the tune and then chooses the tune value that gives the smallest sum of squares per degree of freedom (SSR/DOF) from a linear fit to the BPM data.

The average of the measured tune distribution and the error on the average are required to fit for the average beta function at the shunted quadrupole. Figure 3.13 is a plot of the error on the average tune measurement for each shunt setting at each quadrupole. The error on the average tune measurement is calculated by Eq. (2.10) and is very small, on the order of a few $10^{-5}$. The values shown in Fig. 3.13 agree with the rms measurement spreads observed in the tune distributions reported in Sec. 2.6.2. As noted in Sec. 2.6.2 and shown in Fig. 3.13, the vertical tune distributions have a smaller error on the average tune value and thus a smaller rms measurement spread in comparison to the horizontal tune distributions. The greater spread in the horizontal tune measurement is believed to be due to larger chromatic affects from the pulse-to-pulse momentum variations in the horizontal since the horizontal chromaticity is more negative than the vertical chromaticity.

One final observation of Fig. 3.13 is that the relative values of the error on the average tune for different shunt settings are x-y symmetric. For example, the second shunt setting alternates between large and small values of the error on the average in both the horizontal and the vertical. This behavior is due to the constraint that the
number of scans remaining in both horizontal and vertical measured tune distributions after scans with data acquisition errors have been removed is the same. The $\sqrt{N-1}$ in Eq. (2.10) is the same for both $x$ and $y$ tune measurements. Thus, it is more likely that the shunt settings with larger errors on the average tune actually have less scans in the tune measurement distribution rather than larger $rms$ measurement spreads in these distributions.

A baseline shunt setting of 0 A was applied to each quadrupole to avoid systematic errors due to the tune drifting over the course of the four hour experiment. Figure 3.14 plots the baseline tune measurement at each quadrupole. Quadrupole 10 (SRQU41) was the first quadrupole shunted in the quadrupole perturbation method. The next shunted quadrupole was chosen by convince of the layout of the portable shunt patch board in the Ring Equipment Building (REB). While the next numbered quadrupole was not always the next quadrupole shunted, the general trend in the measurement was in the positive quadrupole number direction, looping around the ring and finishing
with quadrupole 9, SRQF41.

![Baseline Measured Tune, Beta 06/27/2008](image)

**Figure 3.14:** (Color) The horizontal (blue circles) and vertical (green squares) average tune measurement for the baseline shunt value of 0 A at each quadrupole.

It is interesting that there is a systematic drift in the average measured tune at the baseline shunt setting as the beta function measurement progressed from quadrupole 10 to 20 and continued from quadrupole 1 to 9. The baseline measurement was taken with the shunt set to 0 A, so connecting the shunt should not have changed the focusing lattice of the machine. The horizontal baseline average tune drifted \( \sim 0.004 \) over the course of the beta function measurement, while the vertical baseline average tune drifted only \( \sim 0.002 \).

There are two likely parameters responsible for the shift in the baseline shunt tune values: a drift in the momentum output of the linac and effects of quadrupole hysteresis. The possible change in linac energy will be investigated first.

A momentum deviation will affect the focusing properties of the lattice through what is called chromatic aberration. Chromatic aberration is introduce in Sec. 1.1.8. In Sec. 4.2, the horizontal and vertical chromaticities were measured to be \([-4.25, -2.50]\).
So if the drift in the baseline tune was due to a momentum change via chromaticity, a change in tune of −.004 in the horizontal would indicate a fractional change in the momentum of \(-9.41 \times 10^{-4}\). A change in beam momentum would also be observable as a change in the horizontal CO via dispersion. The change in the horizontal CO due to a change in the momentum is \(\Delta x_{CO} = D\delta\), where \(D\) is the dispersion function. The average measured horizontal dispersion function in the PSR is \(-2\) m, Sec. 3.3.3. This would indicate that if the baseline tune drift was caused by a change in beam momentum, the horizontal CO for the baseline shunt value should also drift by an average of -1.88 mm. The CO for the baseline shunt setting at each quadrupole is plotted in Fig. 3.15.

![Figure 3.15: (Color) The CO for the baseline shunt settings at each quadrupole. Each color represents a baseline CO measurement at a different quadrupole. The vertical line separates the horizontal CO (left) and the vertical CO, right.](image)

Figure 3.15 does not show the 1.88 mm average CO shift in the horizontal that is predicted if the baseline tune drift were due to a change in the output energy of the linac. Figure 3.15 does show a very consistent reproduction of the baseline CO which lends more confidence to the RingScan CO measurement. BPMs 1, 7, 13, 14, and 19 are shown to have a bit larger spread in their CO measurements for the baseline shunt value case than the other BPMs, but the spread at these BPMs is still less than
.3 mm. BPM 20 (SRPM92x) is known to yield inconsistent CO measurements, Chap. 2 and Sec. 3.1.1, but Fig. 3.15 indicates that BPM 20 measures a fairly reproducible baseline CO measurement with just 2 outliers in the 20 RingScans. Also shown in Fig. 3.15 is a vertical CO which is as large as 8 mm in BPM 38. This large vertical CO is due to a mis-reset SRVM81, which changed the vertical injection offset from the beginning of the day.

So, a systematic drift in the beam momentum is not believed to be responsible for the drift in the tune through chromatic effects because the baseline shunt setting CO is the same across the entire betatron measurement.

The drift in the baseline tune measurement must then be due to hysteresis in the shunted quadrupoles. While the baseline shunt tune measurement does not change noticeably between two consecutive quadrupole perturbation measurements, the cumulative effects of hysteresis in many quadrupoles can cause the systematic drift in the measured tune observed in Fig. 3.14, Sec. 3.2.

It was shown during a quadrupole hysteresis experiment on October 16, 2008 that the difference in the quadrupole pole tip field at the operational current set point could vary \( \sim 0.5\% \) for the defocusing quadrupoles and \( \sim 0.3\% \) for the focusing quadrupoles depending if the quadrupole was operated on the top or the bottom of the hysteresis curve.

Model simulations indicate that a change of \(-0.11\%\) and \(-0.125\%\) in the focusing and defocusing quadrupole strengths respectively would cause the tune shift that occurred during the course of the quadrupole perturbation method measurement. Since the maximum shunt value was only 25 or 20 A, it is expected that the change in the pole tip magnet field of the quadrupole should be smaller than the difference between the top and bottom hysteresis curves. These facts lead to the conclusion that the drift in the baseline shunt setting tune value was caused by hysteresis in the quadrupoles.
The hysteresis of the quadrupoles will also affect the betatron amplitude functions via Eq. (1.54). The model suggests that for the above changes in the quadrupole strengths (−.11% for focusing and −.125% for defocusing quadrupole) the horizontal beta function changes by less than .05 m in the horizontally focusing quadrupoles and the vertical beta function changes by less than .02 m in the horizontally defocusing quadrupoles. It is believed that these changes are small compared to other systematic errors, but will be set as a lower limit if necessary. Nonetheless, it is believed that this drift does not compromise the betatron tune measurement because a baseline shunt setting of 0 A was taken at each quadrupole.

II Beta function measurement

The average betatron amplitude functions at a perturbed quadrupole are measured as the slope of the betatron tune versus normalized gradient length. The tune measurement is described in the preceding subsection, Sec. 3.4.3 I. The shunt values were recorded by hand during the experiment and the current to gradient length conversions in the model are consulted to obtain the normalized gradient length at the shunted quadrupole. Four shunt values are applied at each quadrupole, and the data is fit to a line. The analytic solution of the fitting scheme Eq. (A.9) for the slope and the fitting error on the slope is shown in Eqs. (A.15) and (A.17) respectively.

The results from the entire betatron amplitude function measurement are plotted in Fig. 3.16. As expected the horizontal beta function is maximum in the horizontally focusing quadrupoles (the odd numbered quadrupoles) and minimum in the horizontally defocusing quadrupoles, the even numbered quadrupoles. The vertical betatron amplitude function is opposite of the horizontal beta function (where the horizontal beta function is maximum, the vertical is minimum) because a horizontally focusing quadrupole is a vertically defocusing quadrupole and because the focusing lattice of
the PSR is what is called a FODO cell: focusing, drift, defocusing, drift.

**Figure 3.16:** (Color) The measured average horizontal (blue circles and solid lines) and vertical (green squares and dashed lines) betatron amplitude functions at each quadrupole in the PSR with one systematic error due to the uncertainty of the current to $KL$ conversion, and the predicted horizontal (red left pointing triangles) and vertical (black right pointing triangles) average baseline model beta function for each quadrupole.

The error bars shown on the measured betatron amplitude functions in Fig. 3.16 are estimates of the systematic errors due to the uncertainty of the fourth order current to gradient length fits. A constant systematic error in the current to $KL$ conversion does not affect the fitted slope, but a constant multiplicative error will change the fitted slope of the tune as a function of $KL$. However, the systematic errors are estimated under the worst case scenario when the baseline and maximum shunt values are either .1% and $-$.1% or $-$.1% and .1% respectively. The average systematic error on the calculation for the maximum and minimum beta functions in the horizontal is .49 and .05 m respectively, and the average systematic error on the maximum and minimum beta functions in the vertical is .40 and .15 m respectively. The systematic errors due to the uncertainty in the gradient length are much larger than the random
calculation uncertainties and dominate the measurement of the betatron amplitude function. The calculation errors on the betatron amplitude function will be discussed later in this subsubsection.

Observe the large beta beating in the vertical especially between the second and fourth quadrupoles (SRQU01 and SRQU11) in Fig. 3.16. The beta beating is 20%, 12.7 m at SRQU11 / 16 m at SRQU01. The beta function beating is largest around the first bend after the foil in the PSR. It is at this location where the 10-fold FODO cell symmetry of the PSR focusing lattice is broken. One of the common 36° horizontal benders was replaced by two C-magnets during the LRIP upgrade to direct H- injection in 1998. The beta beating will cause beam envelope oscillations and can lead to space charge emittance growth as discussed in Ref. [26]. The reduction of this beta beating is one of the applications for an improved model.

Also shown in Fig. 3.16 are the baseline model predictions for the horizontal and vertical beta functions averaged across the length of the quadrupole. The average model beta functions in a quadrupole are calculated as follows. As expected, the model beta functions evolve through a quadrupole as Eq. (1.28) with transfer matrix described in Eq. (1.20). Combining Eqs. (1.28) and (1.20) yields,

$$\beta(s) = \beta_0 \cos^2(\sqrt{K}s) - \frac{2\alpha_0}{\sqrt{K}} \cos(\sqrt{K}s) \sin(\sqrt{K}s) + \frac{1 + \alpha_0^2}{\beta_0 K \sin(\sqrt{K}s)}$$

where the index 0 indicates values at the beginning of the quadrupole.

The average model beta function is calculated by integrating Eq. (3.12) along the length of the quadrupole (L) and then dividing by the length of the quadrupole such that

$$\langle \beta \rangle_{\text{Quad}} = \beta_0 \left( \frac{2 + \sin(2\sqrt{KL})}{4} \right) - \frac{\alpha_0}{KL} \sin^2(\sqrt{KL}) + \frac{1 + \alpha_0^2}{\beta_0 K} \left( \frac{2 - \sin(2\sqrt{KL})}{4} \right)$$

Although the baseline model is not able to predict the measured vertical betatron tune, the baseline model predicts beta functions that agree with the beta functions
measured with the quadrupole perturbation method. This is a great success for the baseline model and might suggest that modifying the quadrupoles in the model may not be the correct way to obtain the measured vertical tune.

It is also of interest to know the relative uncertainty of the beta function measurement. The dominating systematic errors from the uncertainty of the gradient length conversion are investigated in this analysis. Figure 3.17 plots the relative systematic error on the beta function measurements. Because the uncertainty of the current to gradient length conversion is quoted as a percentage, the size of the systematic error for a particular shunt setting depends directly on the gradient length of the quadrupole. This systematic error affects the beta function measurement by changing the domain over which the slope of the tune with respect to $KL$ is calculated. This modification to the length of the domain is greater for larger quadrupole strength.

**Figure 3.17:** (Color) The relative systematic error for the horizontal (blue circles) and vertical (green squares) measured betatron amplitude functions. The systematic error due to the current to $KL$ conversion is estimated in a worst case scenario when the baseline and maximum shunt values are either .1% and −.1% or −.1% and .1% respectively.

Thus, both the horizontal and vertical beta functions in the horizontal focusing quadrupoles (odd numbered quadrupoles) show the largest relative error in Fig. 3.17.
at \( \sim 3.8\% \). Note that quadrupoles 17 and 19 have a slightly different relative error compared to the other focusing quadrupoles. These quadrupoles are SRQF81 and 91 respectively. They have larger bores in their field clamps to accommodate a larger beam pipe for beam extraction from the PSR. The larger holes in the field clamps yield a stronger gradient length when these quadrupoles are operated at the same current as the other focusing quadrupoles because more magnetic field leaks out along the longitudinal increasing the effective length of the quadrupole. To mitigate the increase of gradient length in these quadrupoles, a dedicated shunt, which typically runs 17 A, is applied to each of SRQF81 and 91. The quadrupoles with the large bore field clamps possess steeper current to gradient length conversions compared to the small bore quadrupoles such that the same shunt current will have a larger effect on the gradient length of the large bore quadrupoles. So, the \( \pm 1\% \) systematic error on the current to gradient length conversion has less of a relative affect on the large bore quadrupoles yielding smaller relative systematic errors on the beta function measurement at these quadrupoles.

The defocusing quadrupoles (even number quadrupoles) possess a smaller relative systematic error on the betatron amplitude measurement with a lot of variation due to the uncertainty in the current to gradient length conversion. This is because the portable shunt was able to pull a slightly different amount of current from each defocusing quadrupole, whereas all focusing quadrupoles maxed the shunt at the same current. The shunt was able to pull the most current from quadrupole 12 (SRQU51), 22 A, which has the smallest relative systematic error, and the shunt pulled the least amount of current from quadrupole 2 (SRQU01), 17.6 A. Since all of the PSR quadrupoles are similar in construction, it is believed that the difference in the maximum shunt setting in the defocusing quadrupoles is due to variation of the cable lengths from the quadrupole terminals to the portable shunt patch board. The difference in the current pulled from the quadrupole directly relates to a difference
in the length of the domain over which the tune slope is calculated. The difference in maximum shunt current directly correlates with different domain sizes over which the slope of $\nu(KL)$ is calculated such that the $\pm 1\%$ systematic error has different relative effects on the domain at each defocusing quadrupole.

The random calculated uncertainty on each betatron amplitude function measurement is calculated with Eq. (3.11). The calculation errors on the betatron amplitude function are very small. The average maximum and minimum horizontal beta functions have propagated fitting errors of $4 \times 10^{-3}$ and $9 \times 10^{-3}$ m respectively. The average calculation errors for the maximum and minimum vertical beta functions are $3 \times 10^{-3}$ and $2 \times 10^{-3}$ m respectively. The random calculation error on the betatron amplitude functions are small because the error on the average tune is so small, on the order of $10^{-5}$. So, in the quadrupole perturbation method, the systematic errors associated with the uncertainties in the current to gradient length fits dominate the uncertainty in the beta function measurement.

The relative random uncertainty on the measured beta function, where the horizontal and vertical beta functions are maximum, is most important for considerations of whether Eq. (3.9) may be applied to calculation the betatron amplitude function. The relative fitting error on the beta function is $\sim 0.03\%$ in the horizontal and $\sim 0.02\%$ in the vertical. The relative calculation errors are very small and one should worry about how such a precise result was obtained. Remember that a small angle approximation was applied in the derivation of Eqs. (1.49) and (3.9). The tune shift is $\sim 4$ times larger for the maximum beta functions compared to the minimum. This could possibly introduce nonlinearities into the quadrupole perturbation method because $\Delta \Phi$ may no longer be small.

In order to verify that higher order effects or nonlinearities from $\Delta \Phi \ll 1$ do not affect the betatron amplitude function measurement, the individual fits for the beta function at each quadrupole should be scrutinized. Figure 3.18 plots the fit of the
horizontal beta function at quadrupole 1, SRQF01. The fit shown in Fig. 3.18 yields a large $\chi^2$/DOF of 13.8. However the fit in Fig. 3.18 looks very good. The reason that the $\chi^2$/DOF for this fit is so large is because the measurement error on the average tune is so small. The $\chi^2$/DOF is also large because the model in the fitting scheme of Eq. (A.9) does not include affects from systematic errors such as the uncertainty of the current to gradient length conversion. The effects of these systematic errors were discussed above.

Another way to check the quality of the linear fit to the tune and $KL$ is the coefficient of determination, $R^2$. The $R^2$ for the fit shown in Fig. 3.18 is .9999 indicating that the probability of another data point lying on the fitted line is 99.99%. An $R^2$ very close to 1 also proves that the fit is good and that the large $\chi^2$/DOF is due to a fitting model that does not include the systematic error from the current to gradient length conversion.

SRQF01 is a horizontally focusing quadrupole, so the horizontal beta function is a maximum at this location. According to Eq. (1.49), a quadrupole perturbation

![Graph](image-url)
3.4 Beta Function Measurement

introduced where the betatron amplitude function is larger will produce a larger shift in the tune. The larger tune shift for the same $\Delta K L$ allows for a better fit of the slope. The larger beta function at SRQF01 will also increase the change in the one turn betatron phase advance for the perturbation at SRQF01 via Eq. (1.48). However, the nonlinearities are not observed in Fig. 3.18 because the coefficient of determination ($R^2$) for the fit is .9999.

Figure 3.19 plots the $\chi^2$/DOF and $R^2$ resulting from the beta function fit at each quadrupole. The horizontal beta function at quadrupole 10 (SRQU41) fits with the smallest $\chi^2$/DOF with .86. There does not appear to be a distinction in the $\chi^2$/DOF between focusing and defocusing quadrupoles in the horizontal like there is in the vertical. The larger $\chi^2$/DOF in vertical are for fits of the maximum vertical beta functions. It is expected that the $\chi^2$/DOF is larger in the vertical because the error on the average of the vertical tune distribution is smaller, Fig. 3.13.

Figure 3.19: (Color) The horizontal (blue circles) and vertical (green squares) $\chi^2$/DOF and the horizontal (red left pointing triangles) and vertical (black right pointing triangles) $R^2$ resulting from the betatron amplitude function fit at each quadrupole. The $\chi^2$/DOF values are read on the left vertical axis, while the $R^2$ values are listed on the right vertical axis.

The coefficient of determination for all of the vertical fits is larger than .9985, so
all of the vertical fits are good. The fits for the minimum betatron amplitude function in the horizontal yield the smallest $R^2$. This is because the fit is less constrained since the tune shift is smallest at these quadrupoles. But notice that even the worst $R^2$, which is the horizontal beta function fit at quadrupole 14 (SRQU61), is still .9785, which means that there is a 97.85% probability that another data point would lie on the fitted line. The $R^2$ indicates that all of the quadrupole perturbation method fits are good and that the small error bars on the average tune are not enough to account for the systematic errors in the measurement.

Although slightly improper statistically, the .1% systematic error on the current to gradient length conversion was included in a linear fit for the beta functions as a check of the fit, and it was found that the resulting $\chi^2$/DOF from these fits were 1 or less. The smallest $R^2$ was raised to .993. It was also observed for this alteration of the model of the fit for the beta function that the $R^2$ negatively correlated to the $\chi^2$/DOF as expected. This is of course because both the $\chi^2$ and the coefficient of determination depend on the residuals of the fit

$$\chi^2 = \sum_{i}^{N} \left( \frac{\text{Residuals}_i}{\sigma_i} \right)^2,$$

$$R^2 = 1 - \frac{\sum_{i}^{N} (\text{Residuals}_i)^2}{(N - 1)\sigma_{\text{Data}}^2},$$

where $N$ is the number of data points, in this case $\sigma_i$ is the systematic error due to the uncertainty in the current to gradient length conversation, and $\sigma_{\text{Data}}$ is the standard deviation of all of the measured data. A larger $\chi^2$/DOF indicates a larger residual and thus a smaller $R^2$ via Eq. 3.15. This means that when the fitting model was modified to include the systematic errors due to the uncertainty in the current to gradient length conversation, the $\chi^2$/DOF from the modified fit reproduced the results indicated by the $R^2$ of the modified fit.

The fact that the fits for the beta functions yield an $R^2$ very close to one indi-
cates that the quadrupole perturbation method did not induce higher order affects or nonlinearities by greatly changing the one-turn betatron phase advance so much that the small angle approximation did not apply in the derivation of Eq. (1.49). But to be sure that the betatron amplitude function measurement was not compromised by nonlinearities a few more tests are applied to the analysis.

As a second check to the linearity of the change in the betatron tune versus normalized gradient length, each beta function fit was also fit to a quadratic,

$$\nu = a + bKL + c(KL)^2.$$  \hfill (3.16)

Fitting with the quadratic term changes the instantaneous slope at each data point. The instantaneous slope from the quadratic fit is symmetric about the mid-domain of the normalized gradient length values in the data set. It was observed that the instantaneous slope of the quadratic fit changed across the domain by less than .05 m for the horizontal beta function and less than .12 m for the vertical beta function compared to the linear fit. The average of the instantaneous slopes at each data point in the quadratic fit differs from the linear slope by .002 m or .36% of the linear fit. Fitting the tune versus $KL$ with a quadratic does not significantly change the slope across the domain of interest, suggesting once again that the approximations made in the derivations of Sec. 3.4.1 are verified.

As a final check of the higher order effects and nonlinearities in the beta function measurement, the change in the single turn betatron phase advance due the quadrupole field error is calculated. The greatest change in the one-turn phase advance occurs when the quadrupole field perturbation is located where the beta function is the largest, so the vertical betatron amplitude at quadrupole 2 (SRQU01) is applied to Eq. (1.48) to obtain the largest change in the one-turn betatron phase advance during the quadrupole perturbation measurement. The change in the normalized gradient length at SRQU01 for the largest shunt value is $1.43 \times 10^{-2}$ m$^{-1}$.
Including the average beta function at SRQU01 as measured via the quadrupole perturbation method (15.96 m) and plugging the numbers into Eq. (1.48), the largest change in the one-turn phase advance during the quadrupole perturbation method was \(-.1141\) radians. Thus the \(\cos(-.1141)\) differs from 1 by \(6.5069 \times 10^{-3}\) or \(.65\%\).

Interestingly, \(.65\%\) is twice the change in the slope observed in the quadratic fit to the quadrupole perturbation data, \(.36\%\). This is because the average instantaneous slope of the quadratic fit at each data point is approximately the slope at the mid-domain of the normalized gradient length. Thus, the comparison to the linear slope is then only across half of the domain of the data set, while the above comparison with the largest change in the normalized gradient length is across the entire data set. Doubling the results of the comparison of the linear and quadratic fits to the quadrupole perturbation data agrees with the error derived from the nonlinearity of the largest \(\Delta \Phi\) in the quadrupole perturbation method measurement.

The estimated systematic error due to assuming a small angle approximation in the derivations of Eqs. (1.49) and (3.9) is \(.65\%\). The systematic error due to the nonlinearities in the beta function measurement are \(~40\) times less than the estimated systematic error from the current to gradient length conversion. Thus, it is concluded that the betatron amplitude measurement is not compromised by higher order affects or nonlinearities.

### 3.5 Measurement of the Action and Check of BPM Gains

There are two aspects of the BPM measurement not addressed in Chap. 2. Both the BPM gains and the BPM offsets cannot be verified through a reproducibility measurement. Even if the BPM gains and offsets changed during the course of the
3.5 Measurement of the Action and Check of BPM Gains

experiment, the affect would appear as measurement spread at a particular BPM and not as changes in the gains or offsets. An independent measurement must be made to compare relative gains and offsets at each BPM. In this section, the relative BPM gains are compared by measurement of the action, while a beam-based alignment analysis is employed to check the BPM offsets in Sec. 3.6.

The amplitude of the betatron oscillation about the CO is related to the beta function and the action via Eq. (1.40). The average betatron amplitude function at each quadrupole and the average fitted amplitude of the betatron oscillation at each BPM will be employed in this calculation. The average beta function in each quadrupole measured by the quadrupole perturbation method is described previously in this tech note, Sec. 3.4.3 II. The amplitude of the betatron oscillation can be measured by fitting the turn-by-turn BPM data to a cosine wave, and the average and error on the average may be calculated from the fitted amplitude distribution at each BPM. The average fitted amplitudes for the analysis are from the RingScan reproducibility dataset collected earlier in the June 27, 2008 development and documented in Chap. 2.

From Eq. (1.40), the constant of the betatron motion may be solved

\[ J = \frac{A^2}{2\beta} = \frac{A^2[\text{mm}^2]}{2(1000)^2 \beta[\text{m}]} \]  

(3.17)

where \( J \) is the action, \( A \) is the amplitude of the betatron oscillation measured in [mm], and \( \beta \) is the average beta function measured in [m]. Since the PSR BPMs are situated inside the quadrupoles, where the beta function varies slowly through a quadrupole in a FODO lattice because the first derivative of the beta function is switching sign, the beta function at the BPM may be approximated as the average beta function across the quadrupole measured via the quadrupole perturbation method. Although there is a random error associated with the average beta function in a quadrupole, the systematic error from the uncertainty due to the current to gradient length conversion
3.5 Measurement of the Action and Check of BPM Gains

dominates the calculation. Thus, the random error on the beta function will be ignored, and only the measurement error from the amplitude fitting will be considered. The effect of the systematic errors from the beta function on the action calculation will be analyzed later in this section. The propagation of the random measurement error on the action is

\[ \sigma_J^2 = \left( \frac{A[mm] \sigma_A[mm]}{1000 \beta[m]} \right)^2 \]  

(3.18)

where \( \sigma_A \) is the error on the average of the fitted amplitude distribution at a BPM calculated by Eq. (2.10).

The action describes the phase space area enclosed by the betatron motion of the beam and is constant according to Liouville’s theorem. Because it is constant, the action may be calculated at any point in the accelerator via the Courant-Snyder parameters and the phase space coordinates of the betatron motion of the beam centroid. Since the action is determined by the injection steering into the PSR, a convenient location to calculate the action is at the foil. The relationship between the action and the injection offset was given in Eq. (1.37), yielding

\[ J = \frac{\gamma_0 x_0^2 + 2 \alpha_0 x_0 x'_0 + \beta_0 x'_0}{2} \]  

(3.19)

where \((x_0, x'_0)\) is the measured injection offset and \(\alpha_0, \beta_0, \) and \(\gamma_0\) are the model Courant-Snyder parameters at the foil. For this analysis, the phase space coordinates at the foil are measured with the turn-by-turn BPM data of the reproducibility dataset by the scan method as described in Sec. 2.7.3.

The results of both of the above calculations for the action are shown in Fig. 3.20. The actions calculated via the model Courant-Snyder parameters and the measured injection offset lie on the lower side of the distribution of actions calculated with the amplitudes of the betatron oscillation and the measured beta function. This could be because the baseline model Courant-Snyder parameters are a bit off at the foil. It
is also possible, if unlikely since the correlations become very complicated, that all of
the BPM gains are slightly larger than 1.

![Figure 3.20:](image)

The average systematic error on the action calculation due to the systematic error
in the beta function measurement from the uncertainty of the current to gradient
length fit is $5.3 \times 10^{-9}$ m in the horizontal for both quadrupole families and in the
vertical $8.5 \times 10^{-9}$ m for the focusing quadrupoles and $6.7 \times 10^{-9}$ for the defocusing
quadrupoles. So, the systematic error on the action calculation is about the same as
one random error on the action.

The action is a constant of the betatron motion, so the measurement of the action
at each BPM should yield similar results. The beta function at each BPM was
measured by extracting the tune from turn-by-turn data and is independent of the
gain of the BPM. However, the fitted amplitude parameter is very sensitive to BPM
gain. Thus, BPMs with gains much different than the others will yield calculated
actions much different than the others. BPMs that are outliers in the horizontal or
vertical action measurements have suspicious gains. This method does not calculate
Figure 3.20 shows no obvious horizontal outliers in the calculated action. Actually all points in the horizontal agree with each other within three uncertainties propagated from the random measurement errors.

While perhaps not outliers, there are two BPMs in the vertical that yield calculated actions different than the rest. BPMs 37 and 39 (SRPM81y and 91y) are vertical diamond type BPMs located in quadrupoles 17 and 19 (SRQF81 and 91) respectively. The vertical diamond-type BPMs are known to have larger measurement spread in the fitted parameters compared to the other vertical BPMs, Chap. 2. The stripline electrodes in the diamond type BPMs are not situated up, down, left, and right; they are oriented on the diagonals. Thus, the signal interpreted as the “top” electrode by the BPM measurement program is combined from signals split off the top-left and top-right electrodes. One can easily imagine that over time these connections might loosen or be susceptible to radiation damage. This could explain why the vertical diamond-type BPMs have a gain different than the rest of the vertical BPMs.

The same signal manipulation is applied to acquire “left” and “right” electrode signals for the horizontal diamond type BPMs. However BPMs 17 and 18 (SRPM81x and 91x) calculate actions that agree with the other calculated horizontal actions suggesting that the horizontal diamond-type BPM gains are the same as the other horizontal BPMs.

### 3.6 Beam-Based Alignment

As it turns out, applying the quadrupole perturbation method to measure the betatron amplitude function yields a data set suitable for a beam-based alignment analysis. The beam-based alignment analysis processes beam position data to calculate the distance between the magnetic center of a quadrupole and the electronic center
3.6 Beam-Based Alignment

of a BPM. Beam-based alignment makes use of the fact that if the CO is offset inside a quadrupole and there is a change in the quadrupole strength, the CO will receive a kick proportional to the distance between the CO and the magnetic center of the quadrupole. This kick can be represented as a dipole field error. The beam-base alignment analysis then compares the CO offset in the quadrupole to the CO reading at the BPM to obtain the BPM offset from the quadrupole center. See Fig. 3.21 for a pictorial definition of the distances defined in this derivation.

**Figure 3.21:** (Color) A pictorial description of the distances in the beam-based analysis. Blue vectors are distances between the beam CO and the electronic center of the BPM, green arrows are distances between the beam CO and the magnetic center of the quadrupole, and the red vector is the distance between the electronic center of the BPM and the magnetic center of the quadrupole.

More formally, assume the beam is initially offset in a quadrupole with strength $K_L$. For this derivation, the BPM is assumed to be inside the quadrupole as is the case for a PSR BPM. The measured CO at the BPM in the perturbed quadrupole is assumed to be the average or effective CO throughout the entire quadrupole, and the beta function at the BPM is approximated as the measured average beta function at
the quadrupole measured via the quadrupole perturbation method. The results of the beta function measurement are discussed in Sec. 3.4.3 II.

The initial distance from the beam CO to the magnetic center of the quadrupole is \( x_Q \), and the distance from the beam CO to the electronic center of the BPM is \( x_{CO} \). Now the quadrupole strength is changed from \( KL \) to \( \tilde{KL} \). The beam position at the BPM also changes. The new CO positions relative to the quadrupole center and the center of the BPM are \( \tilde{x}_Q \) and \( \tilde{x}_{CO} \) respectively.

The change in the quadrupole strength will induce a dipole oscillation in the CO. Thus, the difference in the CO everywhere in the ring may be described as a dipole kick, which is described in Eq. (1.61). The strength of the dipole kick depends on the perturbed beam offset in the perturbed quadrupole and may be written as[27]

\[
\theta = -\Delta(KL)\tilde{x}_Q = -\Delta(KL)x_Q - \Delta(KL)\Delta x_{CO}(s_0), \tag{3.20}
\]

where \( \Delta(KL) = \tilde{KL} - KL \), and \( \Delta x_{CO} = \tilde{x}_{CO} - x_{CO} = \Delta x_Q = \tilde{x}_Q - x_Q \). Substituting Eq. (3.20) into Eq. (1.61) yields

\[
\Delta x_{CO}(s) = \frac{\sqrt{\beta(s_0)\beta(s)}}{2 \sin(\pi \nu)} \cos(\pi \nu - \mu_{s_0 \rightarrow s})(-\Delta(KL)x_Q - \Delta(KL)\Delta x_{CO}(s_0)). \tag{3.21}
\]

If Eq. (3.21) is solved by only applying the change in the CO at the perturbed quadrupole such that \( \mu_{s_0 \rightarrow s} = 0 \), the beam offset in the quadrupole \( (x_Q) \) may be written as

\[
x_Q = -\frac{2 + \beta(s_0)\Delta(KL)\cot(\pi \nu)}{\beta(s_0)\Delta(KL)\cot(\pi \nu)} \Delta x_{CO}(s_0), \tag{3.22}
\]

which is equivalent to Eq. (5) in Ref. [27].

Another method to calculate the initial CO offset in the quadrupole, and the means applied in this analysis, is to fit the dipole kick strength to the change in the CO at every BPM in the ring via Eq. (1.61). Fitting the dipole kick strength is preferred because all available data is applied in the analysis. The fitted \( \theta \) is also more robust against bad BPM measurements.
3.6 Beam-Based Alignment

Conveniently, the quadrupole perturbation data set has already yielded average beta functions in each of the quadrupoles. The beta functions are derived via the measured tunes and not the COs, so there is little correlation between these two quantities, as discussed in Sec. 2.6.7. The average offset and error on the average offset calculated via Eq. (2.10) from the turn-by-turn RingScan data are applied in the linear regression fit for $\theta$. $\mu_{s_0 \rightarrow s}$ is the positive difference in the average phase of the betatron motion at the BPM in the perturbed quadrupole and another BPM and is also taken from the cosine wave fit to the turn-by-turn RingScan data. The measurement and calculation errors on $\mu_{s_0 \rightarrow s}$ and $\beta$ are taken to be systematic errors because the same values of these quantities are applied to all fits for $\theta$. The $\chi^2$ applied in the fit for dipole kick strength is

$$\chi^2 = \sum_i \left[ \frac{\Delta x_{CO_i} - G_i \theta}{\sigma_{\Delta x_{CO_i}}} \right]^2,$$

where $G_i = \sqrt{\beta(s_0) \beta_{BPM_i} / 2 \sin(\pi \nu) \cos(\pi \nu - \mu_{s_0 \rightarrow BPM_i})}$

and $\sigma_{\Delta x_{CO}}$ is the propagated random measurement error on the difference in the average fitted offset distributions at the $i^{th}$ BPM, and $G$ is the discrete Green’s function form factor for a dipole kick. The error on the average of the offset distribution in the vertical is always precision limited, and even with only 20 RingScans taken at each shunt setting, the error on the average horizontal offset is very close to being precision limited by the discrete digitization of the ADC.

The $\theta$ that minimizes the $\chi^2$ may be solved explicitly by linear regression (Appx. A.1.1),

$$\theta = \frac{\sum_i G_i \sigma_{\Delta x_{CO_i}}}{\sum_i G_i^2 / \sigma_{\Delta x_{CO_i}}^2},$$

and the fitting error on $\theta$ may be propagated from the error on the average offset measurements and is

$$\sigma^2_\theta = \left( \frac{\sum_i G_i / \sigma_{\Delta x_{CO_i}}}{\sum_i G_i^2 / \sigma_{\Delta x_{CO_i}}^2} \right)^2.$$
Applying the results of the $\theta$ fitting to Eq. (3.20), the initial CO offset in the perturbed quadrupole is found as

$$x_Q = -\theta \frac{\Delta}{\Delta (KL)} - \Delta x_{CO}(s_0)$$

(3.26)

However, the offset of the BPM electronic center compared to the magnet center of the quadrupole is of most interest. The BPM offset is $x_{BPM} = x_Q - x_{CO}$ or

$$x_{BPM} = -\theta \frac{\Delta}{\Delta (KL)} - \tilde{x}_{CO}(s_0)$$

(3.27)

with propagated random error from measurement spreads

$$\sigma_{x_{BPM}}^2 = \left[ \left( -\frac{1}{\Delta (KL)} \sum_i G_i^2/\sigma_{x_{CO_i}}^2 - 1 \right) \sigma_{\tilde{x}_{CO}(s_0)}^2 \right]^2 + \left( -\frac{1}{\Delta (KL)} \sum_{i \neq s_0} G_i \sigma_{\tilde{x}_{CO_i}}/\sigma_{x_{CO_i}}^2 \right)^2$$

(3.28)

where index $s_0$ is evaluated at the BPM in the perturbed quadrupole and the summation of $i \neq s_0$ is over all BPMs except the BPM in the perturbed quadrupole. The uncertainty in the current to gradient length conversion will be treated as a systematic error later in this section and does not contribute to the random calculation uncertainty of the BPM offset in Eq. (3.28).

The strength of the dipole kick due to the CO offset in the perturbed quadrupole is calculated by the change in the CO between the maximum and baseline shunt settings. This comparison will yield the largest and best defined dipole kick strength. An example fit for theta is shown in Fig. 3.22. Figure 3.22 shows one of the strongest dipole kicks in the quadrupole perturbation method with a calculated initial CO offset compared to the quadrupole center of 7.67 mm in the horizontal. This produces a well defined dipole oscillation and a good fit for theta.

According to Fig. 3.22 the vertical CO did not change between the baseline and maximum shunt settings at quadrupole 11 (SRQF51). Even with the maximum shunt...
setting, some BPMs do not show a significant change in the CO at the perturbed quadrupole. This indicates that the initial CO passes through the magnetic center of the quadrupole. But if not careful, the linear regression fit for the dipole kick strength could yield a significant if unconstrained fit with small fitting error for theta. To prevent false theta indications, two quality checks are applied to $\Delta x_{CO}$. First, the change in the CO must be greater than the measurement spreads of the initial and perturbed CO measurements. At least one point of $\Delta x_{CO}$ at a BPM needs to be larger than the combined $rms$ standard deviations of the individual CO measurement distributions,

$$|\Delta x_{CO_i}| > \sqrt{\sigma_{x_{CO_i}}^2 + \sigma_{\tilde{x}_{CO_i}}^2}.$$  

(3.29)
If no points in $\Delta x_{CO}$ are larger than the combined $rms$ spread in the individual CO measurements, the dipole kick strength is defined to possess poor correlation as described below.

The second check on the change in the CO assures that the correlation between the change in the CO and $G$ defined by Eq. (3.23) is large. If the absolute value of the correlation is less than a certain limit, then the change in the CO is too cluttered with noise to resemble $G$, and the dipole kick for this shunt value is defined to possess poor correlation. Figure 3.23 shows the absolute value of the calculated correlation between the change in the CO due to the quadrupole field perturbation and the $G$ of Eq. (3.23). The small correlations indicate that the quadrupole field perturbation does not induce a measurable dipole field error. Because there is no clear gap in the correlation, the question is can a limit in the correlation be applied to distinguish significant dipole kick strengths from noise?

It was found that all quadrupoles which produced correlations of .75 (the horizontal correlation of quadrupole 6, SRQU21) or less resulted in changes in the CO that were less than the combined measurement spread of the individual CO measurements, and thus, the dipole kick strengths for these quadrupoles were set to zero.

The correlation between the change in the CO and $G$ describes the measurement quality of the dipole oscillation. And as will be discussed later, the correlation depends on strength of the dipole kick which in turn depends on the initial CO offset from the center of the quadrupole. Thus, two correlation limits are applied to prevent unconstrained theta fits from affecting the results of the beam-based alignment analysis. The second correlation limit (the lowest) separates $\Delta x_{CO}$’s which do not change significantly with those that have slightly come out of the noise. Since all dipole oscillations with correlations less than .75 were observed not to have significant changes in the CO, the second correlation limit is set at .75.

The first correlation limit (the largest) separates the strong dipole kicks from those
of more questionable quality. The dipole oscillations yielding correlations between the two limits possess $\Delta x_{CO}$’s that have slightly come out of the noise of the CO measurement. But these $\Delta x_{CO}$’s are not large enough to fit a quality dipole kick strength for certain. The fit $\theta$ from these dipole oscillations are not always good, but their correlation with $G$ is too large for the fit to be discounted entirely. So the $\theta$’s that result from these fits are retained, but labeled as possessing moderate or “questionable” correlation. The first correlation limit is set as .95.

In order to gain some insight into the beam-based alignment analysis, the conditions required to obtain a well defined dipole oscillation due to the quadrupole perturbation should be studied in detail. In particular, the smallest measurable dipole kick

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**Figure 3.23:** (Color) The absolute value of the correlation between $\Delta x_{CO}$ and $G$. Quadrupole perturbations yielding well correlated dipole oscillations are shown as blue circles, green squares are moderately correlated dipole oscillations, and poorly correlated dipole oscillations are shown as red triangles. The first and second correlation limits (dashed green and red lines) are .95 and .75 respectively. The vertical line separates correlations from horizontal (left) and vertical (right) data.
and the required initial CO offset from the quadrupole center should be estimated. This analysis starts with the limitations on the CO measurement. The measured maximum change in the CO for each quadrupole perturbation is plotted in Fig. 3.24. Notice that there is a direct relationship between the maximum change in the CO and the calculated correlation between $\Delta x_{CO}$ and $G$ as expected from Eq. (1.61).

**Figure 3.24:** (Color) The maximum $\Delta x_{CO}$ due to each quadrupole perturbation with three random errors. Perturbations yielding well correlated dipole oscillations are blue circles, green squares are moderately correlated, and poor correlations are left pointing red triangles. Black right pointing triangles are results from perturbations in quadrupoles without BPMs. The vertical line separates the horizontal (left) and vertical (right) maximum $\Delta x_{CO}$’s.

It seems that the smallest maximum change in the CO from the dipole kick needed to obtain a well correlated dipole oscillation is $\sim 0.4$ mm in the horizontal. Notice how the three random errors from the kicks with moderate correlation straddle this mark, while the quadrupole perturbations with poor correlation fall short. Demanding a maximum change in the CO of $0.4$ mm for a good dipole kick measurement is not un-
thinkable with the PSR BPMs because .4 mm is about three times the CO measurement spread in the horizontal BPMs (Sec. 2.6.4) invoked to define bad correlations, Eq. (3.29).

It is harder to tell what the smallest maximum change in the vertical CO is because there are less dipole oscillations with moderate or poor correlation in the vertical. However, it can be assumed that the vertical should behave similarly to the horizontal. The three random errors from the moderately correlated vertical dipole oscillations should straddle the smallest maximum CO needed to obtain a good dipole kick measurement, while the three random errors of dipole oscillations with poor correlations fall short of the smallest maximum CO change. Applying these assumptions leads to a smallest maximum vertical CO change of .15 mm for a good dipole kick measurement.

One can apply these estimates of the smallest maximum change in the CO needed to obtain a quality dipole oscillation and the approximated maximum effect of a dipole kick strength to obtain the minimum measurable $\theta$. According to Eq. (1.61), the maximum change in the CO happens when the cosine term goes to 1 and $\beta(s)$ goes to the maximum beta function. It is noted that the beta function may never be maximum when the cosine term is 1, but for the sake of this argument it is assumed that this can happen.

The average maximum horizontal beta function is 14 m, while the average minimum horizontal beta function is 2 m. So, the estimated minimum measurable horizontal $\theta$ in a focusing quadrupole is

$$\theta = \frac{2(.4) \sin(\pi \nu)}{\sqrt{(14)(14)}} = .033 \text{ mrad}$$

and

$$\theta = \frac{2(.4) \sin(\pi \nu)}{\sqrt{(2)(14)}} = .089 \text{ mrad}$$

in the defocusing quadrupoles. This means that the smallest observable horizontal
dipole kick in a defocusing quadrupole is about three times larger (corresponding to
the ratio of maximum and minimum beta functions) than the smallest measurable
horizontal dipole kick in a focusing quadrupole.

The minimum measurable vertical dipole kick strengths can be obtained in a
similar manner as the horizontal. The average maximum and minimum vertical beta
functions are 14 m and 3 m respectively. Thus, the estimated minimum measurable
vertical dipole kick strength in a focusing quadrupole is

\[
\theta = \frac{2(0.15) \sin(\pi \nu)}{\sqrt{14}(4)} = 0.027 \text{ mrad}
\]

and

\[
\theta = \frac{2(0.15) \sin(\pi \nu)}{\sqrt{14}(14)} = 0.013 \text{ mrad}
\]

in the defocusing quadrupoles.

The absolute value of the fitted \( \theta \) for the maximum shunt setting at each quadrupole
is plotted in Fig. 3.25. Note that dipole oscillations with poor correlations with \( G \) fit
\( \theta \)'s very close to 0. The quadrupoles field errors that produce dipole oscillations with
correlations between the first and second correlation limits also yield small \( \theta \)'s. This
can be understood since a small dipole kick strength yields less change in the CO
and is more likely to yield a result either in or just outside the measurement noise
level. Also notice that the estimates of the minimum measurable dipole kick strength
from the above analysis agrees \( \theta \)'s plotted in Fig. 3.25. Dipole kicks yielding good
correlations fit \( \theta \)'s larger than the estimated minimums, the moderate correlations
yield \( \theta \)'s that straddle the minimum measurable dipole kick estimate, and the dipole
oscillations fall short.

The sensitivity of the dipole kick measurement also depends on the family of
the quadrupole that is perturbed and in which dimension the \( \theta \) is fit. The smallest
measurement of the dipole strength can be obtained for a vertical kick in a defocusing
quadrupole. This is because the vertical beta functions are large in defocusing
Figure 3.25: (Color) The absolute value of the fitted $\theta$ for each quadrupole perturbation with three fitting errors. Quadrupole perturbations yielding well correlated dipole oscillations are shown as blue circles, green squares are moderately correlated dipole oscillations, and poorly correlated dipole oscillations are shown as red triangles. The vertical line separates the fitted $\theta$’s from horizontal (left) and vertical (right) data.

quadrupoles and because the vertical CO measurement does not have the added spread due to the pulse-to-pulse momentum variations. Likewise, the hardest dipole kick to measure is a horizontal kick in a defocusing quadrupole because the horizontal beta functions are the smallest and because the horizontal CO measurement has a larger noise level due to the additional measurement spread from the pulse-to-pulse momentum variations.

The above helps to explain why all of the horizontal dipole kicks from defocusing quadrupoles (the even number quadrupoles) were found to either have a poor or moderate correlation with the dipole form $G$.

Ultimately, one would like to correlate the minimum measurable dipole kick strength with the minimum initial CO offset from the quadrupole center for a given change in the normalized gradient length. This can be done by substituting the
change in the CO at the perturbed quadrupole, $\Delta x_{CO}$ of Eq. (1.61) when $s = s_0$, in to Eq. (3.26) and applying the smallest measurable dipole kick strength $\theta_{\text{min}}$,

$$x_Q = -\frac{\theta_{\text{min}}}{\Delta (KL)} - \frac{\beta(s_0)}{2} \cot(\pi \nu) \theta_{\text{min}}. \quad (3.32)$$

The average change in the horizontal normalized gradient length for the focusing quadrupoles is $-.02 \text{ m}^{-1}$ and $.015 \text{ m}^{-1}$ for the defocusing quadrupoles. The change in the vertical $KL$ is the negative of these numbers. Applying the minimum obtainable $\theta$ values to Eq. (3.32) yields the smallest initial CO offset that will produce a measurable dipole kick when the quadrupole is perturbed. The minimum initial horizontal CO offset needed to obtain a quality $\theta$ measurement in a focusing quadrupole is $1.27 \text{ mm}$ and $5.79 \text{ mm}$ in a defocusing quadrupole. The minimum initial vertical CO offset needed to obtain a good fit for the dipole kick strength is $1.61 \text{ mm}$ in a focusing quadrupole and $.84 \text{ mm}$ in a defocusing quadrupole.

The absolute values of the initial CO offsets from the centers of the quadrupoles are plotted in Fig. 3.26. The dipole oscillations that have been identified as having moderate or poor correlations with $G$ obey the estimations for the minimum initial CO needed to obtain a quality $\theta$ fit. None of the initial horizontal CO offsets in the defocusing quadrupoles were near the $5.79 \text{ mm}$ mark needed for a good dipole kick. That is why they have all been identified as poor or questionable dipole oscillations. The horizontal dipole oscillation from quadrupole 3 (SRQF11) has an initial CO offset of $.82 \text{ mm}$ which falls short of the needed offset of $1.27 \text{ mm}$. Lastly, the initial vertical CO offset at quadrupole 5 (SRQF21) was only $.67 \text{ mm}$ when an initial offset of $1.61 \text{ mm}$ is needed to produce a measurable dipole oscillation.

With this analysis of the minimum parameters (initial CO offset, $\theta$, and $\Delta x_{CO}(s)$) needed to fit a quality dipole kick strength to the change in the CO, the limits set on the correlation between $\Delta x_{CO}$ and $G$ seem less arbitrary. Actually, this analysis is self consistent and shows that the limits applied to the correlation of $\Delta x_{CO}$ and $G$
will separate good fits for the dipole kick strength and poor $\theta$ fits. The results found for the minimum parameters are recapped in Tab. 3.7.

Now that the quality of the $\theta$ fits have been scrutinized, the offset of the BPM in relation to the center of the quadrupole may be calculated by applying Eq. (3.27). The offset for each BPM resulting from the beam-based alignment analysis is shown in Fig. 3.27. The beam-based alignment analysis yields significant BPM offsets for most BPMs. The results are about two times larger than the expected BPM misalignments. It is hard to understand how the BPMs can be physically misaligned compared to the quadrupole center on this magnitude, however these BPM offsets could also describe offsets in the BPM electrodes and electronics including possible phase differences in

Figure 3.26: (Color) The absolute value of the calculated initial CO offset in the shunted quadrupole with three random errors. Quadrupole perturbations yielding well correlated dipole oscillations are shown as blue circles, green squares are moderately correlated dipole oscillations, and poorly correlated dipole oscillations are shown as red triangles. The horizontal dashed line indicates zeros. The vertical line separates horizontal (left) and vertical (right) initial CO offsets.
3.6 Beam-Based Alignment

<table>
<thead>
<tr>
<th>Minimum Requirements for Good θ Fits</th>
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<tbody>
<tr>
<td></td>
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<tr>
<td></td>
</tr>
<tr>
<td>QF</td>
</tr>
<tr>
<td>Δx_{CO}(s) [mm]</td>
</tr>
<tr>
<td>θ [mradian]</td>
</tr>
<tr>
<td>x_{CO}(s_0) [mm]</td>
</tr>
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Table 3.7: The minimum values for the initial CO offset, θ, and maximum Δx_{CO}(s) needed to produce a quality dipole oscillation. QF and QU stand for focusing and defocusing quadrupole respectively.

the cables before the AM-PM conversion. A strenuous systematic error analysis will follow later in this section to verify the results of the beam-based alignment.

Many of the quadrupole perturbations that yield dipole oscillations that poorly correlated with \( G \) calculate BPM offsets equal to the CO at the perturbed quadrupole. This is as expected. The initial CO offset is too small to produce a well defined dipole oscillation, the fitted θ is very small, and the BPM offset is equal to negative the CO. This result also lends confidence to the definition of θ in Eq. (3.20). However, some of the BPM offsets from poorly correlated dipole oscillations are not within three measurement errors of the negative initial CO. These are cases where the θ fit is bad giving a bad BPM offset measure. Most of these poor θ measurements occur in the horizontal with a perturbed defocusing quadrupole where the dipole kick strength is hardest to measure. None of the differences between the negative of the initial CO and the calculated BPM offsets are larger than the minimum initial CO offset required for a good θ fit. This verifies that θ fits for these dipole oscillations do not yield trustworthy dipole kick strengths.

The dipole oscillations yielding moderate correlations between Δx_{CO} and \( G \) also calculate BPM offsets where the difference in the negative initial CO and the BPM offset is smaller than the initial CO offset required for a good θ fit. These BPM offsets
3.6 Beam-Based Alignment

Figure 3.27: (Color) The calculated BPM offsets from the beam-based alignment analysis with three random errors and minus the initial CO at each quadrupole with three measurement errors, black dashed line. Quadrupole perturbations yielding well correlated dipole oscillations are blue circles, green squares are moderately correlated, and poor correlations are red triangles. The vertical line separates the horizontal (left) and vertical (right) BPM offsets.

are also suspicious because their $\theta$ fits may not be good.

The dipole oscillations with good correlations between the change in the CO and $G$ represent well fit $\theta$'s and meaningful BPM offset calculations. The calculation error on the BPM offset measurement is calculated via Eq. (3.28). The horizontal BPM offset calculation error depends on quadrupole family and has large variation in the defocusing quadrupoles. The average horizontal BPM offsets are .04 mm and .13 mm in the focusing and defocusing quadrupoles respectively. The vertical BPM offset calculation error does not seem to have dependence on quadrupole family. The average vertical BPM offset calculation error is .02 mm. The much larger calculation error in the horizontal BPM offsets for the defocusing quadrupoles is due to the small dipole kicks introduced by the quadrupole perturbation. With such small random
errors on the BPM offset one might expect the systematic error from the current to
gradient length conversion is the dominating uncertainty in the BPM offset calcu-
lation, but interestingly enough, it does not dominate the uncertainty of the BPM
offset calculation.

It was found that the systematic error due to the uncertainty in the current to
gradient length conversion on the BPM offset random is comparable or smaller than
one calculation error. This was a surprising result but can be understood by the
equation for the BPM offset. There are two terms in Eq. (3.27) that depend on
the gradient length, $\Delta(KL)$ and $G$. $G$ is a function of the measured beta functions
which in turn are a function of $KL$. The systematic errors on $G$ and $\Delta(KL)$ seem to
cancel leaving less than a $\pm 0.1\%$ systematic error in most BPM offsets. Calculated
BPMs offsets close to zero (especially BPMs 32, 33, 34, and 38) obviously have
larger relative systematic error. The systematic errors on the horizontal BPM offset
calculation due to the uncertainty of the current to $KL$ conversion are $2.5 \times 10^{-3}$ mm
and $6.4 \times 10^{-3}$ mm in the focusing and defocusing quadrupoles respectively, which is
about ten times less than the calculation error. The systematic errors on the vertical
BPM offset calculation are $2.7 \times 10^{-3}$ mm and $1.4 \times 10^{-2}$ mm in the focusing and
defocusing quadrupole respectively.

The initial CO offset is also shown in Fig. 3.27. The initial CO offset is the
distance from the calculated BPM offset to the negative of the CO. While the BPM
offsets calculated from well correlated dipole oscillations yield clear conclusions, the
initial CO offset can help retrieve information from the BPM offsets calculated from
moderately or poorly correlated dipole oscillations. For instance, BPM 31 yields a
BPM offset very close to the negative of the baseline CO as expected. The baseline
CO is 2.68 mm, but the minimum initial CO offset needed to obtain a quality dipole
oscillation in the vertical from a focusing quadrupole is 1.61 mm. This indicates
that BPM 31 is offset negatively enough such that a 2.68 mm baseline CO does not
induce a significant dipole oscillation when the quadrupole is shunted. Unfortunately because the dipole kick is too weak to be measured, the best measurement of the BPM offset for BPM 31 is negative the baseline CO with a measurement threshold error of the needed initial CO offset to obtain a measurable dipole oscillation, $2.68 \pm 1.61$ mm, which is better than throwing away the data obtained from a poorly correlated dipole oscillations all together.

The same procedure can be performed for all of the BPM offset from poorly or moderately correlated dipole oscillations. Unfortunately, this procedure does not help to shed any light on the horizontal BPM offsets for BPMs in the defocusing quadrupoles. This is because the measurement threshold error on the horizontal BPM offset for the defocusing quadrupoles is $5.79$ mm and the baseline horizontal CO in the defocusing quadrupoles did not exceed $2$ mm.

Although the quadrupole perturbation method data was ideal for the beam-based alignment analysis, the PSR setup was not optimized for this procedure. If a dedicated experiment was desired to collect beam-based alignment data, it would be best to set up the PSR with large CO offsets in both dimensions. This can easily be done in the horizontal by adjusting the energy output of the linac or by changing the strength of the bending magnets in the ring. Large initial CO offsets are desired for good dipole oscillations when the quadrupoles are perturbed, thus a $\pm 8$ to $10$ mm horizontal CO and $\pm 5$ to $7$ mm vertical CO should work well. At these extreme COs, it is also suggested that the beam is injected near-on-axis, $2$ mm in the horizontal and $3$ mm in the vertical with some amount of angle to avoid the large measurement spread in the fitted amplitude.
3.7 Summary

In this chapter, I discussed the experimental procedures and in-depth analysis of the three supporting measurements. The first measurement involving the hysteresis of the PSR magnets resulted in affirmation that the orbit response matrix measurement and center method for beam momentum measurement would not suffer from dipole hysteresis. However, it was found that hysteresis in the quadrupoles could affect quadrupole strength by as much as .5%. Also during this analysis, it was discovered that the horizontal CO measurement of BPM 20 (SRPM92x) was not reproducible.

I compared two methods for measuring the beam momentum and found that the procedure employing the model momentum compaction factor was the superior in application to the PSR. A third hybrid method of momentum measurement was introduced for fast momentum measurements which relate the beam momentum to the measured position at LDPM03.

It was found that the baseline model was able to successfully predict the measured dispersion function. A much smaller vertical dispersion function was also observed.

I introduced the quadrupole perturbation method and carried out beta function measurements by this method. The results of the beta function measurements were found to be dominated by a systematic error due to the uncertainty in the current to gradient length conversion of the quadrupoles. It was also observed that the baseline tune drifted over the course of this measurement by about .005 due to the hysteresis in the quadrupoles. This result is verified by the baseline model and actual quadrupole hysteresis measurements.

The measured betatron amplitude functions and the measured amplitudes of betatron oscillation were combined to obtain the beam action. This calculation was performed for each BPM, revealing relative gain differences between the BPMs.

Lastly, I reported the results from a beam-based alignment calculation. The out-
come of this analysis suggests that the BPMs in the PSR may be offset electronically by a few millimeters.
Chapter 4

PSR Model Improvement Experiments

Now finally onto the experiments that test some aspect of the baseline model. Recall that the baseline model systematically mistreatments the vertical betatron phase advance. In this thesis, I investigate three possible sources of mistreatment of the vertical focusing: the quadrupoles, the fringe fields of the PSR extract septa, and the edge focusing of the rectangular dipoles. I employ an orbit response matrix (ORM) analysis method to test the model quadrupoles, Sec. 4.1. In Sec. 4.2, I measure the magnetic multipole components of the PSR extraction septa as observed by the beam. These measured multipole components will be included in the improved model of the PSR. I report the results of ray tracing through simulated 3D magnetic field, Sec. 4.3.

Now finally with all possible sources of additional vertical focusing in the baseline model investigated, in Sec. 4.4 I may construct an improved model for the PSR. I apply measurements from the supporting experiments to first establish the improved model as a better model and then to verify the improved model at PSR operational set points far from nominal. Lastly, I apply the ORM analysis to the improved model
and show the fitted model is very similar to the improved model, further verifying the
improved model. With the improved model established and verified, I will once again
apply the ORM analysis, but this time to the improved model. The LOCO fitted
model from this final test with the improved model as the input will show whether
the improved model has moved in the proper direction in model parameter space.

4.1 Orbit Response Matrix (ORM) Analysis

An orbit response is the change in the closed orbit (CO) due to a dipole error, kick,
or bump[28], as described in Eq. (1.61). The orbit response is linearly related to
the dipole kick by the Greens Function solution to Hills equation for a dipole error.
By executing CO bumps with each corrector and measuring the corresponding CO
difference at each BPM, a system of equations may be formed, where because it is a
Greens function, $R$ is the unique ORM for the machine.

$$\Delta \vec{x}_{BPM} = R \vec{\theta}_{Corrector}$$  \hspace{1cm} (4.1)

One can perform the same experiment in a model and compare the resulting
measured and model ORM, which is a function of several model parameters. Because
the ORM is unique, if the model properly describes the real machine, the model
and measured ORM will be the same. If model and measured ORMs do not agree,
one can iteratively minimize the difference by changing the model parameters i.e.
gradients, rolls, positions, currents. . . . The Matlab code Linear Orbits from Closed
Orbits (LOCO)[29] was applied in the analysis to iteratively minimize the $\chi^2$ between
model and measured ORMs.

LOCO was written for and has been successfully applied to many electron storage
rings around the world[30]. However, unique complications arise when attempting
to apply an ORM method to an accumulator ring such as the PSR. First, typically
for the electron storage rings, the ORM is measured during a single beam store, which can last up to tens of hours. The storage time in the PSR is only hundreds of microseconds, so many different beam pulses with slightly different momentum and injection offset and intensity are needed to measure the many COs necessary for the ORM measurement. The variations in the many beam pulses of course lead to much larger measurement spreads than those observed in the electron storage rings. Second, each corrector bump modifies the injection offset for that ORM kick. The action can become quite large for some corrector bumps leading to large amplitude betatron oscillation, which in turn can yield BPM saturation and beam scraping. However, as was shown in Chap. 2, quality results may be obtained if a sufficient number of RingScans are collected for each corrector setting.

4.1.1 Limitations on the LOCO fitting

Before applying the LOCO ORM method for model improvement to the PSR, many questions need to be answered: is the CO measurement in the PSR good enough for LOCO, can the data acquisition errors of the PSR BPMs compromise the ORM measurement, and how close does the baseline model need to be for LOCO to converge to the proper solution?

LOCO will fit the model quadrupole gradients as well as the BPM gains and corrector kick strengths. It is of interest to know the maximum allowable deviation between the baseline model and the real machine for LOCO to converge to a solution. Two accelerator toolbox (AT) models were created for this test, one representing the “real” machine and the other for the “model”. AT is a Matlab based accelerator modeling code. LOCO requires input in the form of AT output. This is why AT is employed in the study instead of a more common code such as MAD. The “model” was exactly the same as the “real” save for quadrupole strengths which were multiplied
by some gain factor. The “real” machine AT model generated simulation data for the LOCO analysis.

It was found that LOCO could converge the “model” to the “real” solution in a single iteration when a single quadrupole differed by 10%. However, LOCO was unable to converge to the solution when all quadrupoles were off by 10%. The limits on the baseline model quadrupole gradients were found to be ±4%. A 4% error in the gradients correspond to ~20 A and ~12 A errors in the currents for the focusing and defocusing quadrupoles respectively.

Next the maximum deviation in the corrector kick strength in which LOCO converges to the solution was tested in the same manner as the quadrupole gradients. This time the difference between the “model” and “real” AT models was the corrector kick strength. No maximum deviation was found for the horizontal kickers. The investigation halted after LOCO was observed to converge when there was a 6 mrad difference in all of the horizontal correctors between “model” and “real”. Next, a ±5 mrad random error was introduced to each horizontal corrector. LOCO was able to converge to this solution within .002 mrad while leaving the quadrupole gradients and BPM gains unchanged.

The vertical corrector magnets were studied similarly. LOCO was able to reproduce a constant 5 mrad offset in all kicks and was able to converge for a ±5 mrad random offset for each kick. LOCO was also able to identify the two different types of vertical correctors. In the case of the ±5 mrad random error, LOCO was able to converge to within .015 mrad for the 9″ correctors and .065 mrad for the 11″ correctors.

The next step was to fit both horizontal and vertical corrector errors. Fitting both horizontal and vertical correctors each with a random error between ±5 mrad provides a solution which recovers the random input errors within the limits stated above, less than .002 for horizontal and .065 and .015 mrad for the 9″ and 11″ vertical correctors.
respectively. Limiting the random input error to ±2 mrad, LOCO can recover this error to within .01 mrad.

When the 4% quadrupole errors are included with the corrector kick strength errors, LOCO was able to recover the random input error of the horizontal correctors to within .01 mrad, but does not do such a great job with the vertical. LOCO recovers random input error of the vertical correctors to within .2 mrad. This may not be good enough for our purposes, as for a typical 1 mrad kick, this is a 20% error. However a 5 mrad error in the corrector strengths is very unlikely. When the random corrector strength error was reduced to ±2 mrad, which is still possibly a 100% error, LOCO was able recover both horizontal and vertical random input errors to within .08 mrad.

The last quantity needed to be examined is the BPM gain. The BPM gain in which LOCO converges was studied in a similar manner as the quadrupole gradients and the corrector kick strengths. The first task was to set the gain of BPM 7 to +10% in the “real” machine. LOCO easily found the gain of BPM 7 as 10% high while also fitting for quadrupole gradients and corrector kick strengths which were the same between “model” and “real”. Next BPM 11 was added with a gain of −20% to the “real” machine, and LOCO was able to find both bad BPMs.

LOCO was also tested with the introduction of random BPM CO measurement noise of either .25 or .5 mm rms standard deviation. LOCO was able to fit either case yielding $\chi^2$/DOF of ~1 in either case. LOCO was also able to find both bad BPM gains under these conditions, though some of the other fitting parameters began to change with the .5 mm random errors.

The final test was to see if LOCO could converge to the solution when all BPMs possess multiplicative errors and noisy data. Each BPM was given a random multiplicative error between ±10% and a random noise error with rms standard deviation of .5mm. And things got a little messy. LOCO is able to fit this situation with the Levenberg-Marquardt method but not the Gauss-Newton method, which jumps past
the solution minimum, and then the $\chi^2$ explodes.

It is all very unlikely that the BPMs would have errors of 10%, but LOCO is being tested under extreme conditions. Additive BPM errors are not considered in this analysis because they will cancel out when the change in the CO is found between the baseline and corrector kicked measurement. A summary of the parameter derivation limitations between the model and real machine in which LOCO converges to a solution is posted in Tab. 4.1

| Summary of LOCO Input Parameter Deviation Limitations |
|-----------------|----------|
| Quadrupole Gradient | 4%       |
| Horizontal Correctors | $\pm2$ mrad |
| Vertical Correctors | $\pm2$ mrad |
| BPM Gain Errors | $<10\%$ |
| BPM Random Errors | $<.5$ mm |

**Table 4.1:** Results of the study of the maximum deviation between the model parameters and the real machine in which LOCO could converge to the solution.

### 4.1.2 Lessons learned from previous ORM measurements in the PSR

The ORM was previously measured in the PSR on December 12, 2002 and January 24, 2003[24, 25]. An in-house ORM analysis code from Indiana University was created to analyze the data. Although the results of the ORM analysis were inconclusive, many lessons may be learned from the measurement setup and data analysis and may be applied to this measurement of the ORM to be analyzed with LOCO.

The first lesson is that only one RingScan and Save Accel dataset were captured for each kick. One scan at each BPM is definitely not enough information to fill the measurement distributions and average out the measurement spread especially the
pulse-to-pulse variations. Acquiring so few RingScans for each ORM corrector kick also runs the risk of leaving holes in the measured ORM due to the take out of BPM data acquisition errors.

The second lesson yields a means to mitigate the main compromise to the 2003 ORM dataset. It was discovered that shunt for SRBM01 drifted by 2 A. This introduces a CO kick in addition to the ORM corrector kick. The reason why the magnet drift had such a large affect on the ORM measurement was because only two baseline measurements were made in the 2003 ORM measurement experiment, one for the vertical correctors and one for the horizontal correctors. The baseline measurement was made and then the orbit response was measured for all of the correctors in one direction. So hours could separate the baseline and corrector kick measurement allowing the magnet drift to compromise the data. The effect of drifting magnets may be mitigated by taking a baseline measurement for each corrector kick.

As a test of the data analysis, the 2003 ORM data was analyzed with LOCO taking into account the affect of the drifting magnet. LOCO produced a fit yielding a $\chi^2$/DOF of .7 and realistic fitted parameters. All of the LOCO fitted BPM gains were between .9 and 1.1. LOCO was also able to identify that the normalized gradient of SRQU41 was much different between the baseline model and the real machine. This is because the SRQU41 in the baseline model has a large bore field clamp while the quadrupole that was SRQU41 in 2003 did not. The quadrupole that was SRQU41 in 2003 was replaced by a quadrupole with an electron detector after this previous ORM measurement. It is surprising and encouraging that LOCO was able to distinguish these two quadrupoles.
4.1 Orbit Response Matrix (ORM) Analysis

4.1.3 ORM measurement setup and data analysis

The ORM was measured during the June 27, 2008 accelerator development. The PSR was setup for single shot near-on-axis injection as described in Sec. 2.2.

The orbit response of each corrector magnet was measured at the 18 BPMs in the PSR. Eleven of the 12 PSR horizontal bending magnets were applied as horizontal correctors. RIBM09 was not invoked because RIBM09 controls the horizontal injection into the PSR. All 9 vertical correctors were employed for the ORM measurement. Three bump settings were applied at each corrector magnet: baseline, plus, and minus. The plus and minus bump settings applied $\sim \pm 1$ mrad CO bump. This corresponds to a 2 A change in the current for the common horizontal benders and a 4 A current change in the horizontal C-magnets and the vertical correctors. Some correctors were unable to kick their full amount due to beam scraping, so smaller kicks were applied when necessary. Ten RingScans and a Save Accel data set were collected at each bump setting.

The RingScan data is analyzed as described in Sec. 2 and the average and error on the average CO is extracted. The corrector currents are read from the Save Accel data set. Typically in the bi-directional ORM measurement method, the baseline CO is subtracted from the plus and minus bump COs, and the resulting $\Delta$COs weighted by the strength of the plus and minus kicks are averaged. But why average when the data may be fit? Thus, the average CO at each BPM as a function of the current in the corrector magnet is fit to a line. The slope of this line is the orbit response per unit corrector kick produced by 1 A current change in the corrector. Linear regression is applied for this fit, Appx. A.1. While the error on the average CO measurement is considered in the fit, the current read back is assumed to be errorless. The fitting error of the orbit response is calculated by an ML error analysis, Appx. B. The fitting error on the slope is constrained to be larger than the intrinsic BPM resolution.
Because the ORM is the change in the CO per unit kick produced by a 1 A change in the corrector current, the initial kick strength guess inputted into LOCO is the current to milliradian conversions for the corrector magnets. The conversion for the 7" vertical correctors is .24631 mrad A$^{-1}$ and .23419 mrad A$^{-1}$ for the 11" vertical correctors[11]. The conversion for the horizontal correctors is calculated from the slope of the current to $\int Bdl$ magnetic field maps evaluated at the design magnetic field divided by the design $B\rho$. The current to magnetic field map for Nancy (SRBM31) is applied to all of the common horizontal benders, while the C-magnets have their own current to $\int Bdl$ magnetic field maps. The current to $\int Bdl$ magnetic field maps may be found in Ref. [11].

Putting everything together yields a 40 × 20 measured ORM. The missing BPMs were removed from the ORM for LOCO analysis. Data from SRPM92 was also removed from the measured because the horizontal CO measurement of BPM 20 was rather irregular. LOCO is then applied to a ORM with 680 measured points ($BPMs \times Correctors = 34 \times 20$). LOCO was instructed to fit the 34 BPM gains, 20 corrector kick strengths, and 20 quadrupole gradients, so there are 74 fitting parameters in the LOCO. Thus, the LOCO fit should be well constrained with 606 degrees of freedom. Although the full ORM (including horizontal response to vertical kicks and vice versa) was inputed into LOCO, BPM coupling and corrector roll were not included as fitting parameters in the LOCO analysis.

One of the first checks on the quality of the ORM measurement is the baseline CO for all correctors. Figure 4.1 plots the measured baseline CO at every BPM for each corrector with 0 current change. This can be thought of as the stability of the CO measurement over the course of the ORM measurement. Note that there are two families of vertical COs. There are two different vertical baseline COs because VM81 was not reset correctly after it applied a CO bump for the ORM measurement. The original baseline set point for SRVM81 was 1.13 A, but after its CO bump, SRVM81
was set to \(-.13\) A. This is the event that made the angle of the vertical injection offset small and introduced the large measurement variation in the amplitude fitting parameter discussed in Sec. 3.4.2.

Aside from there being two vertical baseline COs, the vertical CO is very reproducible over the course of the entire ORM measurement. The horizontal baseline CO is also fairly reproducible. The spread in the horizontal baseline CO is much larger than the vertical baseline CO measurement spread due to the pulse-to-pulse momentum variations. The consistency in the baseline CO shows that corrector hysteresis does not interfere with the ORM measurement as expected from Sec. 3.1. Note that the largest horizontal CO measurement spread is in BPM 20.

The performance of BPM 20 is worst for this ORM measurement. Most scans at BPM 20 were fit with offsets of 15 mm or larger, so only one or two scans remain at BPM 20 for each corrector kick. With so few scans remaining, neither BPM 20 or BPM 40 (SRPM92) can be applied in the LOCO analysis. There were 620 scans taken at SRPM92 during the ORM experiment. All but 269 (43.4\%) of these scans were removed for possessing data acquisition errors or as outliers in the fitted parameters.
4.1 Orbit Response Matrix (ORM) Analysis

Since a baseline CO measurement was made for each corrector kick, the change in the baseline current set point of SRVM81 will not compromise the ORM measurement. Figure 4.1 also shows that there was no CO drift due to magnet power supply drift or energy drift during the ORM measurement.

Another complication encountered during the 2003 ORM measurement is a drifting tune. Reference [25] cites the tune drift as $\sim5 \times 10^{-3}$, which is about 10 times larger than the measurement spread observed in Sec. 2.6.2. The measured tunes for each corrector kick during the 2008 ORM measurement is plotted in Fig. 4.2.

**Figure 4.2:** (Color) The measured betatron tunes for each corrector kick: baseline horizontal (blue circles) and vertical (green squares), minus horizontal (red right pointing triangles) and vertical (black left pointing triangles) kick, and plus horizontal (magenta up pointing triangles) and vertical, cyan down pointing triangles. The vertical line separates the horizontal (left) and vertical (right) correctors.

The drift in the tune is also observed in the 2008 ORM measurement, but only when the CO was bumped by a corrector. The baseline tune measurements are very consistent across the ORM measurement. Bumping the CO changes the path length around the machine (Eq. (1.65)), so a previously on-momentum particle on the old orbit is off-momentum on the new bumped orbit. The change in the tune can then be
identified as a chromatic affect. The tune change due to the corrector kicks is small.

4.1.4 ORM measurement results

The orbit response to a corrector is found by fitting the CO from the baseline, minus, and plus corrector kick settings as a function of corrector current. The slope of this line is the orbit response for a kick produced at the corrector due to a 1 A change in current. Figures 4.3 and 4.4 plot the orbit response fit at SRPM01 (BPMs 1 and 21) due to a corrector kick at SRBM01.

Figure 4.3: (Color) The measured horizontal CO with 3 measurement errors (blue circles) produced by the ORM bumps at SRBM01, and the line fit, red. This fit produces a $\chi^2$/DOF of 5.092 and an $R^2$ of 1.

Since SRBM01 is a horizontal bender, it is expected that the range of the horizontal CO is larger than the vertical. The orbit response measurement at BPM 1 due to a kick at SRBM01 moves the horizontal CO by about 14 mm, which constrains the slope of the fitted line very well. Even though the change in the horizontal CO is so large, the orbit response is very linear, fitting a coefficient of determination of 1. The line fit yields fitted points within three measurement errors of each CO measurement,
but the measurement errors on the CO measurement are all less than .05 mm, which produces the larger $\chi^2$/DOF. The larger $\chi^2$’s indicate that certain systematic errors not included in the line fit model are larger than the measurement error. Since there are three data points and two fitted parameters, there is only one degree of freedom for these fits.

Figure 4.4 graphs the vertical CO response to the horizontal CO kick at SRBM01. As expected the change in the vertical CO is small compared to the horizontal. But what is not expected is the correlation between the vertical CO and the current. It appears that there may be some coupling in the corrector kick or the beam measurement. Since most of the vertical BPMs do not exhibit the correlation between the vertical CO and the horizontal corrector current, it must be something to do with SRPM01. The correlation could be explained by roll in the alignment of SRPM01. The roll is of course small, because the change in the vertical CO is only 2.59% of the horizontal CO range. This translates into a $1.48^\circ$ roll. It is unlikely that the
observable orbit response in the vertical due to a horizontal corrector is due to elec-
tronic drift because the CO measurements were taken out of order. Another possible
explanation is that saturation in the horizontal BPM signal could couple into the
vertical via Eq. (2.3) and produce a correlation in the measured CO.

Neglecting coupling produced by corrector misalignment, BPM roll, or electronic
issues in the data acquisition, a horizontal CO bump should only affect the horizontal
CO and the same for the vertical. This means that the vertical orbit response due
to horizontal corrector kicks and vice versa is not well constrained in the linear fit
because the change in the CO is mostly due to measurement variation. The orbit
response, which is not well defined, together with the small CO measurement error
lead to larger $\chi^2$/DOF than expected in the coupling quadrants of the ORM.

The coefficient of determination is a better means to quantify the quality of the
orbit response fits. The $R^2$ can be small or even negative for orbit response fits cou-
pling vertical and horizontal. This is expected because the fit is not well constrained
and dominated by measurement variation. However, the $R^2$ is close to 1 for all but a
few fits of the orbit response in the x-x and y-y quadrants of the ORM, Fig. 4.5. Note
that there are a few fits in the horizontal with very poor $R^2$’s. As it turns out, these
fits, like the fits in the coupling quadrants of the ORM, are also not well constrained.
This is because the betatron phase advance between the corrector and the BPM is
such that the cosine term of Eq. (1.61) goes to zero.

The resulting measured orbit response per unit current kick in the corrector is
plotted in Fig. 4.6. As expected the coupling x-y and y-x quadrants of the measured
ORM are small indicating that there is little coupling between the BPMs and the
correctors. The x-x and y-y quadrants of the ORM plot the Green’s function for a
dipole kick, Eq. 1.61. There are more oscillations in the x-x quadrant than the y-y
quadrant because the horizontal tune is 3.19 while the vertical tune is 2.19.

The x-x orbit response is about 50% larger than the vertical. The average max-
Figure 4.5: The coefficient of determination ($R^2$) for every orbit response fit in the x-x and y-y quadrants of the ORM.

The measured ORM displayed in Fig. 4.6 was inputed into the LOCO analysis routine. Orbit responses at the missing BPMs (BPMs 10, 15, 30, and 35) and SRPM92 (BPMs 20 and 40) were not included in the LOCO fit. LOCO was setup to fit 34 BPM gains, the 20 corrector kicks (or in this case the milliradian kick per amp in the corrector), and the 20 PSR quadrupole gradient strengths. The corrector roll and BPM coupling
The measured ORM. The black lines separate the ORM into the four quadrants with the x-x quadrant in the forefront and the y-y quadrant in the back.

are not fit in this LOCO analysis.

The orbit response per unit current in the corrector was fit with LOCO. When LOCO is employed in this manner, it fits for the corrector current to kick strength conversion instead of the absolute corrector kick strength. Fitting directly for the current to corrector kick strength is preferred for direct comparison with Ref. [11].

LOCO converges to a result in two iterations. The initial comparison of the measured and baseline model ORMs yields a $\chi^2$/DOF of 188. The difference in the measured and LOCO fitted model ORMs after three iterations is shown in Fig. 4.7. When compared to the measured ORM, the final LOCO fitted model ORM yields a $\chi^2$/DOF of 4.89.

LOCO has reduced the difference between the x-x and y-y quadrants of the measured minus the model ORM to the level of the measurement noise in the coupling quadrants of the ORM, which are unchanged in the LOCO fit since the BPM coupling and quadrupole and corrector rolls for not considered in the LOCO fit. The largest differences between the measured and LOCO fitted ORM lie in the diamond type BPMs and in the coupling quadrants of the ORM where the orbit response is domi-
nated by measurement spread. The average absolute value of the difference between the measured and LOCO fitted ORM is .027 mm. This result is outstanding. The larger \( \chi^2/\text{DOF} \) must be due to the estimate of the BPM measurement error which was limited by the intrinsic BPM resolution.

The LOCO fitted BPM gains are plotted in Fig. 4.8. All of the LOCO fitted BPM gains are within 10\%. The BPMs with the gains furthest from one are the horizontal diamond-type BPMs, BPMs 17 and 19. This is not too surprising considering the complicated splitting and combining of the BPM electrode signals. What is surprising is that the vertical diamond type BPMs (BPMs 37 and 39) fit BPM gains very close to one. Also note that the horizontal BPMs mostly fit gains lower than one, while most vertical BPMs fit gains slightly larger than one.

Remember that the BPM gains were independently measured by comparing the measured betatron amplitude function and betatron oscillation amplitude at each BPM, as discussed in Sec. 3.5. Compare the BPM gains from the LOCO fit with the relative BPM gains in the form of an action calculation shown in Fig. 3.20. The action calculation of Sec. 3.5 suggests that the vertical diamond-type BPMs are outliers in
4.1 Orbit Response Matrix (ORM) Analysis

Figure 4.8: The LOCO fitted BPM gains for all BPMs included in the LOCO analysis. The vertical line separates horizontal (left) and vertical BPMs (right), and the horizontal dashed line plots a gain of one.

The LOCO BPM gain fit does not distinguish between the 4″ and 6″ BPMs or between BPMs where the betatron oscillation amplitude is larger and BPMs where the betatron oscillation is small. This could be because the BPM gain does not depend on beam signal strength on the BPM electrodes.

The LOCO analysis of the ORM also fits the corrector kick strength employed during the measurement of the ORM. But for this analysis, the ORM and all of the corrector kicks have been normalized to a one amp change in the corrector. So the results of the LOCO fit are the current to milliradian kick conversions for each corrector, plotted in Fig. 4.9.

The initial current to milliradian kick conversions for the horizontal correctors were calculated from the slope of the current to \( \int Bdl \) magnetic field conversion evaluated at the design operating current value. While the two C-magnets have their own current to \( \int Bdl \) magnet field conversions, all of the other horizontal correctors were evaluated with the only existing current to \( \int Bdl \) magnetic field map for the common PSR bending magnet, Nancy. Corrector 1 (SRBM01) has a different current
4.1 Orbit Response Matrix (ORM) Analysis

Figure 4.9: (Color) The initial (blue circles) and LOCO fitted (green squares) corrector current to milliradian kick conversions. The vertical line separates the horizontal (left) and vertical correctors, right.

The LOCO analysis fits all of the horizontal corrector current to milliradian kick conversions about 20% less than the initial values. While this could be a result of the history of the corrector current and may be influenced by hysteresis, the systematic mistreatment seems to indicate that the initial current to corrector kick strength in the baseline model is incorrect.

The LOCO fit also yields smaller current to kick conversions for the vertical correctors. It is interesting that LOCO does not distinguish between the 7" and 11" vertical correctors. It is definitely intuitive that the two different types of vertical correctors would have different current to kick conversions, so why is LOCO unable to fit that result? The studies in Sec. 4.1.1 definitely suggest that LOCO is capable to kick conversion than the other common PSR benders because the operational set point for SRBM01 is 100 A less.

The vertical corrector conversions are based solely on the vertical corrector type either 7" or 11" to accommodate larger beam pipes. Correctors 14, 18, and 19 are the 11" vertical correctors.
4.1 Orbit Response Matrix (ORM) Analysis

of such distinction. Perhaps the LOCO fit does not converge to the parameters of the real machine.

The last and most important set of LOCO fitted parameters are the quadrupole gradients. The initial quadrupole gradients are derived from the current to normalized gradient length field maps. Figure 4.10 plots the percentage difference between the initial and LOCO fitted quadrupole gradients. LOCO fits an average change in the focusing quadrupoles (odd number quadrupoles) of \(-2.2\%\), while the average change in the defocusing quadrupoles is \(-2.2\%\).

\[
\begin{align*}
\text{LOCO Change in Quad Strengths [\%]} \\
\text{Focusing Quadrupoles} \\
\text{Defocusing Quadrupoles}
\end{align*}
\]

It was expected that in order to recover the vertical tune LOCO would change the defocusing quadrupole strengths more than the focusing quadrupole strengths. Actually recall that, one of the methods to obtain the proper betatron tunes with the baseline model was to multiply the focusing and defocusing quadrupole strengths by a few percent. This result is obtained by the LOCO fit.

While the difference in the initial and LOCO fitted quadrupole gradients in the focusing quadrupoles may be recovering the uncertainty in the current to gradient
length conversions, as explained in Sec. 3.2, there is no explanation for the difference of quadrupole gradients in the defocusing quadrupoles. It could be that the quadrupole strengths were the wrong model parameter to fit in the ORM analysis.

4.1.6 Comparison of the LOCO fitted and baseline models

In order to verify the LOCO fitted model as the improved model of the PSR, the LOCO fitted model needs to be compared to the baseline model and tested against the measured betatron tune and betatron amplitude functions. The tune comparison is shown in Tab. 4.2 and the beta function comparison is displayed in Tab. 4.3.

<table>
<thead>
<tr>
<th>Measured, Baseline, and LOCO Fitted Model</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Tune Comparison</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Measured</td>
</tr>
<tr>
<td>Horizontal</td>
<td>.19150</td>
</tr>
<tr>
<td>Vertical</td>
<td>.19793</td>
</tr>
<tr>
<td>( rms ) measurement spread ( [3.447 \times 10^{-4}, 3.238 \times 10^{-4}] )</td>
<td></td>
</tr>
</tbody>
</table>

**Table 4.2:** The measured horizontal and vertical betatron tunes compared with predictions from the baseline and the LOCO fitted models.

The LOCO fitted model corrects for the baseline model’s over focusing in the vertical and produces a predicted vertical tune that matches the measurement. The LOCO fitted model also improves the horizontal tune prediction compared to measured. The remaining difference between the LOCO fitted model and measured tunes can be a result of quadrupole hysteresis incurred during the PSR tuning process. Remember that it was observed that a change of only 20 A in the quadrupole would cause the quadrupole hysteresis to change the tune by \( \sim 0.005 \), Sec. 3.4.3 I. So, the LOCO fitted model passes the first test of model verification by predicting better tune values than the baseline model.
The second test for the LOCO fitted model is the beta function prediction. Since there are many measured beta functions, a $\chi^2$/DOF comparison between the measured and model predicted beta functions is employed to quantify the goodness of the model predictions. There is no fitting in this comparison between the measured and LOCO fitted model predicted beta functions. The $\chi^2$ is employed as a means to compare 2 vectors of data. However statistically improper, the systematic error due to the uncertainty in the quadrupole current to gradient length conversions on the measured beta functions is applied as the measurement error in the $\chi^2$ calculation. The results of the $\chi^2$/DOF comparison for both the baseline and the LOCO fitted model are displayed in Tab. 4.3.

<table>
<thead>
<tr>
<th>$\chi^2$/DOF between Measured and Model Predicted Beta Functions</th>
<th>Baseline</th>
<th>LOCO</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Beta Functions</td>
<td>11.965</td>
<td>12.679</td>
</tr>
<tr>
<td>Horizontal Beta Functions</td>
<td>19.574</td>
<td>19.003</td>
</tr>
<tr>
<td>Vertical Beta Functions</td>
<td>4.3556</td>
<td>6.3550</td>
</tr>
<tr>
<td>Systematic Error Large Beta [.4960 m, .4035 m]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Systematic Error Small Beta [.0504 m, .1153 m]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 4.3:** Comparison of the measured and model beta functions represented as a $\chi^2$. The systematic error on the measured beta function is applied as the measurement error in the $\chi^2$ calculation.

Although the LOCO fitted model predicts better betatron tunes than the baseline model, it does not predict the measured beta functions as well as the baseline model. The LOCO fitted model is able to predict slightly better horizontal beta functions than the baseline model, but to do so, the LOCO fitted model does 50% worse in yielding the measured vertical beta functions. In order to obtain the correct vertical betatron tune, the LOCO fit decreased the defocusing quadrupole strengths about $\sim2.5\%$, but this also leads to a worse model prediction of the vertical beta functions.
The LOCO fitted model does not pass the second test for model verification and is not the improved model that was hoped for. But how can this be when LOCO has been resulted in positive improvements to the models at so many other accelerators[30]? Well, LOCO not yielding an improved model is not due to a problem with the LOCO algorithm. It could be that the baseline model is so incorrect that LOCO is not able to produce the correct parameters. This is not likely because the baseline model is able to predict both the dispersion and beta functions well. Most likely, the above result is because the element that produces the extra vertical focusing in the model was not included in the LOCO fit. There are two other possible locations for the missing vertical focusing, the fringe fields of the PSR extraction septa and the edge focusing of the PSR benders.

4.1.7 ORM and LOCO summary

The orbit response matrix was successfully measured in the PSR. LOCO was applied to modify the baseline model quadrupole gradient strengths in order to fit the measured and model ORMs. The LOCO results indicated a $\sim 2.5\%$ systematic decrease in the defocusing quadrupole strengths. This $\sim 2.5\%$ discrepancy was not found in the current read backs of the defocusing quadrupole power supply or its current output.

A LOCO fitted model was created by applying the LOCO fitted quadrupole strengths to the baseline model. The LOCO fitted model was able to reproduce the measured vertical betatron tune within $10^{-4}$ while slightly improving the horizontal tune prediction. However, the LOCO fitted model was unable to preserve the quality of the baseline model’s prediction of the beta functions because the LOCO fitted model produced beta functions further from measurement than the baseline model. This result indicates that the LOCO fitted model is not the improved model of the PSR that was hoped for.
The accelerator element that is mishandled in the baseline model is not the quadrupole. The search of the additional vertical focusing will continue in the study of the extraction septa fringe fields and in the edge focusing of the rectangular PSR dipoles.

4.2 Septum Characterization

Although the baseline model of the PSR is able to reproduce the measured betatron amplitude functions (Sec. 3.4), it does not predict the measured vertical betatron tune. There are three possible sources of vertical focusing in the PSR that are either not in the baseline model or may not be handled properly in the model. The possible sources of additional vertical focusing are the PSR quadrupoles (Sec. 4.1), the fringe fields of the extraction septa, and the edge focusing from the rectangular horizontal bending magnets, Sec. 4.3. This section will concentrate on the focusing properties of the extraction septa fringe fields.

There are two extraction septa in the PSR. RODM01 is a true septum magnet with a current DC copper sheet septum to separate the circulating beam path and the extracted beam path. RODM02 is a window frame dipole with more iron on the inner side of the magnet than the outer side. Both PSR extraction septa bend the beam harder than the circulating beam such that the extracted line is located inside the ring of the PSR. The first non-septum bending magnet in the Ring Out (RO) line is a vertical dipole magnet that bends the beam up and out of the PSR area. The circulating beam passes along side both of the PSR extraction septa. Actually, part of the outer iron of RODM02 was removed to allow room for the circulating beam pipe, as shown in Fig. 4.11.

The circulating beam feels the affects of the fringe fields and leakage fields of the PSR extraction septa and receives a kick each revolution around the ring. The fringe
fields originate from the magnetic field escaping the ends of the magnets, which is not surprising because the main coils extend longitudinally past the iron in an unclamped Rogowski-type configuration, while the leakage fields are derived from the magnetic field lines that issue from the iron generally on the sides of the magnets due to the finite permeability of the iron. However in this thesis, fringe field and leakage field will be applied interchangeably for the magnetic fields of the extraction septum felt by the circulating beam. This is because the beam feels the effects of the integral magnetic field, so it is impossible to untangle the influence of the fringe fields from the leakage fields in these beam-based measurements.

Both PSR extraction septa are equipped with a trim coil to augment or diminish the amount of magnetic field penetrating the iron. The trim coils are meant to minimize the leakage fields from the extraction septa but may be operated to enhance such an effect. While the operation set point for the current of the PSR extraction septa is relatively fixed, the trim coil current may be corrected to tune for the desired effect. Both extraction septa trim coils are energized by a ±10 A power supply.

The lowest three magnetic multipole components of the PSR extraction septa fringe fields were measured by the beam. The beam measures the integrated dipole, quadrupole, and sextupole strengths of the kick received by both the fringe and

**Figure 4.11:** The 3D model of RODM02 employed in simulations of the multipole components of the fringe fields. Courtesy of D. Barlow.
leakage fields of the extraction septa. It is of operational interest to measure the multipole components of the septa fringe fields as a function of trim coil current. This yields a means to minimize the multipole component of choice.

This section is divided into the following subsections: Sec. 4.2.1, *Measurement setup and data analysis*, describes the experimental setup of the PSR and the procedure applied to calibrate LDPM03 and measure the natural chromaticity as well as the magnet multipole components of the extraction septa fringe fields as a function of trim coil current and introduces the analysis applied to the data in order to obtain the multipole components of the septa fringe fields including fitting schemes to calibrate LDPM03 and to measure the dispersion function, natural chromaticity, and chromaticity along with possible sources and mitigations of systematic errors; Sec. 4.2.2, *Results*, describes the quality of the calibration of LDPM03 and the results of the baseline dispersion and natural chromaticity measurements and reports the results of the multipole component measurement of the extraction septa fringe fields as a function of trim coil current along with fourth order polynomial fits for inclusion in the model of the PSR, lastly the measured multipoles of the RODM02 fringe fields are compared with 3D magnetic field simulations; and Sec. 4.2.3, *Summary*, restates the highlights of this section.

### 4.2.1 Septum characterization measurement setup and data analysis

Three measurements of the magnetic multipole components of the PSR extraction field were performed during the 2008 and 2009 run cycles. These include the accelerator developments of November 14, 2008, July 26, 2009, and September 26, 2009. The beam position monitor (BPM) data from the November 14, 2008 development was of poor quality with large amplitude variations and offset drifts across the scans
of the turn-by-turn data and not usable. Due to limited time during the July 26, 2009 development shift, only one current polarity of the trim coils was measured for an incomplete data set. Finally, a complete dataset with good BPM data was obtained during the September 26, 2009 development. Thus, the data and analysis presented in this section will be limited to the September 26, 2009 development.

The PSR was setup for single shot accumulation and near-on-axis injection at 20 Hz as in the RingScan reproducibility measurement described in Sec. 2.2. First a RingScan reproducibility measurement of 100 RingScans was acquired as a means to check the performance of the data acquisition programs, the BPM measurement, and the measured injection offset obtained by the central control room (CCR) BPM program.

A dispersion function measurement was performed second with the center the CO method described in Sec. 3.3.3. The purpose of this dispersion measurement was to calibrate LDPM03, a BPM in the high dispersion section of the transport from the linac to the PSR. Many measurements of the momentum are needed to measure the chromaticity as a function of trim coil current, and the quickest way to measure the momentum is via a position measurement with a calibrated LDPM03.

For the dispersion measurement, three different momentum settings were applied by changing the phase of the last two accelerating modules in the linac (Mod 47 and Mod48): a baseline momentum setting and both a plus and minus momentum setting with average changes in the horizontal CO in the PSR of $-1.8$ mm and $3$ mm respectively. An increase in beam momentum will push the horizontal CO out, which is negative in the coordinate system of the PSR. A Save Accel ring documentation and 20 RingScans were taken for each momentum setting. Fifty beam positions at LDPM03 were also recorded for each momentum setting. This dispersion measurement also revealed the range of momentums that could be applied for the later chromaticity measurements.
After data was taken to calibrate the dispersion function and on-momentum position at LDPM03, measurements were made to characterize the magnetic multipole components of the PSR extraction septa. The absolute multipole components as a function of trim coil current are desired, so it was necessary to take data with both septa off and their trim coils set to zero current as a baseline measurement. In order to run the PSR with both extraction septa turned off, the extraction septa were bypassed in the run permit. The repetition rate was also reduced to 4 Hz to protect the current sheet septum of RODM01 and other beam line components that the extracted beam crashes into with the extraction septum magnets turned off. Turning both septa off perturbed the CO moving the horizontal average CO from 0 mm to \(-0.8\) mm. The CO was not corrected because most of the septa characterization measurements would be performed with one septum on and closer to the setup parameters.

With both PSR extraction septa off and the trim coils set to zero current, a natural chromaticity measurement and baseline dispersion function measurement were performed by applying six different momentum settings. Once again the beam momentum was varied by moving the phase set point of Mod 47 and Mod 48. A baseline momentum setting, two plus, two minus, and a return to the baseline momentum setting were applied in the natural chromaticity and baseline dispersion function measurement. These momentum settings correspond to average horizontal COs in the PSR of \(-0.8\) mm for the baseline momentum setting, \(0.6\) mm and \(2.3\) mm for the two minus momentum settings, \(-2.2\) mm and \(-2.6\) mm for the two plus momentum settings, and \(-0.7\) mm for the return to baseline momentum setting. For each momentum setting, a Save Accel machine documentation was recorded, 20 RingScans were taken, and 25 position measurements at LDPM03 were saved.

It is interesting that the return to baseline momentum setting and the original baseline setting are not at the same momentum even through Mod47’s and Mod48’s phase set points were returned to their original values. This could be evidence of a
sort of “hysteresis” in the accelerating cavities, but more likely this effect is due to a systematic shift in the output energy of the linac during the course of the natural chromaticity and baseline dispersion measurements, which took about 30 min. The shift in momentum between the baseline and return to baseline momentum settings was found to be very small with a shift in $\delta = -3.1198 \times 10^{-5}$ with respect to the design momentum.

Next, with the return to baseline momentum setting from the natural chromaticity and baseline dispersion measurement, RODM02 was reset to its operational set point of 2349 A. Then data was taken to probe the dipole and quadrupole multipole components as a function of the trim coil current. The trim coil power supplies operate between $\pm 10$ A. Nine different trim coil currents were applied to measure the dipole and quadrupole multipole components of the extraction septum fringe fields, $0$ A, $\pm 2.5$ A, $\pm 5$ A, $\pm 7.5$ A, and $\pm 10$ A. For each trim coil current setting a Save Accel ring documentation was saved, 20 RingScans were made, and 25 position measurements at LDPM03 were recorded to measure the beam momentum. It took about 50 minutes measure the beam at the nine different trim coil current settings, which is less than the magnitude of the pulse-to-pulse momentum variations, Sec. 2.8.

To measure the sextupole component of extraction septa fringe fields felt by the circulating beam, the same measurements for the dipole and quadrupole multipole components were collected (a Save Accel ring documentation, 20 RingScans, and 25 position measurements at LDPM03) at the same trim coil currents ($0$ A, $\pm 2.5$ A, $\pm 5$ A, $\pm 7.5$ A, and $\pm 10$ A) but at different momentums. Aside from the return to baseline momentum setting, where the data was taken during the measurement of the dipole and quadrupole magnetic components, both minus momentum settings and one positive momentum setting from the natural chromaticity and baseline dispersion measurement were applied, corresponding to average horizontal COs of .6 mm, 2.3 mm and $-2.2$ mm when both septa were off. Due to experimental time considerations,
only one positive momentum setting was applied.

After all of the characterization measurements were performed for RODM02, RODM02 was turned off and the trim coil current set back to zero. A single baseline momentum setting measurement was made for RODM01, a Save Accel ring documentation, 20 RingScans, and 25 position measurements at LDPM03. Then RODM01 was set to its operational set point of 2025 A and the magnetic multipole characterization measurements performed for RODM02 (described above) were repeated for RODM01.

The characterization of the extraction septa leakage fields is the most complex experiment reported in this thesis. Many different measurements are needed to obtain the dipole, quadrupole, and sextupole multipole components of the extract septa fringe fields. The resolution of the sextupole component is most involved because baseline dispersion function, natural chromaticity, and chromaticity (which in turn needs tune and momentum) measurements are required. The RingScan data provides CO information for measurements of the dipole component and the baseline dispersion function as well as tune data for measurements of the quadrupole component, natural chromaticity, and chromaticity measurements. Information of the fractional momentum deviation is also needed for the dispersion function, natural chromaticity, and chromaticity measurements. Since so many momentum measurements are required for characterization of the extraction septa fringe fields, it was decided that converting position data from LDPM03, a BPM in the high dispersion region of the transport from the linac to the PSR, would be the fastest way to obtain the needed fractional momentum deviations. In turn, the calibration of LDPM03 requires the CO information from the RingScan measurements as well as TOF measurements.

The RingScan data was fit to a cosine wave and the data acquisition errors were identified and removed from the dataset as described in Chap. 2. The average CO at each BPM and the average betatron tune in each direction were extracted from the
4.2 Septum Characterization 267

cosine wave fits. The error on the measurement average ($\sigma_{\text{ave}}$) is calculated by Eq. (2.10).

The data at LDPM03 is a series of position measurements, so no fitting is applied to this data. The set of position data is simply averaged and the error on the average position at LDPM03 is calculated by Eq. (2.10). The intrinsic position resolution of LDPM03 is .0106 mm and .0103 mm in the horizontal and vertical respectively, Sec. 2.1. If the error on the average position at LDPM03 is less than the intrinsic BPM resolution, the error on the average position is set to the intrinsic position resolution of LDPM03.

As with the energy correction in the setup of the PSR, the TOF measurements needed for the calibration for LDPM03 were measured on an oscilloscope of the fast current monitor SRWC41 in CCR. The TOF is a delay between the $n^{\text{th}}$ zero crossing of the “moving” 2.8 MHz (the 72.07 subharmonic of the 201.25 MHz linac frequency) revolution frequency of the PSR and the $n^{\text{th}}$ beam passage. For the calibration of LDPM03, 1000 turns were employed in the TOF measurement.

The center the CO method was employed to convert the TOF measurement to a fractional momentum deviation. Unfortunately, this experiment was performed before the experiment comparing the momentum compaction factor and center the CO methods for measuring the momentum reported in Sec. 3.3.3. As reported in Sec. 3.3.3, the momentum compaction factor would have been quicker and suffered from less systematic error.

One of the main causes of systematic error in the center the CO method is due to the assumption that the baseline CO lies on the reference trajectory, is at the design momentum, and that this baseline CO can be reproduced by centering the CO for all of the momentum settings. However, the baseline TOF measurement for the calibration of LDPM03 reveals that the beam is late compared to the “moving” 2.8 MHz revolution frequency by 33.6 ns. How can this be when the energy was
corrected during setup of the PSR just 10 minutes prior to this baseline dispersion measurement?

The horizontal CO is well centered with an average radius of $2.38 \times 10^{-2}$ mm. So, what most likely happened during the setup of the PSR is that the CO was centered, and then the energy was corrected, which drove the horizontal CO off center. The CO was again centered using the orbit control knob SRMP999, but the path length of this last centered CO was different enough to create a 33 ns time delay after 1000 turns. The energy should have been corrected again to obtain the design orbit and momentum. The $-33$ ns TOF measurement for the baseline momentum corresponds to a fractional momentum deviation from the design momentum of $-3.206 \times 10^{-4}$ and an average horizontal CO radius of .5 mm for the design momentum orbit. When the CO was centered for a second time, the change in the energy of the central orbit was probably overlooked due to the small change in the design momentum orbit and the baseline orbit.

While a $-33$ ns TOF for 1000 turns is acceptable during operations, this does present a unique problem when applying the center the CO method for momentum measurement. The center the CO method assumes a centered on-momentum baseline measurement. Using the present baseline momentum with $\delta = -3.206 \times 10^{-4}$ will introduce a systematic error to all momentum measurements on the order of the baseline fractional momentum deviation from the design. However, the center the CO method does not care what the design momentum is, so as a means of circumventing this systematic error, the fractional momentum deviations are calculated with respect to the baseline momentum and then converted to fractional momentum deviations with respect to the design momentum.

As an example, assume a design momentum $p_0$ and a baseline momentum which is different than the design momentum $p_b$ such that the fractional momentum deviation
of a third momentum, \( p \), with respect to the design momentum is

\[
\delta_0 = \frac{p - p_0}{p_0}.
\] (4.2)

and the fractional momentum deviation with respect to the baseline momentum is

\[
\delta_b = \frac{p - p_b}{p_b}.
\] (4.3)

Defining the fractional momentum deviation of the baseline momentum with respect to the design momentum as

\[
\delta_{0b} = \frac{p_b - p_0}{p_0},
\] (4.4)

the fractional momentum deviation with respect to the design momentum can be written as a function of \( \delta_b \) and \( \delta_{0b} \),

\[
\delta_0 = \delta_b + \frac{p}{p_0} \delta_{0b}.
\] (4.5)

The derivation of Eq. (4.5) makes use of an expansion that assumes that \( \delta_{0b} \) is small.

The systematic errors due to the baseline momentum setting not being equal to the design momentum may be avoided by applying Eq. (4.5) to the fractional momentum deviation results of the center the CO method, \( \delta_b \).

The dispersion function and on-momentum position at LDPM03 can be found by fitting a line to the beam position at LDPM03 as a function of the fractional momentum deviation with respect to the design momentum. Interestingly, both of these quantities possess random measurement errors. To take both the horizontal and vertical measurement errors into account in the fitting of a line, the fitting scheme introduced in Appx. A.2 is applied to the data. The \( \chi^2 \) for this fitting scheme, Eq. (A.18), is repeated here for convenience

\[
\chi^2 = \sum_i \left[ \left( \frac{x_i - \eta_i}{\xi_i} \right)^2 + \left( \frac{y_i - f(\eta_i; \vec{a})}{\epsilon_i} \right)^2 \right].
\] (4.6)
A maximum likelihood (ML) error analysis is performed to obtain the fitting errors of and correlations between the on-momentum CO, dispersion function, and the fitted fractional momentum deviations.

Once the dispersion function and on-momentum position at LDPM03 have been calculated, LDPM03 is calibrated and may be employed to measure the fractional momentum deviation by converting position data at LDPM03 to $\delta$. For this process, the fitting errors on the on-momentum position and dispersion function from calibrating LDPM03 become systematic errors.

While a calibrated LDPM03 may be employed to calculated the fractional momentum deviation which then may be applied to a line fit by Eq. (4.6), it is better to only use measured quantities in such an analysis. Thus for the baseline dispersion measurement, a line is fit to the CO position in the PSR as a function of the horizontal position at LDPM03. The fitting scheme for this analysis is a slightly modified version of Eq. (4.6) derived in Appx. A.4. The xy fitting scheme in Appx. A.4, Eq. (A.30), is repeated here for convenience,

$$
\chi^2 = \sum_i \left[ \left( \frac{x_{LDi} - \eta_i}{\sigma_{LDi}} \right)^2 + \left( \frac{x_{PSRi} - a - b\eta_i}{\sigma_{PSRi}} \right)^2 \right].
$$

Likewise, a line may be fit to the measured tune as a function of horizontal position at LDPM03 for measurements of the chromaticity and natural chromaticity. A thorough study of employing a calibrated LDPM03 as a momentum measurement momentum in application to the dispersion function fit is reported in Appx. A.4.

In order to limit extreme beam steering from the unknown dipole component between septum-on and septum-off measurements, the CO was centered with both septa on. One particular concern was extreme injection offsets yielding large betatron oscillations and possible BPM saturation due to the dipole kick changing the CO at the foil, the injection point. Since most of the measurements in the characterization of the septa fringe fields would be performed with one septum on, it was believed
that this method would yield the best beam quality for the RingScan measurement. However, this means that a dipole kick was introduced when the septa were turned off for the baseline measurements of the characterization of the multipole components of the septa fringe fields. The initial CO for these baseline measurements were [−6.91 mm, 2.50 mm] at RODM01 and [−4.35 mm, 1.78 mm] at RODM02. Thus, the magnetic field expansion is referenced to these initial CO coordinates and not to the center of the beam pipe, which is more desired.

The initial COs for the baseline measurements at both septa is negative in the horizontal indicating that the beam is further out and away from the source of the septa leakage fields. This indicates that the dipole component of the magnetic field expanded about the reference trajectory at the center of the beam pipe will be larger than that measured in this experiment because the reference trajectory is closer to the source of the fringe fields. If one assumes a decay in the magnetic field of one over the transverse distance from the septum (1/r) or one over the transverse distance from the septum square (1/r^2), it can be stated that the both the quadrupole and sextupole components will also be stronger at the reference trajectory then measured in the expansion around the initial baseline COs in this experiment because the first and second derivatives are larger for smaller r.

The measurement of the dipole component of the fringe field employs CO data at the baseline momentum. All changes in the CO are referenced to the baseline CO when both septa are off. The other CO data was taken with one septum on and at nine different trim coil currents: ±10 A, ±7.5 A, ±5 A, ±2.5 A, and 0 A. The difference in two COs due to a dipole kick may be equated by Eq. (1.61). Equation (1.61) is applied to fit a dipole kick strength (θ) to the change in the CO as was done in the beam-based alignment analysis of Sec. 3.6, Eq. (3.23). This fit can be solved analytically by linear regression. The measured average tune and the baseline model beta functions and betatron phase advances are applied in the fit.
The quadrupole component of the fringe field is measured by,

\[ \Delta(KL) = \frac{4\pi \Delta \nu}{\beta(s_0)}, \]  

(4.8)

where \( \Delta(KL) \) is the normalized gradient length in units of \([m^{-1}]\). Equation (4.8) may be derived from the betatron tune shift due to a quadrupole field error, Eq. (1.49). Like the dipole component measurement, the measurement of the quadrupole component only employs data taken at the baseline momentum. The changes in the betatron tune are referenced to the baseline tune measurement with both septa off. The other tune measurements were taken with one septum on and nine different trim coil currents: \( \pm 10 \text{ A}, \pm 7.5 \text{ A}, \pm 5 \text{ A}, \pm 2.5 \text{ A}, \) and \( 0 \text{ A} \). The model beta function at the septum is applied in the calculation of Eq. (4.8) for the quadrupole component strength.

The strength of the sextupole component of the septa fringe fields can be solved by recognizing the natural chromaticity term in Eq. (1.59) and assuming the only sextupole in the ring is at the location of the septum fringe fields,

\[ K_2(s_0) = \pm \frac{4\pi}{\beta(s_0)D(s_0)}(\mathcal{C} - \mathcal{C}_n), \]  

(4.9)

where the \( \pm \) is for horizontal and vertical sextupole strengths respectively representing the fact that a horizontally focusing quadrupole field is a vertically defocusing quadrupole fields, \( D(s_0) \) is the baseline dispersion function at the septum (measured at the upstream BPM (SRPM91) and propagated to the septum), \( \mathcal{C} \) and \( \mathcal{C}_n \) are the chromaticity and natural chromaticity respectively. The assumption that the only sextupole in the ring is at the location of the septum fringe fields is not a bad assumption. While there are most likely sextupole magnetic field components in all of the PSR benders (especially in the C-magnets), these sextupole components have already been accounted for in the measurement of the “natural” chromaticity.

The chromaticity at each trim coil current was measured at four different momentum settings corresponding to average horizontal COs of \(-.8 \text{ mm} \) (baseline), \( .6 \)
mm, 2.3 mm and −2.2 mm when both septa were off. A line was fit to the tune as a function of the horizontal position at LDPM03 by the fitting schemes described in Eq. (4.7).

### 4.2.2 Septum characterization results

The characterization of the multipole components of the PSR extraction septa requires an accumulation of many measurements of varied quantities. Tallied in this section are the results of such measurements, including: the calibration of LDPM03; measurements of the baseline dispersion function and natural chromaticity; the dipole, quadrupole, and sextupole multipole component strengths as a function of trim coil current; and finally comparison of the multipole measurements of RODM02 with 3D magnetic field simulation.

#### I  Calibration of LDPM03

The dispersion function and on-momentum position at LDPM03 were calibrated to provide quick measurements of the beam momentum for the rest of the septa fringe field characterization experiment. The quality of the calibration of LDPM03 is crucial for measurement of the sextupole component of the septa fringe fields because the baseline dispersion function, natural chromaticity, and chromaticity, which are all fit to momentums measured by the calibrated LDPM03, are applied in the calculation of the sextupole strength. Any error in the calibration of LDPM03 will propagate to the rest of the experiment as systematic errors.

The center the CO method (Sec. 3.3.1 II was employed to measure the fractional momentum changes for the calibration of LDPM03. The systematic errors involved with this procedure are discussed in detail in Sec. 3.3.3 II. It is believed that all of the systematic errors are much smaller than the measurement uncertainty from the
TOF measurement. CO data was not taken for the centered COs where the TOF measurements were made for comparison, but if the reproducibility of the centered CO discussed in Sec. 3.3.3 II was the same for this measurement, this systematic error is well below measurement error. The largest possible systematic error for the calibration of LDPM03 is due to the off-design momentum baseline measurement. However, it is believed that this systematic error was mitigated by applying the center the CO method to calculated fractional momentum deviations with respect to the baseline momentum and then converting to fractional momentum deviations with respect to the design momentum by applying Eq. (4.5) as discussed in Sec. 4.2.1.

To calibrate the dispersion function and on-momentum position at LDPM03, a line is fit to the horizontal position data at LDPM03 as a function of the $\delta$’s measured via the center the CO method by the fitting scheme in Eq. (4.6). The line fit calibrating LDPM03 is shown in Fig. 4.12. The fit finds the dispersion function at LDPM03 to be $-4.26 \pm 0.16$ m, and the fitted on-momentum position is $0.10 \pm 0.21$ mm. The fit for the dispersion at LDPM03 yields a very good $\chi^2$/DOF of 0.87263. The difference between the fitted ($\eta$’s) and measured $\delta$’s is only a few times $10^{-5}$, which is on the order of the systematic error from the center the CO method for beam momentum measurement. Note that the initial guess does not differ much from the fitted values. The difference in the $\delta$ initial guess and the fitted values is on the order of $10^{-6}$. This indicates that although the fit is nonlinear, the initial guess given by a linear calculation is very good.

The calibration of LDPM03 can be compared with the LDPM03 calibration discussed in Sec. 3.3 which was found during a different accelerator development session. During the December 22, 2009 accelerator development the dispersion function and on-momentum position at LDPM03 were found to be $-4.56 \pm 0.13$ m and $-8.96 \pm 0.18$ mm respectively with the center the CO method to measure the beam momentum.

The dispersion function at LDPM03 from both calibrations only differs by about
two fitting errors. It is quite possible that the dispersion function at LDPM03 changed by .3 m in the three months between these measurements. The largest difference in the calibrations is the result from the on-momentum position at LDPM03. The calibration from the December development yields a value of \(\sim -9 \text{ mm}\) indicating that the beam was not well centered in the beam pipe at LDPM03. The calibration of LDPM03 for the septa fringe field characterization experiment shows a well centered beam at LDPM03 for the design momentum. The fitted on-momentum position is mostly affected by the steering in the transport and not directly linked to a lattice function.

Interestingly, the measured dispersion functions in the PSR from the December 22, 2009 accelerator development discussed in Sec. 3.3.3 also agree with the dispersion function measured while calibrating LDPM03 for the characterization of the extraction septa fringe fields. The ring documentation records also show that the PSR
magnets were operating at the same set points to within 1 A. This seems to indicate that the dispersion function in the PSR is reproducible over the course of a run cycle.

The data from the calibration of LDPM03 may also be employed to give a measure of the chromaticity with both septa operating at production set points. The trim coil current of RODM01 was $-1$ A, and the RODM02 trim coil current was set to $-5$ A. The measured chromaticity of the PSR as found was $-4.25 \pm .16$ in the horizontal, and in the vertical the measured chromaticity with both septa on was $-2.510 \pm 9.6 \times 10^{-2}$.

II Baseline dispersion function and natural chromaticity measurements

Once LDPM03 was calibrated, both PSR extraction septa were turned off, the trim coil currents set to zero, and the baseline dispersion and natural chromaticity measurements were performed. Due to time constraints, only one measurement of the baseline dispersion and natural chromaticity was made, so both extraction septa share the same baseline data. The fitting scheme described in Eq. (4.7) is employed to combine the measured horizontal position at LDPM03 and the CO or tune in the PSR to measure the dispersion function and natural chromaticity. These baseline measurements of the dispersion function and natural chromaticity will be applied to the calculation of the sextupole component of the septa fringe fields.

Figure 4.13 plots the resulting baseline dispersion measurement. The baseline dispersion function measured with both PSR extraction septa off agrees with the dispersion function measured while calibrating LDPM03 with both septa on indicating that the septa’s quadrupole component, which affects the lattice functions, is small as expected. Note that the dispersion function is negative. This is because in the coordinate system of the PSR, out is negative in the horizontal direction. Thus, a particle with a positive fractional momentum compared to the design momentum will
swing out in orbit and more negative in the horizontal coordinate.

Unfortunately the dispersion function is only measured at the BPMs, so it is necessary to propagate the measured dispersion function to the location of the extraction septa to obtain the $D(s_0)$ required in Eq. (4.9) to calculate the sextupole strength. The dispersion function measured at the two BPMs immediately upstream of the extraction septa (SRPM82 and 91) is employed in the calculation. The BPM immediately downstream of the extraction septa (SRPM92) is not employed because of its questionable horizontal CO and thus dispersion measurement.

In the straight sections of the PSR, where the bending radius is infinite, the dispersion function propagates just like normal phase space coordinates, so model transfer matrices are applied to describe the motion from SRPM82 to an intervening dipole (SRBM91) and from SRBM91 to SRPM91. However, when the bending radius is not infinite, the dispersion function receives something quite analogous to a dipole kick, and the nominal model transfer matrix cannot be applied to completely describe the propagation of the dispersion function. Additional terms describing the differ-
ent paths for particles of different momentum are required. The propagation of the dispersion function through a sector dipole is described in Eq. (1.70). One needs to be careful because of the different horizontal coordinate system of the PSR where out is negative compared to Eq. (1.70), where out is positive. Thus the measured dispersion functions are multiplied by $-1$, applied in Eq. (1.70), and then the results are multiplied $-1$ to transform back to the coordinate system of the PSR.

Combining the model transfer matrices and the transfer matrix in Eq. (1.70) describing the change in the dispersion function around a bend, the total transfer matrix from SRPM82 to SRPM91 may be obtained. The model transfer matrix from SRPM82 to SRPM91 allows one to solve for the first derivative of the dispersion function with respect to the longitudinal at SRPM91, $D'$. Another model transfer matrix is applied to propagate the measured dispersion function and its inferred first derivative from SRPM91 to the location of the septa fringe fields. The baseline dispersion functions, when both septa were off, at the septa were found to be $-2.78$ m and $-2.38$ m at RODM01 and 02 respectively. Note that as expected, the propagated baseline dispersion functions at the septa are between the measured dispersion functions at BPMs 19 and 20 (SRPM91 and 92) $-2.78$ m and $-1.47$ m respectively, Fig. 4.13.

Fitting the slope of the measured tunes versus the horizontal position at LDPM03 with both septa off and the trim coil currents set to 0 A yielded natural chromaticities of $-3.063 \pm 2.2 \times 10^{-2}$ in the horizontal and $-2.761 \pm 2.2 \times 10^{-2}$ in the vertical. Compare the measured natural chromaticities to the model chromaticities for the baseline linear model, $-2.6043$ for the horizontal and $-2.8943$ for the vertical. While the baseline model makes a fairly close prediction for the vertical chromaticity, the baseline model prediction for the horizontal chromaticity does not come close to the measured value. This is not too surprising as the baseline model cannot predict the vertical tune either. An additional difference between measured and model is that the measured “natural” chromaticity includes all sextupoles components in the PSR.
lattice except for those belonging to the extraction septa fringe fields. Additional sextupole components most likely arise in the dipole magnets, especially the C-magnets. The higher order multipoles of the horizontal bending magnets are not included in the baseline model of the PSR.

III Dipole component

The dipole component of the PSR extraction septa fringe fields is least important magnetic multipole for the purposes of modeling the ring because to first order the dipole field only affects beam steering and not lattice functions. During this analysis it has been assumed that the extraction septa are always operated at the same current value or very close to the design set point such that the multipole components of the fringe fields are dictated by the current of the septa trim coils. Because it describes the constant offset of the magnetic field, the dipole component is most affected by any change in the operating set point of the extraction septa.

Operationally, the kick received by the circulating beam due to extraction septa fringe fields is compensated by applying dipole kicks at all of the horizontal benders.

The dipole component of the septum fringe field is calculated by relating the change in the CO to the dipole oscillation form factor $G$ of Eq. (1.61). The change in the CO is measured with respect to the baseline CO measured with both septa off and the trim coil currents set to 0 A. The baseline model beta functions and phase advances from the septum to the BPMs are applied to obtain the dipole oscillation form factor for each septum. The baseline model beta functions at RODM01 are 11.53 m and 3.76 m for the horizontal and vertical respectively, while the horizontal and vertical beta functions at RODM02 are 6.28 m and 5.86 m respectively. It is believed that the systematic error introduced in the dipole strength calculation is small because the baseline model has been shown to reproduce measured beta functions.
4.2 Septum Characterization

It is of interest to know the minimum dipole kick strength that can be measured due to the septum fringe field. Following the procedure laid out in the beam-based analysis section of Sec. 3.6, it can be assumed that the maximum change in the CO needed to obtain a measurable dipole oscillation is .4 mm in the horizontal and .15 mm in the vertical. These are reasonable limits because they are between two and three $rms$ standard deviations in the CO measurement spread.

The minimum dipole kick strength may be calculated from the necessary maximum change in the CO. The dipole form factor $G$ may be approximately maximized by setting the cosine term of Eq. (1.61) to one and by applying the maximum beta function for $\beta(s)$. Applying these approximations, the minimum measurable dipole strengths are $3.54 \times 10^{-2}$ mrad in the horizontal and $2.33 \times 10^{-2}$ mrad in the vertical dipole kicks produced by the fringe fields of RODM01 and $4.80 \times 10^{-2}$ mrad in the horizontal and $1.86 \times 10^{-2}$ mrad in the vertical dipole kick due to the fringe fields of RODM02.

An example of the CO response to a dipole kick due to the leakage fields of RODM02 is shown in Fig. 4.14. Figure 4.14 shows the change in the horizontal and vertical COs for a trim coil current of $-10$ A. This dipole kick has produced very large oscillations in the horizontal with a maximum change in the CO 8 mm. The horizontal dipole is fit to be 1 mradian. Although the dipole kick appears to fit well in Fig. 4.14, the $\chi^2$/DOF is large, 48. This is because the propagated random measurement error on the change in the CO is nearly precision limited by the digitization of the BPM ADC, on the order of a few $10^{-2}$ mm. However, the coefficient of determination ($R^2$) is very close to one, .996. The dipole kick due to RODM02 is the reason why the baseline CO is off the reference trajectory at the locations of the extraction septa.

On the other hand, the vertical dipole oscillation is very small. The largest change in the vertical CO is .4 mm, which should be large enough to obtain a quality dipole kick strength as discussed above. The fitted vertical dipole kick strength is very
small (−.04 radians), indicating almost no vertical dipole affects in the fringe fields of RODM02. This result is expected because the magnetic field lines of the leakage fields should mostly be oriented vertically in the plane of the reference trajectory producing a horizontal dipole kick and not a vertical one. There is still a large $\chi^2$/DOF for the vertical dipole kick fit. The reasons for this are the same as for the horizontal fit; the calculation error on the change in the CO is very small.

Although the $\chi^2$/DOF is large for all of the dipole kick fits, the coefficient determination ($R^2$) is close to one for all but two of the horizontal fits, Fig. 4.15. The horizontal fit with the smallest $R^2$ occurred for the dipole oscillation from the fringe fields of RODM01 with the trim coil currents set to −2.5 A. The CO measurement for this trim coil setting happened during an extreme momentum deviation, a hickup, in the output of the linac. The beam position at LDPM03 was recorded to have changed by .6 mm compared to the baseline, corresponding to an average horizontal CO change of .3 mm in the PSR. This introduces a constant offset in the residuals of
the dipole kick fit. While there are small offsets for all of the dipole kick fits due to momentum variations, this affect is compounded by the small dipole kick such that the magnitude of the offset in the residuals is comparable to the maximum change in the CO. Thus, the sum of squares of residuals (SSR) becomes larger than the variance of the change in the CO, and the $R^2$ is negative.

![Graph showing $R^2$ from dipole kick fits](image)

**Figure 4.15:** (Color) The $R^2$'s resulting from the dipole kick fits to the dipole oscillations induced by the fringe fields of RODM01 (horizontal and vertical are blue circles and green squares respectively) and RODM02 (horizontal and vertical are red left pointing triangles and black right pointing triangles respectively) for each trim coil current.

The other horizontal dipole kick fit that yields an $R^2$ less than .9 fits the smallest dipole kick from the fringe fields of RODM02. This fit is less well constrained due to the weak dipole kick strength.

All of the vertical fits for RODM01 possess $R^2$'s close to one, while the vertical fits for RODM02, fitting vertical dipole strengths about half those of RODM01, are less well constrained and thus have smaller $R^2$'s.

The dipole kick strength as a function of trim coil current resulting from this analysis is fit to a fourth order polynomial. The fit of the fourth order polynomial
4.2 Septum Characterization

provides a tool for an operational model to convert a trim coil current input to a dipole strength. The fourth order polynomial is fit to the data by linear regression, and a ML error analysis is performed to obtain the fitting errors on the coefficients of the fourth order polynomial. The strengths of the dipole components of the fringe fields for RODM01 and RODM02 and their fourth order polynomial fits are shown in Figs. 4.16 and 4.17. The fitted polynomial coefficients are displayed in Tab. 4.4 for RODM01 and Tab. 4.5 for RODM02.

As expected the strengths of the vertical dipole component of the both extraction septa fringe fields is small, less than .1 mradian in absolute magnitude. Also note that the vertical dipole strength varies little as a function of trim coil current. This is also expected as the trim coils modify the amount of magnetic field leaking from the iron along the side of the magnet. It is most likely that the vertical dipole component results from the small vertical offset in the baseline CO compared to the reference trajectory. Because the baseline CO is not in the plane of the reference trajectory
4.2 Septum Characterization

<table>
<thead>
<tr>
<th>Coefficients of the 4th Order Fit to the Dipole Strengths of the Fringe Fields of RODM01</th>
<th>Horizontal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_0$ [mradian]</td>
<td>$-3.6584 \times 10^{-2} \pm 7.3500 \times 10^{-4}$</td>
</tr>
<tr>
<td>$p_1$ [mradian $A^{-1}$]</td>
<td>$-2.3517 \times 10^{-2} \pm 1.5403 \times 10^{-4}$</td>
</tr>
<tr>
<td>$p_2$ [mradian $A^{-2}$]</td>
<td>$1.7351 \times 10^{-4} \pm 4.0456 \times 10^{-5}$</td>
</tr>
<tr>
<td>$p_3$ [mradian $A^{-3}$]</td>
<td>$-1.0519 \times 10^{-5} \pm 1.9364 \times 10^{-6}$</td>
</tr>
<tr>
<td>$p_4$ [mradian $A^{-4}$]</td>
<td>$-2.6718 \times 10^{-7} \pm 3.8188 \times 10^{-7}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Vertical</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_0$ [mradian]</td>
</tr>
<tr>
<td>$p_1$ [mradian $A^{-1}$]</td>
</tr>
<tr>
<td>$p_2$ [mradian $A^{-2}$]</td>
</tr>
<tr>
<td>$p_3$ [mradian $A^{-3}$]</td>
</tr>
<tr>
<td>$p_4$ [mradian $A^{-4}$]</td>
</tr>
</tbody>
</table>

Table 4.4: The fitted coefficients of the 4th order polynomial fit to the dipole strength of the fringe fields of RODM01 as a function of trim coil current with fitting error.

or the elevation mid-plane of the extraction septa, there exists a horizontal magnetic field (which produces the vertical dipole component) in the extraction septa fringe fields. As proof of this, note that the strength of the dipole kick due to the fringe fields of RODM01, where the baseline CO 2.5 mm, is greater than that of RODM02, where the vertical baseline CO is 1.78 mm. It is interesting that both the vertical dipole strength and vertical baseline CO of RODM01 are about twice as large as their counterparts for RODM02.

In addition, the geometry of the end fields of the extraction septa is complicated because the main coils extend longitudinally past the iron of the magnet. Since the extraction septa are oriented at a slight angle compared to the circulating beam pipe, it is easy to imagine that the longitudinal end fields of septa are observed as horizontal
magnetic fields by the circulating beam.

The horizontal dipole kick strength of the RODM02 fringe fields was expected to be strongest because not only is RODM02 oriented at a larger angle to the circulating beam pipe such that more end field is seen by the circulating beam but some of the iron on the outside of RODM02 was removed to allow the circulating beam pipe to run alongside the magnet. The horizontal dipole kick produced by the fringe fields of RODM02 can be as large as 1 mradian when the trim coil current is $-10 \text{ A}$. This corresponds to a maximum CO response of 8 mm, Fig. 4.14. In order to make more room for the circulating beam pipe, the thickness of the iron on the circulating beam side of the RODM02 is less than the inner side of the magnet. It appears that even with the trim coils operating at maximum current (10 A), the beam still feels a horizontal dipole kick due to the fringe fields of RODM02.

The horizontal dipole kick strength of the fringe fields of RODM02 appears to be linear for large positive trim coil currents, between 5 and 10 A. For trim coil currents
4.2 Septum Characterization

Table 4.5: The fitted coefficients of the 4th order polynomial fit to the dipole strength of the fringe fields of RODM02 as a function of trim coil current with fitting error.

<table>
<thead>
<tr>
<th>Coefficients of the 4th Order Fit to the Dipole Strengths of the Fringe Fields of RODM02</th>
<th>Horizontal</th>
<th>Vertical</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_0$ [mrad]</td>
<td>0.75062 ± 3.4191×10^{-4}</td>
<td>$p_0$ [mrad]</td>
</tr>
<tr>
<td>$p_1$ [mrad A^{-1}]</td>
<td>-4.8250×10^{-2} ± 7.1678×10^{-5}</td>
<td>$p_1$ [mrad A^{-1}]</td>
</tr>
<tr>
<td>$p_2$ [mrad A^{-2}]</td>
<td>-2.3734×10^{-3} ± 1.8832×10^{-5}</td>
<td>$p_2$ [mrad A^{-2}]</td>
</tr>
<tr>
<td>$p_3$ [mrad A^{-3}]</td>
<td>5.7767×10^{-5} ± 9.0206×10^{-7}</td>
<td>$p_3$ [mrad A^{-3}]</td>
</tr>
<tr>
<td>$p_4$ [mrad A^{-4}]</td>
<td>2.5621×10^{-6} ± 1.7791×10^{-7}</td>
<td>$p_4$ [mrad A^{-4}]</td>
</tr>
</tbody>
</table>

less than 2.5 A the dipole strength seems to saturate and then levels out around 1 mradian for trim coil currents between -7.5 and -10 A. The trim coils drive the magnetic field leaking from the iron for negative currents, but at some point the effect of the few amp trim coil saturates the system, yielding very little change to the leakage fields for trim coil currents less than -5 A. The horizontal dipole kick strength of the fringe fields of RODM02 is the only dipole strength observed to possess this nonlinear behavior with respect to the current of the trim coil.

The strength of the horizontal dipole kick of the fringe fields of RODM01 is considerably weaker than that of RODM02 but still large compared to the vertical dipole kick strengths of either septa fringe field. The horizontal dipole kick strength of the fringe fields of RODM01 cross zero as a function of trim coils current, and like the
vertical dipole strengths of both extraction septa fringe fields is rather linear.

One method to get a handle on the quality of the fourth order polynomial fits is to compare the fitting error with the magnitude of the coefficients. For instance, the constant term of the polynomial fit \( p_0 \) is well constrained for all of the polynomial fits because the fitting error on \( p_0 \) is at least two orders of magnitude smaller than the fitted \( p_0 \). Likewise, the higher order polynomial coefficients tend to be less well constrained. As an example, the fitting error on the vertical \( p_4 \) for RODM02 is more than half of the fitted magnitude of that coefficient, while the vertical \( p_4 \) for RODM01 possesses a fitting error larger than itself. These findings agree with what is observed graphically in Figs. 4.16 and 4.17; the horizontal dipole strength of RODM02 is the only dipole strength with some amount of curvature as a function of trim coil current, while the other dipole strengths are mostly linear with respect to trim coil current.

If it is the interest of operations to minimize the horizontal dipole component of the extraction septa fringe fields, the trim coil current of RODM01 should be set to \(-1.54\) A, and the trim coil current of RODM02 should be set to maximum, \(10\) A. Compare the trim coil currents that are measured to minimize the dipole component of the septa fringe fields with the set points suggested in the PSR hand book[31]: RODM01 trim coil current \(7.10\) A and RODM02 trim coil current \(10\) A. The current recommended by the PSR Handbook for the trim coil of RODM02 is supported by this measurement. However, this experiment suggests a different current set point for the trim coil of RODM01 than that of the PSR Handbook.

It is important to remember that the dipole components of the extraction septa fringe fields are dependent on the current set point of the extraction septa. It may be that the value suggested for the RODM01 trim coil current in the PSR Handbook is appropriate for a different current set point for the main coil of RODM01.
IV Quadrupole component

The quadrupole components of the PSR extraction septa are the most important multipole because the quadrupole term affects the lattice functions (betatron amplitude function and dispersion function) and the betatron tune. Remember the hope is to find a strong quadrupole term in the extraction septa fringe fields to explain the deviation in the measured and model prediction of the vertical betatron tune.

Any tune shift caused by the fringe fields of the extraction septa are compensated during operations with the main quadrupole power supplies. But the betatron amplitude modulation introduced by this additional quadrupole field is not corrected for production. It is doubtful that the quadrupole multipole component of the septa fringe fields can explain the $\sim 20\%$ beta-beat observed in the vertical betatron amplitude function because the baseline model, which does not include the affects of the septa fringe fields, reproduces the measured beta-beat.

An additional quadrupole field due to the septum fringe fields will induce a shift in the betatron tune proportional to the beta function at the location of the extraction septum and the strength of the quadrupole field, as defined in Eq. (1.49). The kicks observed by the circulating beam due to the fringe fields of the septa are represented as thin lens kicks in the improved model located at the upstream corner (the corner closest to the circulating beam pipe) of the extraction magnets because this is where the magnitude of the fringe fields is largest. Following this assumption, the baseline model beta functions at the upstream corners of the extraction septa are applied to Eq. (4.8) in order to calculate the quadrupole components. As with the measurement of the dipole components, it is believed that the baseline model beta functions do not introduce significant systematic errors in the calculation.

Like the dipole component strengths, the quadrupole component strengths as a function of trim coil current are fit to a fourth order polynomial to be included in
the improved model of the PSR. The fitting errors and correlations of the fitted polynomial coefficients are calculated through a ML error analysis. The results of the quadrupole component strengths as a function of trim coil current and the fourth order fits are plotted in Figs. 4.18 and 4.19. The fitted coefficients for the fourth order polynomial fits are reported in Tabs. 4.6 and 4.7 for RODM01 and RODM02 respectively.

Figure 4.18: (Color) The horizontal (blue circles) and vertical (green squares) quadrupole component strengths of the fringe fields of RODM01 with three uncertainties. The 4th order polynomial fits to the horizontal and vertical quadrupole strengths are the solid red and black lines respectively, while the fits corrected for the off-momentum baseline tune measurement are the dashed cyan (horizontal) and magenta (vertical) lines.

The horizontal quadrupole strength should be equal and opposite the vertical quadrupole component because a horizontally focusing quadrupole field is vertically defocusing, $K_x = -K_y$ where $K$ is the normalized gradient of a quadrupole field. However, this is not observed in Figs. 4.18 and 4.19. The horizontal and vertical quadrupole strengths should be symmetric about zero, but there seems to be a constant offset of $\sim -0.002 \text{ m}^{-1}$. This offset can be calculated by averaging both the
4.2 Septum Characterization

Table 4.6: The fitted coefficients of the 4th order polynomial fit to the quadrupole strength of the fringe fields of RODM01 as a function of trim coil current with fitting error.

| Coefficients of the 4th Order Fit to the Quadrupole Strengths of the Fringe Fields of RODM01 | Horizontal          | Vertical          |
|                                                                                         |                     |                   |
| $p_0 \ [\text{m}^{-1}]$                                                               | 7.8897×10^{-3} ± 1.6514×10^{-5} | -7.1088×10^{-3} ± 1.1276×10^{-4} |
| $p_1 \ [\text{m}^{-1}\text{A}^{-1}]$                                                  | 5.0544×10^{-4} ± 3.5048×10^{-6} | -3.7443×10^{-4} ± 2.6782×10^{-5} |
| $p_2 \ [\text{m}^{-1}\text{A}^{-2}]$                                                  | -1.0108×10^{-5} ± 9.1237×10^{-7} | -1.5836×10^{-5} ± 6.2999×10^{-6} |
| $p_3 \ [\text{m}^{-1}\text{A}^{-3}]$                                                  | -3.2425×10^{-7} ± 4.3676×10^{-8} | -2.1656×10^{-6} ± 3.5952×10^{-7} |
| $p_4 \ [\text{m}^{-1}\text{A}^{-4}]$                                                  | 4.4141×10^{-8} ± 8.6046×10^{-9} | -3.7736×10^{-9} ± 6.1776×10^{-8} |

horizontal and vertical quadrupole strengths. Amazingly enough, the offset can also be estimated by the measured natural chromaticity. The offset in the quadrupole component strengths corresponds to the chromatic affect on the baseline tune because the baseline measurement was taken off design momentum.

The asymmetry in the quadrupole strengths can be corrected by defining the on-momentum betatron tunes from the fits of the natural chromaticity measurement as the baseline tune values. Multiplying the natural chromaticities by the baseline fractional momentum deviation with respect to the design yields a change in the baseline tune compared to the design tune of 9.8184×10^{-4} in the vertical and 8.9023×10^{-4} in the horizontal. Applying these baseline $\Delta \nu$’s to Eq. (4.8) results in offsets in the horizontal and vertical quadrupole strength of 1.9659×10^{-3} and 1.9077×10^{-3} respect-
Figure 4.19: (Color) The horizontal (blue circles) and vertical (green squares) quadrupole component strengths of the fringe fields of RODM02 with three uncertainties. The 4th order polynomial fits to the horizontal and vertical quadrupole strengths are the solid red and black lines respectively, while the fits corrected for the off-momentum baseline tune measurement are the dashed cyan (horizontal) and magenta (vertical) lines.

tively. The average of the horizontal and vertical offsets is the observed offset from zero in the measured quadrupole strengths of the fringe fields in Figs. 4.18 and 4.19. The chromatic affect on the baseline tune measurement can account for $\sim 90\%$ of asymmetry in the quadrupole multipole strength measurement. Correcting for the asymmetry due to the offset only effects the constant coefficient of the fourth order polynomial fit, $p_0$. The offset corrected constant coefficients are reported in Tabs. 4.6 and 4.7. Note, the constant coefficients for the horizontal and vertical fits are nearly equal and opposite.

Note that the strongest quadrupole error is produced at RODM01 when the trim coil current is set to 10 A. The normalized gradient length of the quadrupole component of the fringe field of RODM01 at this setting is $\pm 0.013 \text{ m}^{-1}$, focusing in the horizontal while defocusing in the vertical. The strength of this quadrupole error cor-
4.2 Septum Characterization

| Coefficients of the 4th Order Fit to the Quadrupole Strengths of the Fringe Fields of RODM02 |
|-------------------------------|------------------|
| **Horizontal**                |                  |
| $p_0$ [m$^{-1}$]              | $-5.7039 \times 10^{-3}$ $\pm$ $3.4847 \times 10^{-5}$ |
| $p_1$ [m$^{-1}$A$^{-1}$]      | $7.9370 \times 10^{-4}$ $\pm$ $6.6011 \times 10^{-6}$ |
| $p_2$ [m$^{-1}$A$^{-2}$]      | $5.9659 \times 10^{-5}$ $\pm$ $1.7992 \times 10^{-6}$ |
| $p_3$ [m$^{-1}$A$^{-3}$]      | $-9.5264 \times 10^{-7}$ $\pm$ $8.1789 \times 10^{-8}$ |
| $p_4$ [m$^{-1}$A$^{-4}$]      | $-1.6356 \times 10^{-7}$ $\pm$ $1.6564 \times 10^{-8}$ |
| **Vertical**                  |                  |
| $p_0$ [m$^{-1}$]              | $4.9970 \times 10^{-3}$ $\pm$ $4.0120 \times 10^{-5}$ |
| $p_1$ [m$^{-1}$A$^{-1}$]      | $-8.3768 \times 10^{-4}$ $\pm$ $8.8116 \times 10^{-6}$ |
| $p_2$ [m$^{-1}$A$^{-2}$]      | $-3.2997 \times 10^{-5}$ $\pm$ $2.3078 \times 10^{-6}$ |
| $p_3$ [m$^{-1}$A$^{-3}$]      | $1.8076 \times 10^{-6}$ $\pm$ $1.1557 \times 10^{-7}$ |
| $p_4$ [m$^{-1}$A$^{-4}$]      | $7.3430 \times 10^{-8}$ $\pm$ $2.2360 \times 10^{-8}$ |

**Table 4.7:** The fitted coefficients of the 4th order polynomial fit to the quadrupole strength of the fringe fields of RODM02 as a function of trim coil current with fitting error.

responds to the strength of the quadrupole errors introduced at the third shunt setting for the large beta function quadrupoles and somewhere between the third and maximum shunt setting for the small beta function quadrupoles during the quadrupole perturbation method for measuring the PSR betatron amplitude functions.

The change in the betatron tune between the fitted on-momentum tune values from the natural chromaticity fits and the tunes measured at this maximum strength quadrupole component is $1.1315 \times 10^{-2}$ in the horizontal and $-2.8117 \times 10^{-3}$ in the vertical. The horizontal tune change is much larger than the vertical because RODM01 is adjacent to a horizontally focusing quadrupole (SRQF91) where the horizontal beta function is large and the vertical beta function is small. The change in the horizontal tune is quite significant. However, the change in the vertical is comparable to
4.2 Septum Characterization

the observed change in the tune during the quadrupole perturbation method due to quadrupole hysteresis effects.

The largest change in the vertical tune in comparison to the fitted on-momentum tune from the natural chromaticity measurements occurred when the trim coils of RODM02 were set to $-10$ A. The largest change in the vertical tune due to the extraction septa fringe fields was found to be $-4.721 \times 10^{-3} \pm 6.4 \times 10^{-5}$. While this change in the vertical tune is about twice that observed due to the hysteresis in the quadrupoles during the quadrupole perturbation method measurement discussed in Sec. 3.4, it is also an order of magnitude smaller than the deviation between baseline model and measured vertical tunes. So it appears that the missing vertical focusing term in the baseline model is not due to the exclusion of the fringe fields of the extraction septa in the baseline model of the PSR.

Also note, the additional wiggles in the fourth order fit to the vertical quadrupole component of the fringes fields for RODM01, Fig. 4.18. The wiggles are not observed in the horizontal fit, which should be equal but opposite of the vertical fit because for quadrupoles $K_x = -K_y$. It could be that these wiggles are artifacts of the fitting. They may be an indication that too many orders are applied in the polynomial fit. To verify this, an F-test or equivalent method should be employed to determine the correct number of fitting parameters needed to fit the data. A fourth order polynomial fit was arbitrarily chosen for the operations conversion of trim coil current to multipole component and no statistical test applied. The fourth order polynomial fit was chosen because the current to gradient length conversions for the PSR quadrupoles are also fourth order fits[10, 11].

The ratios between the fitted polynomial coefficients and their fitting errors can lend some insight into the source and concreteness of the wiggles. Observe that the largest deviation between the horizontal and vertical fitted coefficients for the quadrupole strengths of the RODM01 fringe fields in Tab. 4.6 is in the coefficient for
the third order of the polynomial, \( p_3 \). The vertical fit has a surprisingly large fitted value for \( p_3 \) compared to the horizontal. It is believed that this is the source wiggles in the vertical fit. This order of magnitude difference between the fitted horizontal and vertical \( p_3 \) coefficient does not appear in the fit for the quadrupole strengths of the fringe fields at RODM02.

All the same, the wiggles still are not observed in the horizontal fit to the quadrupole strength of the fringe fields at RODM01, which should be a mirror image of the vertical. The quadrupole strength fits for the RODM02 fringe fields do not show wiggles either even though there is much more curvature in these fits. This indicates that there is a difference in the horizontal and vertical tune measurements for data at RODM01. Actually, it appears that the measured vertical quadrupole strengths are fairly linear for negative trim coil currents like their horizontal counterparts. However, some nonlinearity in the quadrupole strength as a function of trim coil current seems to slip in for positive trim coil currents. This also correlates with a decrease in the vertical injection offset. The smaller injection offset is enough to send the amplitude of the vertical betatron oscillation in the BPMs with small beta functions to less than 1 mm. The measured turn-by-turn data becomes dominated by the single turn BPM measurement error, and the fitted phase parameter is not well constrained, which yields a large error on the tune measurement through correlation.

The small amplitudes seem to affect some BPMs more than others. The BPMs where the vertical beta function is large are not affected. But of the BPMs located where the vertical beta function is smallest, the tune measurement at BPMs 33, 37, and 39 (SRPM61y, 81y, and 91y) was affected most when the RODM01 trim coils were set to positive currents. BPMs 37 and 39 are the vertical diamond-type BPMs and have been shown to posses larger spreads in the fitted parameters. The vertical diamond-type BPMs also have the largest intrinsic BPM position resolution due to the digitization of the voltage from the analog front end (AFE). It is unknown
why BPM 33 should also yield bad tune measurements other than the amplitude of betatron oscillation is smallest at this BPM. However, all scans at these BPMs fit small vertical betatron oscillation amplitudes. The tune measurement at BPM 33 was consistently less than the tune measurements at the other vertical BPMs, while the tune measurements at BPMs 37 and 39 were consistently larger than the tune measurements at the other vertical BPMs. Such behavior in the vertical tune measurement can yield a measured tune distribution with two uneven shoulders on either side, Fig. 4.20.

![Figure 4.20](image_url)

**Figure 4.20:** A histogram of the measured vertical tune when the RODM01 trim coil current was set to 10 A. The average of the tune distribution is $1.8662 \pm 8.8619 \times 10^{-5}$, and the rms standard deviation is $1.6025 \times 10^{-3}$.

Although it is easy to suppose that the uneven shoulders on either side of the measured vertical tune distribution can shift the average of the distribution and cause the wiggles in the vertical fit of the quadrupole component of the RODM01 fringe fields, there are too many tune measurements in the main body of the distribution for the average to be offset by a mere 20 or 40 scans in the shoulders. As proof of this, a routine was added to the RingScan analysis scripts to check the fitted amplitude of betatron oscillation before the scan’s fitted tune was included in the total measured tune distribution. When BPMs with small fitted amplitudes are not included in the
total measured tune distribution, the average of the distribution changes only slightly and does not take the wiggles out of the vertical quadrupole strength of the RODM01 fringe fields.

A more likely explanation for the wiggles is the introduction of higher order affects in the tune measurement. The betatron tune is defined as the number of betatron oscillations per revolution. If the path length where to change considerably compared to the baseline measurement say due to the dipole kick of the extraction septa fringe fields, the measurement of the quadrupole strength would include this additional chromatic term describing the beam being off-momentum on this new orbit. To prevent this, the CO could be centered for each trim coil current. However, this was not done because centering the CO and removing the centering can be a messy business likely to introduce more systematic error than remove, and as shown in Fig. 4.18, the chromatic effects are small and not observable in the horizontal.

The change in path length due to a dipole kick is equal to the dispersion function at the location of the kick multiplied by the dipole kick strength, Eq. (1.65). The change in path length did not affect the tune measurement during the quadrupole perturbation method to measure the betatron amplitude functions reported in Sec. 3.4, but the dipole kick strengths due to the fringe fields of the extraction septa are an order of magnitude stronger than the dipole kicks received due to an off-center CO in the perturbed quadrupole. There is the additional complication of the dipole kick from the RODM01 fringe fields with positive trim coil current significantly altering the horizontal injection. The angle of horizontal injection was measured to be as large as $-0.9$ mrad. This induced large, 12 mm, amplitude oscillations in the horizontal which will also augment the orbit path length.

If it is the desire of operations to minimize the quadrupole component of the PSR extraction septa fringe fields, the trim coil current of RODM01 should be set to $-10$ A, and the trim coil current of RODM02 should be set to $5.33$ A. It may be that min-
4.2 Septum Characterization

imizing the quadrupole component of the extraction septa fringe fields is the correct set point for operations. Reference [26] concludes that the space charge emittance growth in the PSR is driven by the \( n = 4 \) harmonic of the envelope oscillations or beta-beating. If it can be assumed that minimizing the beta-beating will also minimize the \( n = 4 \) harmonic, then setting extraction septa trim coil currents to minimize quadrupole component should also lower the space charge emittance growth.

V Sextupole component

The sextupole multipole moment is the lowest order magnetic component that is nonlinear in the position from the center of the field expansion. Thus, it is not very important for an improved linear model of the PSR. However, data for the sextupole component was collected while the opportunity presented itself during this experiment. It would be a wonderful achievement if the improved model of the PSR could also predict the measured chromaticities.

The chromatic affects and thus the sextupole’s ability to correct the chromaticity are important during operations. During production, the beam is stored within a radio frequency (rf) bucket provided by a harmonic one bunching cavity. During the accumulation cycle, beam slowly rotates in longitudinal phase space exchanging a phase deviation with the synchronous particle for fractional momentum deviation with respect to the synchronous particle. The bucket height, or maximum \( \delta \) which can be stored in the rf bucket of the PSR, is determined primarily by the buncher voltage. During operations the buncher voltage is typically set to 7 kV, so the momentum spread in the accumulated beam can be as large as \( \pm 4.436 \times 10^{-3} \). The momentum spread in the stored beam translates to a tune spread of
±1.8843×10^{-2} in the horizontal and ±1.1075×10^{-2} in the vertical. Thus, the tune shift due to the large momentum spread and chromatic affects are much larger during production and important for operations.

The chromaticity was measured at each of the nine trim coil current settings with different momentum settings. The chromaticity is measured by fitting a line to the measured tune versus the position at LDPM03 exactly like what was done to measure the natural chromaticity. Although the sextupole strength as a function of trim coil current is the ultimate goal for inclusion in the improved model of the PSR, the measured chromaticity is a more intuitive quantity and will be discussed first in order to gain insight into the quality of the measurement.

The measured chromaticities as a function of the trim coil current for RODM01 and 02 are shown in Figs. 4.21 and 4.22. The measured natural chromaticities discussed in Sec. 4.2.2 II are [−3.0625, −2.7613]. The horizontal and vertical chromaticities measured with both septa on and the RODM01 trim coil current set to −1 A and the RODM02 trim coil current set to −5 A are −4.2479 and −2.4966 respectively as discussed in Sec. 4.2.2 I.

Comparing Figs. 4.21 and 4.22, it is apparent that turning the extraction septa on reduces the magnitude of the vertical chromaticity while increasing the horizontal chromaticity. It is also evident that the sextupole component of the fringe fields of RODM01 contributes most of the difference in the measured chromaticity between septa on and septa off. It is interesting that the chromaticities as a function of trim coil current generally trend negatively for RODM02 while the chromaticities trend in different directions at RODM01. The effects of the trim coils at RODM01 and 02 on the vertical chromaticity mostly cancel so that the difference in vertical chromaticity between septa on-off is much smaller than the difference in the horizontal chromaticity where the effects of the RODM01 and 02 trim coils add.

The chromaticity measurement is further complicated by the chromatic effects in
the tune measurement due to the path length change introduced in Sec. 4.2.2 IV. While it is true that the trim coil current may change the path length of the CO due to dipole kicks in the septa fringe fields, the change in momentum applied in the chromaticity measurement will most certainly affect the path length. The chromatic affect does not appear in the nonlinearity of the linear fits for the chromaticity measurement because the $R^2$ for every fit is above .98. However, the change in path length could result in a constant shift in the tune, in which case would not show in the statistics of the linear fits.

Knowing the fractional momentum deviation from the horizontal position measurement at LDPM03, the baseline model momentum compaction factor may be applied to calculate the change in path length due to the change in momentum. Also, the measured dipole kick strengths from Sec. 4.2.2 III and the propagated baseline
dispersion function from measurement at the location of the septa may be combined to find the change in the path length due to the dipole kick of the extraction septa fringe fields. These results may be included to correct the tune measurement for a given trim coil current and momentum setting. This correction has not been included in the analysis scripts for the sextupole measurement.

The chromatic affect from the change in path length effecting the tune measurement is most likely the cause of the jumps in the measured chromaticity as a function of trim coil current. This characteristic of the chromaticity measurement will continue into the calculation of the strength of the sextupole component of the septa fringe fields. An additional difficulty with the chromaticity fits is that the momentum data for a particular trim coil current was taken over the span of a four hour period. Many aspects of the machine may have changed in that time such as magnet power supply

**Figure 4.22:** (Color) The horizontal (blue circles) and vertical (green squares) chromaticities as a function of RODM02 trim coil current with three uncertainties, the horizontal and vertical natural chromaticities (red left and black right point triangles respectively) with three fitting errors, and the chromaticities at production set points (cyan up and magenta down pointing triangles respectively) with one fitting error.
4.2 Septum Characterization

One more concern for the chromaticity fits is that the septum trim coil current was exercised across the range of the trim coil power supply, ±10 A. This may have caused hysteresis effects in the septum magnet, where perhaps a trim coil current set point does not produce the same fringe field. The measurement procedure to take data at all trim coil current settings at one momentum setting before changing momentum settings was decided upon because it was believed that the trim coil current set point was more reproducible than the beam momentum.

The sextupole strength as a function of trim coil current was calculated by Eq. (4.9). Like the dipole and quadrupole multipole components, the sextupole strength as a function of trim coil current is fit to a fourth order polynomial for inclusion in the improved model of the PSR. A ML error analysis is employed to calculate the fitting errors on the fitted polynomial coefficients. The measured sextupole strengths and the fourth order polynomial fit for both extraction septa are plotted in Figs. 4.23 and 4.24. The fitted coefficients for the fourth order polynomial fits to the sextupole

![Diagram of Sextupole Strength](Image)
4.2 Septum Characterization

Coefficients of the 4th Order Fit to the Sextupole Strengths of the Fringe Fields of RODM01

<table>
<thead>
<tr>
<th></th>
<th>Horizontal</th>
<th>Vertical</th>
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</thead>
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<tr>
<td>$p_0$</td>
<td>$-3.2168 \times 10^{-1}$ ± $9.9179 \times 10^{-3}$</td>
<td>$-5.9000 \times 10^{-1}$ ± $2.7781 \times 10^{-2}$</td>
</tr>
<tr>
<td>$p_1$</td>
<td>$-1.0888 \times 10^{-2}$ ± $2.1348 \times 10^{-3}$</td>
<td>$-3.5462 \times 10^{-2}$ ± $6.0868 \times 10^{-3}$</td>
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<td>$p_2$</td>
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<tr>
<td>$p_3$</td>
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<td>$9.2547 \times 10^{-6}$ ± $7.8187 \times 10^{-5}$</td>
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<tr>
<td>$p_4$</td>
<td>$-7.3946 \times 10^{-6}$ ± $5.0839 \times 10^{-6}$</td>
<td>$1.0039 \times 10^{-5}$ ± $1.4732 \times 10^{-5}$</td>
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Table 4.8: The fitted coefficients of the 4th order polynomial fit to the sextupole strength of the fringe fields of RODM01 as a function of trim coil current with fitting error.

The results of the sextupole strength calculation shows that the extraction septa fringe fields are capable of producing significant sextupole component strengths.

It is expected, from Eq. (4.9), for the horizontal and vertical chromaticities to yield the same results for the sextupole strength. The calculated sextupole strengths for negative trim coil current agree, but the sextupole strength results diverge quite dramatically for positive trim coil currents. The most positive trim coils introduce large dipole kicks compared to where the PSR was optimized during the tuning process. In addition the momentum changes take the PSR far away from where the “natural” chromaticities were measured. Thus, it is believed that the samples additional sextupole fields at locations in the machine other than the extraction septa. These

strengths are recorded in Tabs. 4.8 and 4.9.
4.2 Septum Characterization

Figure 4.24: (Color) The horizontal (blue circles) and vertical (green squares) strengths of the sextupole component of the fringe field of RODM02 with three uncertainties. The 4th order polynomial fits to the horizontal and vertical quadrupole strengths are shown as the solid red and black lines respectively.

additional sextupole fields are not taken into account in the “natural” chromaticity measurement because the PSR has moved far away in parameter space.

Again the RODM01 vertical tune measurements are affected by the small vertical injection offsets when the trim coil current is positive. The vertical injection offsets decreased further for the positive momentum settings.

The strength of the sextupole component is definitely the most difficult multiple component to measure due to compound effects of dipole kicks, injection offset and momentum changes, tune shifts, large time delays between measurements, and possible hysteresis all from the fringe fields of the extraction septa which were being characterized. All of these effects combine to create the jumps in the measured sextupole strength as a function of trim coil current observed in Figs. 4.23 and 4.24.

As stated before, the sextupole component of the extraction septa fringe fields is not important for an improved linear model of the PSR. However, these measurements will most likely be revisited when the model is in a position to include sextupole
### 4.2 Septum Characterization

#### Table 4.9: The fitted coefficients of the 4th order polynomial fit to the sextupole strength of the fringe fields of RODM02 as a function of trim coil current with fitting error.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>$p_0$ [m$^{-2}$]</td>
<td>$-2.6123 \times 10^{-1} \pm 2.1445 \times 10^{-2}$</td>
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<tr>
<td>$p_1$ [m$^{-2}$A$^{-1}$]</td>
<td>$-3.6473 \times 10^{-2} \pm 4.5710 \times 10^{-3}$</td>
</tr>
<tr>
<td>$p_2$ [m$^{-2}$A$^{-2}$]</td>
<td>$-1.7128 \times 10^{-3} \pm 1.1897 \times 10^{-3}$</td>
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<tr>
<td>$p_3$ [m$^{-2}$A$^{-3}$]</td>
<td>$1.5838 \times 10^{-4} \pm 5.7533 \times 10^{-5}$</td>
</tr>
<tr>
<td>$p_4$ [m$^{-2}$A$^{-4}$]</td>
<td>$1.3366 \times 10^{-5} \pm 1.1273 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

Operationally, one might want to reduce the chromaticity of the PSR and thus the sextupole component of the septa fringe fields in order to reduce tune spread of the stored beam and nonlinear resonances introduced by the sextupole fields. If this is the case, the trim coil currents of RODM01 should be set to $-10$ A, while the trim coil current of RODM02 is set to $-5.32$ A.

### VI Comparison with simulation of RODM02

Reference [32] reports 3D simulation results of the multipole components of the fringe fields of RODM02. The multipole components were simulated with the main coils of RODM02 operating at 2300 A and at three different trim coil currents: 0 A, 5
A, and 10 A. The strengths of the multipole components reported in Ref. [32] are in units of absolute magnet field, i.e. units of Tesla. However, the measured multipole component strengths reported in this section are in units normalized to the magnetic rigidity \( (B\rho) \) of the beam. So in order to match the units of the multipole strengths reported in Ref. [32] with those measured and reported in this section, the multipole strengths reported in Ref. [32] have been divided by the PSR design \( B\rho \) and displayed in Tab. 4.10. For comparison the measurement results for the RODM02 fringe fields are restated in Tab. 4.11. There are several difficulties in comparing the measured results with the simulation from Ref. [32], all of which will be discussed below.

### Simulation Results for the Magnetic Multipole Strengths of the Fringe Fields of RODM02 from Ref. [32]

<table>
<thead>
<tr>
<th>Trim Coil Current</th>
<th>0 A</th>
<th>5 A</th>
<th>10 A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dipole [mrad]</td>
<td>-.4951</td>
<td>-.052</td>
<td>.2732</td>
</tr>
<tr>
<td>Quadrupole [m(^{-1})]</td>
<td>-.0082</td>
<td>-.0029</td>
<td>.0014</td>
</tr>
<tr>
<td>Sextupole [m(^{-2})]</td>
<td>-.1019</td>
<td>-.049</td>
<td>.00013</td>
</tr>
</tbody>
</table>

Table 4.10: The results of the multipole strengths of the fringe fields of RODM02 at three different trim coil currents divided by the PSR design \( B\rho \) with main coils operating at 2300 A from a 3D simulation reported in Ref. [32].

### Measured Magnetic Multipole Strengths of the Fringe Fields of RODM02

<table>
<thead>
<tr>
<th>Trim Coil Current</th>
<th>0 A</th>
<th>5 A</th>
<th>10 A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dipole [mrad]</td>
<td>.7295</td>
<td>.4567</td>
<td>.1141</td>
</tr>
<tr>
<td>Quadrupole [m(^{-1})]</td>
<td>-.0076</td>
<td>-.0027</td>
<td>.0034</td>
</tr>
<tr>
<td>Sextupole [m(^{-2})]</td>
<td>-.1828</td>
<td>-.4611</td>
<td>-.5187</td>
</tr>
</tbody>
</table>

Table 4.11: A restatement of the measurement results for the magnetic multipole components of RODM02 for trim coil currents of 0 A, 5 A, and 10 A from Secs. 4.2.2 III, IV, and V.

First compare the reported dipole component strengths from Tabs. 4.10 and 4.11. The first observation is that the dipole strengths are opposite in polarity and offset
from one another because the simulation results cross zero while the measured dipole strengths do not. The offset between the measured and simulated dipole strengths is most likely due to a difference in the operating current of the main coil of RODM02. RODM02 was set to 2300 A for the simulation, while the RODM02 main coil set point was 2347.72 A for the experiment. This is a 2.5% discrepancy in the current, and since the dipole component is the constant term of the magnetic expansion, it is most sensitive to the magnitude of the magnetic field present at the point of expansion. The amount of magnetic field at the expansion point is clearly a strong function of the operation current of the main coil of RODM02.

The difference in sign between measurement and simulation is due to an incomplete conversion from the field reported in Ref. [32] and the dipole kick strength, \( \theta \). Because of the coordinate system of the PSR where positive is in, the conversion from the integral field is

\[
\theta = -\frac{1}{B_\rho} \int \Delta B_y(l) dl
\]

with a minus sign. The minus sign is not present for calculating the dipole kick in a right-handed coordinate system [4].

Taking these facts into account, it is still hard to compare the measured and simulated dipole component because the difference in the current set point of the main coil of RODM02 has such a large influence on the dipole component strength. However, only the constant component of the leakage field should be affected by the difference in current of the main coil of RODM02. The shape of the decay in the leakage field as a function of transverse distance from the extraction magnet should be preserved so the higher order multipoles should not be affected.

The measured and simulated strengths of the quadrupole component of the magnetic field expansion agree well. The difference between measurement and simulation is only a few \( 10^{-4} \, \text{m}^{-1} \) for trim coil currents of 0 A and 5 A. The difference jumps to a few \( 10^{-3} \, \text{m}^{-1} \) for a trim coil current of 10 A. This is where, according to Fig. 4.19, the horizontal quadrupole strength is approximately equal to the saturated vertical quadrupole strength from negative trim coil currents. The large difference between
the simulated and measured quadrupole strength for a trim coil current of 10 A could be due to the mechanics of how the quadrupole strength saturates in the simulation model. The difference in main coil current could also affect this saturation. Nonetheless, agreement with simulation is encouraging for the measurement of the quadrupole strengths to be included in the model of the PSR.

Lastly, the strengths of the sextupole component do not compare between measurement and simulation, but recall that the sextupole measurement looses quality for positive trim coil currents because the PSR departed so far from the optimized tuned settings. The measured sextupole strengths are much stronger than the strengths from the simulation. The measured sextupole strengths also increase with greater trim coil current while the sextupole strengths from simulation decrease to almost nothing with a trim coil current of 10 A. The smaller sextupole strengths predicted by simulation are not able to reproduce the difference in the measured chromaticities between septa on and septa off.

There are additional complications in the comparison of measured and simulated sextupole strengths; the question revolves about which point the magnetic field is expanded in the Taylor series and how the components of the multipole expansion vary about different points of expansion. The multipole components of the fringe fields of RODM02 were measured about the baseline CO, which was \(-4.35\) mm in the horizontal and \(1.78\) mm in the vertical, while the simulation results are the expanded about the reference trajectory. However, Ref. [32] shows that the shape integral of the fringe field along the reference trajectory does not vary significantly within a \(-4\) mm deviation in the horizontal from the reference trajectory. This is probably why the measured and simulated quadrupole strengths agree so well.

There remains a final complication in the comparison of the measured and simulation. The coordinate system of the simulation is centered about the reference trajectory of the extracted beam at the longitudinal center of RODM02. The hor-
horizontal x-axis of the simulation is thus tangent to this point which differs from the horizontal axis of the reference orbit of the circulating beam by 10.7° because the extraction beam has been bent by this amount. This will stretch out the unit of length in the horizontal direction of the simulation compared to the measurement coordinate system based on the reference trajectory of the circulating beam.

### 4.2.3 Septum characterization conclusions

A full set of data was collected to characterize the magnetic multipole components of the PSR extraction septa fringe fields as observed by the beam. The magnetic multipole components of the septa fringe fields were expanded about an initial baseline CO with both septa off and trim coil currents set to zero via the Beth representation of the 2D transverse magnetic fields. Beam-based experiments to measure the lowest three multipole components of the septa fringe fields were performed. The magnetic multipole components were measured as a function of trim coil current and fit to a fourth order polynomial for inclusion as an operational parameter in the improved model of the PSR.

It was found that RODM01 possessed the strongest quadrupole component of .013 m\(^{-1}\) when the trim coil was set to 10 A. The largest shift in the vertical tune occurred between septa off and an RODM02 trim coil current of \(-10\) A. Unfortunately, the largest vertical tune shift was only \(4.453 \times 10^{-3}\) which is an order of smaller than that needed to correct the baseline model’s prediction of the vertical tune. To minimize the quadrupole component of the septa fringe fields, the trim coil currents should be set to \(-10\) A for RODM01 and 5.33 A for RODM02.

The additional vertical focusing element needed to correct the baseline model’s vertical tune prediction is not in the fringe fields of the extraction septa. However, the quadrupole component of the septa fringe fields is still included in the improved model
of the PSR. The baseline model does not include the focusing fields of the extraction septa fringe fields, so including these quadrupole fields in the model automatically provides a better model of the PSR.

4.3 Ray Tracing through the Edge Focusing

The last location where the baseline model’s treatment of the vertical focusing may be different than the real machine is in the edge focusing of the PSR benders. All of the horizontal bending magnets in the PSR are rectangular dipoles, so the beam is vertically focused as it enters and exits each dipole.

As introduced in Eq. (1.26) of Sec. 1.1.6 I, the edge focusing in the MAD model of a dipole is determined by three independent parameters: the angle between the normal of the magnet edge and the reference trajectory (edge angle), the gap height of the dipole, and the fringe field integral. The edge angle and magnet gap height are physical quantities and can be measured with a ruler and protractor. The fringe field integral is a bit more elusive. The values of the fringe field integral in the baseline model are derived from performing the fringe field integral in the 3D magnetic field simulations.

The baseline model includes effects of the edge focusing. Since the horizontal benders are rectangular dipoles, the edge angles in the baseline model are defined as one half of the total dipole bend angle. The gap height and fringe field integrals in the baseline model are derived from results of the 3D magnet field simulation results reported in Ref. [33].

As a means to check the edge focusing in the baseline model, parallel rays were traced through 3D magnetic field simulations of the PSR dipoles. The parallel rays were initially positioned on a transverse grid at the longitudinal center of the dipole. The grid spanned 4 cm in the horizontal and vertical with a ray every centimeter.
4.3 Ray Tracing through the Edge Focusing

So 25 parallel rays ranging between ±2 cm in the horizontal and vertical were traced through the downstream half of the PSR dipoles. All rays were given on-momentum velocities for the simulation. D. Barlow of AOT-RFE carried out the ray tracing simulations from a TOSCA 3D simulation of the PSR horizontal bending magnets. The results of the ray tracing simulation are reported in Refs. [34, 35, 36, 37, 38].

The results of the parallel ray tracing through a common PSR bender is shown in Fig. 4.25. The coordinate system of the simulation is locked to the center of the dipole. The axis of interest is the vertical axis, which is reported as is y-axis in the simulations. All of the parallel rays are initially aligned with the z-axis and perpendicular to the x-axis, which points out such that the rays are bent toward the −x-axis direction. The ray tracing simulation is iterated along the ray trajectory (s-axis) and records the ray’s position every centimeter.

The right plot of Fig. 4.25 shows the rays traced through the downstream half of
4.3 Ray Tracing through the Edge Focusing

a common PSR dipole and $\sim 0.8$ m of drift space in the 3D coordinate system of the simulation described above. The edge of the dipole is about $-20$ cm in $x$ and 135 cm in $z$.

The left plot of Fig. 4.25 displays a ray’s vertical position as a function of distance along the ray’s trajectory. The edge of the dipole is around 130 cm on the $s$-axis. It is at this point that the rays with nonzero initial vertical position are focused due to the dipole edge focusing. As expected from the discussion of rectangular dipoles in Sec. 1.1.6 I, the edge focusing provides a quadrupole-like focusing affect in the vertical with focusing strength depending linearly on vertical position. This is observed in Fig. 4.25 because the rays with initial vertical position of $\pm 2$ cm are bent about twice as hard as the rays with $\pm 1$ cm initial vertical position.

The focal length of the edge focusing maybe calculated from the ray tracing data. A line is fit by linear regression to the bent trajectory of the ray, roughly the portion of the ray between $s = 140$ and the end of the ray tracing. The focal length for the focusing of the ray is then the location in $s$ where the fitted line equals zero minus the location in $s$ where the fitted line equals the initial vertical position (where the ray receives the angular kick from the edge focusing). The calculated focal lengths from tracing rays though a common PSR bender are shown in Fig. 4.26.

The rays with initial vertical positions of zero are not focused because they propagate through the center of the quadrupole-like edge focusing field and thus do not lead to focal lengths of zero. For each initial horizontal starting position, the focal lengths are symmetric about the 0 cm initial vertical position. Notice that the focal length varies across the grid of initial positions. This magnetic aberration is small for a given initial horizontal position and is expected in real magnets. Note that the grid of initial positions extends to $\pm 2$ cm, further than the calibrated region of the BPM measurement, and much further than where the CO is located during typical operations.
4.3 Ray Tracing through the Edge Focusing

The focal lengths resulting from the ray originating from initial horizontal position 0 cm and initial vertical position ±1 cm should be compared with the focal length of the edge focusing in the baseline model. Fig. 4.26 shows that the model edge focusing focal length is about half a meter shorter than the focal length resulting from the ray tracing data. A shorter focal length indicates stronger focusing and a larger betatron tune.

The focal length of the edge focusing may be constrained in the model by altering any one or all of the three parameters that affect the edge focusing: edge angle, gap height, and fringe field integral. For simplicity only one of the parameters will be modified to impose the focal length results from the ray tracing simulation in the improved model.

The baseline model value for the edge angle may be incorrect because the edge

---

**Figure 4.26:** (Color) The focal lengths calculated from rays traced through a common PSR bender with initial positions horizontal positions 2 cm (blue circles), 1 cm (green squares), 0 cm (red left pointing triangles), −1 cm (black right pointing triangles), and −2 cm, magenta up pointing triangles. The dashed cyan line is the focal length of the edge focusing from the baseline model at the momentum of the ray tracing simulation.
angle should really be between the normal of the magnetic field gradient and the reference trajectory. Since the main coils of the PSR bending magnet extend further longitudinally than the iron, the magnetic field gradient and the edge of the magnet may not be parallel. However, the edge angle is not modified because inverting Eq. (1.25) results in a transcendental equation.

The baseline model value for the magnet gap height may be incorrect because the gap is not constant transversely and because there are shims at the magnet ends, which reduce the gap height. The shims are in place to limit higher order magnetic multipoles. Since imposing the focal lengths from the ray tracing can modify the gap height by several centimeters, it is not chosen as the model parameter to modify.

The fringe field integral is chosen somewhat arbitrarily as the parameter to constrain in order to impose the focal length results of the ray tracing simulation in the improved model. A comparison of the focal lengths and fringe field integrals for the different PSR horizontal benders and models is shown in Tab. 4.12.

<table>
<thead>
<tr>
<th>Coefficients of the 4\textsuperscript{th} Order Fit to the Quadrupole</th>
<th>Focal Length [m]</th>
<th>Fringe Field Integral</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnet</td>
<td>Baseline</td>
<td>Improved</td>
</tr>
<tr>
<td>Common</td>
<td>13.208</td>
<td>13.733</td>
</tr>
<tr>
<td>SRBM01</td>
<td>13.208</td>
<td>13.733</td>
</tr>
<tr>
<td>SRBM11</td>
<td>32.186</td>
<td>32.389</td>
</tr>
<tr>
<td>SRBM12</td>
<td>30.823</td>
<td>30.889</td>
</tr>
<tr>
<td>RIBM09</td>
<td>234.17</td>
<td>238.96</td>
</tr>
</tbody>
</table>

Table 4.12: The focal lengths and fringe field integrals for each type of PSR horizontal bender from the baseline and improved models.

The largest fractional difference in the edge focusing focal length occurs for the common PSR benders. Modifying the common PSR dipole edge focusing focal length has the most effect on the model vertical tune. There are several reasons for this: one,
most of the PSR benders are the common dipoles, two, the common PSR dipoles have the largest bend angle of the horizontal benders, and three, the edge focusing focal lengths are shortest for the common dipoles. The ray tracing simulations suggest a dramatic modification to the fringe field integral for the common PSR benders. However, a fringe field integral of .90291 is more representative of the unclamped “Rogowski” type geometry of the common PSR benders, Tab. 1.1.

The ray tracing data suggests the largest change in the edge focusing focal length at RIBM09. However, this modification has little effect on the model vertical tune because the edge focusing from RIBM09 is so weak due to the small bend angle, 6.8°.

Interestingly, the ray tracing simulation suggests that there is a very small additional horizontal focusing element in the edge focusing of the PSR benders. The horizontal focal length was found to be greater than 300 m and was not included in the improved model. This result indicates that the PSR benders are really rectangular dipoles with an edge angle equal to one half of the total bend angle such that the horizontal focusing of the sector dipole is completely canceled by the edge focusing.

The focal length results from ray tracing simulations are systematically longer than the focal lengths from the baseline model indicating an over focusing in the vertical at all of the PSR benders. This provides for the systematic mistreatment of the vertical phase advance observed in Fig. 1.6. Observe that the difference in measured and model vertical phase in Fig. 1.6 seems random and about the same amplitude as the horizontal difference in phase until BPM 26. BPM 26 (SRPM22) is the downstream BPM in section 2, the section immediately after the first common PSR dipole encountered after the foil. From this point onward, there is a systematic mistreatment of the model vertical phase advance, observed in Fig. 1.6 as a constant drift negative. It is interesting that this should coincide with the location of the first common PSR bending magnet encountered after the foil.

Note the fringe field integral parameter for SRBM01 in the improved model, Tab.
4.12. SRBM01 a common 36° bender but is operated at 100 A less than the other common PSR benders in order to bend the beam only 32.8°, which allows “room” for the merging magnet, RIBM09. A fringe field integral greater than one would suggest that the vertical component of the magnetic field in the fringe field would change sign. It is hard to understand how this could be, but it is a complicated situation since the main coils of SRBM01 extend well past the iron. Perhaps the normal of the gradient of the magnetic field is at a different angle than the iron such that SRBM01 is not really a rectangular dipole. However, the AT model is able to handle a fringe field integral parameter greater than one, and the model is constrained to produce the same focal length in the edge focusing as observed in the ray tracing data.

The difference in the measured betatron phase and the phase predicted from an improved model with edge focusing constrained to the values indicated by the ray tracing data is shown in Fig. 4.27.

![Figure 4.27:](Color) The measured horizontal (blue circles) and vertical (green squares) betatron phase minus the baseline model prediction referenced to the betatron phase at the first BPM after the foil, SRPM02. The red line marks zero.

The difference between the focal lengths from the 3D magnetic field integral cal-
calculation and the constrained focal lengths from the ray tracing has not been resolved. The same magnetic fields from the TOSCA simulation were applied in both calculations. Could it be that the model for the edge focusing applied in the AT program (Eqs. (1.25) and 1.26) is not the correct model for the PSR dipoles? The mechanics of the edge focusing of the PSR benders should be investigated further in the future. Nonetheless, the model focal length of the edge focusing from each PSR dipole may be constrained to the same as the focal lengths observed from the ray tracing. This modification to the baseline model improved the treatment of the vertical phase advance as observed in Fig. 4.27.

One also needs be careful when calculating the focal length from the ray tracing data as to make sure to find the focal length with respect to the coordinate along the beam trajectory ($\hat{s}$) and not the initial longitudinal direction of the simulation, $\hat{z}$. If the focal length of the edge focusing is found along the $z$-direction, the focal lengths in the baseline model are obtained. The difference between $\hat{s}$ and $\hat{z}$ is greatest for the case of the common PSR benders because their bending angle is the greatest.

### 4.4 Improved Model Verification

Three accelerator elements were identified as possible sources of addition vertical focusing in the baseline model: the quadrupoles, the fringe fields of the PSR extraction septa, and the edge focusing of the dipoles. Each of the previous sections in this chapter have addressed the vertical focusing in one of these accelerator components.

Now, combining the results from the previous sections, I can construct an improved model for the PSR. This improved model will be introduced (Sec. 4.4.1), established as an improved model by comparison of the baseline model’s predictions an the results of the supporting measurements (Sec. 4.4.2), and finally experimentally verified at three different PSR operational set points with additional supporting measurements,
4.4 Improved Model Verification

Sec. 4.4.3.

4.4.1 Improved PSR model

An enhanced model of the PSR may be created by considering the results of the model improvement experiments documented in this chapter and the lattice function measurements described in Chap. 3. The improved model of the PSR is founded on the baseline model, which is discussed in Sec. 1.3. Up to this point, only the baseline model has been applied for comparison with measurements in this thesis. All considerations made in the baseline model are also in the improved model. However, the improved model benefits from the experimental results described previously in this chapter were as the baseline model does not. The combined results of the ORM (Sec. 4.1), extraction septa characterization (Sec. 4.2), and ray tracing through the dipole edge focusing (Sec. 4.3) indicate the presence of two types of vertical focusing elements in the real machine that are not handled properly in the baseline model.

The LOCO fitted quadrupole strengths are not included in the improved model because the LOCO fitted model was not able to predict the betatron amplitude functions as well as the baseline model.

The PSR extraction septa characterization experiment measured quadrupole fields in the fringe fields of the extraction septa. The circulating beam observes these quadrupole focusing fields and receives the appropriate kick each revolution. Although the quadrupole fields of the septa fringe fields were found to be about .1 the strength of the PSR quadrupoles, the septa fringe fields were observed to modify the betatron tune by .005, which is about the prediction accuracy hoped for in the tune by the improved model. Thus, the improved model enhances the baseline model with the inclusion of the quadrupole component of the PSR extraction septa fringe fields. The septa fringe fields are modeled as thin quadrupoles located at the upstream
4.4 Improved Model Verification

outer corners of the exaction septa. The model makes use of the measured and fit extraction septa trim coil current to magnetic moment strength from the septa characterization experiment. Thus, the septa fringe fields are modeled operationally. The improved model reads a file containing the septa trim coil current, consults the current to gradient length fit, and defines a thin lens quadrupole with corresponding focal length.

The second modification between the baseline and improved PSR models is the handling of the edge focusing of the horizontal rectangular dipole magnets. The ray tracing through the PSR dipoles reveals focal lengths for the edge focusing different from those applied in the baseline model. The difference in the focal lengths is only about .5 m in the common PSR benders but this is enough to lower the model predicted tune to measured values. The focal lengths measured in the ray tracing are constrained in the improved model by defining a fringe field integral different than that of the baseline model.

4.4.2 Baseline and improved models compared with measurement

As with the LOCO fitted model in Sec. 4.1, the improved model must be shown to enhance the predictions of the baseline model in comparison to measured quantities before it is heralded as a better model. The baseline and improved models of the PSR are linear models, so the quantities of interest are the betatron tunes, betatron amplitude functions, and the dispersion function. The comparisons of the baseline and improved models with the measured tunes, beta functions, and dispersion function are displaying in Tab. 4.13.

The measured data compared in Tab. 4.13 is collected from three different measurements during two different development periods. The betatron tunes and betatron
<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>LOCO Fitted</th>
<th>Improved</th>
</tr>
</thead>
<tbody>
<tr>
<td>Betatron Tune:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>rms Measurement Spread</td>
<td>3.447×10^{-4}, 3.238×10^{-4}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Horizontal Error</td>
<td>−.005840</td>
<td>.00457</td>
<td>−.01210</td>
</tr>
<tr>
<td>Vertical Error</td>
<td>−.04716</td>
<td>−.03868</td>
<td>−.007477</td>
</tr>
<tr>
<td>Betatron Phase:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean Measurement Spread</td>
<td>.20 mradian, .18 mradian</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total $\chi^2$/DOF</td>
<td>0.700</td>
<td>0.397</td>
<td>0.189</td>
</tr>
<tr>
<td>Horizontal $\chi^2$/DOF</td>
<td>0.072</td>
<td>0.088</td>
<td>0.084</td>
</tr>
<tr>
<td>Vertical $\chi^2$/DOF</td>
<td>1.328</td>
<td>0.707</td>
<td>0.294</td>
</tr>
<tr>
<td>Beta Function:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean Systematic Error, Large Beta</td>
<td>.4960 m, .4035 m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean Systematic Error, Small Beta</td>
<td>.0504 m, .1153 m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total $\chi^2$/DOF</td>
<td>12.770</td>
<td>13.321</td>
<td>12.025</td>
</tr>
<tr>
<td>Horizontal $\chi^2$/DOF</td>
<td>20.928</td>
<td>21.932</td>
<td>18.553</td>
</tr>
<tr>
<td>Vertical $\chi^2$/DOF</td>
<td>4.611</td>
<td>4.710</td>
<td>5.496</td>
</tr>
<tr>
<td>Dispersion Function:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean Fitting Error</td>
<td>.05564 m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\chi^2$/DOF</td>
<td>24.543</td>
<td>20.014</td>
<td>17.272</td>
</tr>
</tbody>
</table>

**Table 4.13:** The baseline, LOCO fitted, and improved model predictions compared to the measured betatron tunes and phase (from the RingScan reproducibility dataset, Chap. 2) betatron amplitude functions (quadrupole perturbation method, Sec. 3.4), and the dispersion function ($\alpha_c$ method, Sec. 3.3). The \textit{rms} measurement spread, systematic and fitting error were applied in the $\chi^2$ calculations for the phase, beta and dispersion functions respectively.
phases are from the RingScan reproducibility dataset taken during the July, 26, 2008 accelerator development discussed in Chap. 2. The \textit{rms} spread of the fitted phase parameter at each BPM is applied in the calculation of the $\chi^2$. The measurement spread is employed instead of the error on the average phase to wash out possible affects of running the models at a different energy than the measured data was taken at. The beta functions were measured with the quadrupole perturbation method also during the July, 26, 2008 accelerator development and are discussed in Chap. 3.4. The systematic error on the beta function measurement due to the uncertainty of the quadrupole current to gradient length conversion is the dominating error in the measurement and is thus applied, however statistically incorrect, in the $\chi^2$ calculation for the beta functions. Lastly, the measured dispersion function compared with the models in Tab. 4.13 is from the December 22, 2009 accelerator development dataset and was measured with the $\alpha_c$ method, Sec. 3.3.1 I. The dispersion function $\chi^2$ calculation employs the fitting error on the dispersion function. The results of the LOCO fitted model are repeated here for convenience.

The first thing to note in Tab. 4.13 is that the vertical tune prediction for the improved model is closer to the measured value by .04. In other words, the improved model has fixed the deficiency in the baseline model’s vertical tune prediction. One may note that the improved model predicts a much better vertical tune at a slight expense of the horizontal tune prediction. It is a little disappointing that the improved model predicts a horizontal tune with error twice as large as the baseline model, however, this may be understood as quadrupole hysteresis. The model predicted betatron phase at each BPM should be consulted to quantify the worsening of the horizontal tune prediction by the improved model.

Although the baseline model yields a small $\chi^2$/DOF when compared to the measured betatron phases, the improved model makes a much more accurate prediction, specially in the vertical. Observe in the horizontal and vertical breakdowns of the to-
4.4 Improved Model Verification

The slightly larger error in the improved model’s horizontal tune prediction is not very troublesome after all. Additionally, there is the consideration that the PSR quadrupoles were not ramped prior to the RingScan reproducibility measurement. Thus, magnet hysteresis may play a considerable role in the $\sim 0.01$ horizontal tune error of the improved model. However, if the improved model consistently yields a horizontal tune prediction .01 off from measured, the improved model may possess its own deficiencies. Keep this in mind during the model verification experiments discussion in Sec. 4.4.3.

The improved model passes the first test of model enhancement, the prediction of the betatron tunes.

The second test to prove that the improved model is better than the baseline model and the correct linear model to be applied to the PSR is the prediction of the betatron amplitude functions. According to the results posted in Tab. 4.13, the improved model enhances the horizontal beta function prediction while slightly decreasing the accuracy of the vertical beta function prediction. It is interesting that the improved model should yield a much better tune prediction while yielding a slightly worse beta function prediction in the vertical. One would believe that the quality of these two parameters would be more correlated. However, the best overall beta function prediction goes to the improved model. Thus, the improved model also passes the second test to prove it is a better model than the baseline model. Recall
that the LOCO fitted model was not established as an improved model because it was not able to predict the beta functions as well as the baseline model.

The last comparison in Tab. 4.13 is the dispersion function. While not necessarily a test of model improvement, the dispersion function is a linear lattice function which should be predictable by a good model of the PSR. The dispersion is a defined by all of the previously compared machine parameters, Eq. (1.69). The improved model produces the better dispersion function prediction.

Thus, with the comparisons in Tab. 4.13 made, the improved model has shown that it is actually an enhancement of the baseline model for these measurements of the betatron tunes, phases, amplitude functions, and dispersion functions at this set point for the PSR.

4.4.3 Model verification experiments

Now that the improved model has been shown to be a better model for the PSR than the baseline model at a particular set point, the improved model needs to be tested far away from the nominal operation set point in order to verify its superiority to the baseline model. Three RingScan reproducibility dataset were collected during the September 25, 2010 accelerator development to verify the quality of the improved model. Each RingScan reproducibility dataset was taken at different PSR quadrupole settings yielding different tunes and beta functions compared to normal operations. The quadrupole power supplies were ramped to maximum before the collection of each reproducibility dataset to ensure that the quadrupoles operated on the top hysteresis curve where the magnet mapping occurred.

Due to a shortage of remaining development time, fifty RingScans instead of the nominal 100 RingScan were collected during each reproducibility measurement. Likewise, the CO was centered, the injection offset lowered to near-on-axis, and the energy
was corrected only for the first reproducibility dataset and not for the second or third datasets to save time.

The baseline and improved models produce tunes and betatron phases for direct comparison for with results from the RingScan reproducibility measurements. Comparing the model predicted beta and dispersion functions directly with the fitting parameters from the turn-by-turn RingScan data is a little more difficult because the quadrupole perturbation method was not employed to measure the real betatron amplitude functions nor were momentum measurements made to measure the real dispersion function. However, the model predicted beta and dispersion functions may still be compared with measurement, even if the real beta and dispersion functions were not measured directly.

The measured fitted amplitude of the betatron oscillation may be related to the model predicted beta functions by Eq. (1.40). In this case the action must be fit. Linear regression will serve well for the fit of the action. This is a one parameter fit, so the $\chi^2$/DOF comparing the model predicted beta functions and the fitted amplitudes from the RingScan data has one less degree of freedom than the number of BPMs.

Likewise, the fitted offset $rms$ measurement spread can be related to the model predicted dispersion function by Eq. (2.16). Like for the beta functions, the model dispersion function must be fit, see Eq. (2.17). In order to save one degree of freedom, the BPM measurement error may be constrained to equal the intrinsic BPM resolution, .02 mm. Linear regression is employed to fit for the covariance between the momentum and BPM error and the pulse-to-pulse momentum variations. The sum of squares of residuals (SSR) is applied as the statistic to compare the model and measured dispersion functions because the uncertainty on the CO measurement spread is unknown, thus the $\chi^2$ is undefined. The SSR/DOF which compares the model to measurement has two less degrees of freedom than the number of BPMs.
4.4 Improved Model Verification

The measurements from the first RingScan reproducibility dataset with tunes [3.2266, 2.2192] are compared with results from the baseline and improved models in Sec. 4.4.3 I, data from the second reproducibility dataset with tunes [3.8002, 2.3826] is discussed in Sec. 4.4.3 II, and lastly, the comparison of the third dataset with tunes [2.6539, 3.5829] is shown in Sec. 4.4.3 III.

I Reproducibility 1 with tunes [3.2266, 2.2192]

The first reproducibility dataset was collected at the production operating set point found at the beginning of the September 25, 2010 development period. The CO was centered, the injection offset lowered to [−3.313 mm, .456 mrad] in the horizontal and [2.168 mm, .847 mrad] in the vertical. The energy was corrected. The quadrupole power supplies were ramped, the magnets allowed to saturate at maximum, and the currents were brought down to their production set points. The focusing quadrupole power supply (BEMP02) was set to 462.8 A, and the defocusing quadrupole power supply (BEMP03) was set to 282 A. The measured betatron tunes for these quadrupole settings are [3.2266, 2.2192]. These tunes are a bit high for nominal PSR production, but it is how the machine was running during production September 2010. The baseline and improved model predictions are compared with the results of the RingScan measurement in Tab. 4.14.

The improved model produces closer predictions than the baseline model for all cases in Tab. 4.14. The improved model reduces the error on the tune prediction by about a factor of ten compared to the baseline model. The enhanced tune predictions by the improved model are also indicated in the betatron phase comparison. Although the error on the tune prediction by the improved model is about ten times better, the $\chi^2$/DOF in the horizontal phase comparison is only slightly improved. The $\chi^2$/DOF for the phase comparisons are much larger than those displayed in Tab. 4.13. This is
### Table 4.14:

The baseline and improved model predictions compared to the results of the RingScan analysis. The \( \text{rms} \) measurement spread is applied in the \( \chi^2 \) calculations for the phase and beta functions.

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Improved</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Comparison of Model and Measured RingScan data and, ( \nu_x = 3.2266, \nu_y = 2.2192 )</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Betatron Tune:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>rms Measurement Spread ( [8.763 \times 10^{-4}, 3.927 \times 10^{-4}] )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Horizontal Error</td>
<td>.004154</td>
<td>.0006517</td>
</tr>
<tr>
<td>Vertical Error</td>
<td>-.03591</td>
<td>.002878</td>
</tr>
<tr>
<td><strong>Betatron Phase:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean Measurement Spread ( [.037 \text{ mradian}, .017 \text{ mradian}] )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total ( \chi^2/\text{DOF} )</td>
<td>86.112</td>
<td>19.929</td>
</tr>
<tr>
<td>Horizontal ( \chi^2/\text{DOF} )</td>
<td>5.561</td>
<td>4.308</td>
</tr>
<tr>
<td>Vertical ( \chi^2/\text{DOF} )</td>
<td>166.663</td>
<td>35.549</td>
</tr>
<tr>
<td><strong>Beta Function:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean Meas. Spread, Large Amp. ( [.3201 \text{ mm}, .1785 \text{ mm}] )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean Meas. Spread, Small Amp. ( [.1369 \text{ mm}, .0896 \text{ mm}] )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total ( \chi^2/\text{DOF} )</td>
<td>3.636</td>
<td>3.484</td>
</tr>
<tr>
<td>Horizontal ( \chi^2/\text{DOF} )</td>
<td>3.395</td>
<td>3.361</td>
</tr>
<tr>
<td>Vertical ( \chi^2/\text{DOF} )</td>
<td>3.878</td>
<td>3.608</td>
</tr>
<tr>
<td><strong>Dispersion Function:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SSR/DOF</td>
<td>.003139</td>
<td>.003153</td>
</tr>
</tbody>
</table>
because the large amplitudes of betatron oscillation are about 8 mm which produces less spread in the phase measurement compared to the few millimeter amplitude from the RingScan reproducibility measurement discussed in Chap. 2 and compared with measurement in Tab. 4.13. The smaller measurement spread calculates a larger $\chi^2/\text{DOF}$ for the same residual between predicted and measured.

Likewise, the increased amplitude also increases the amplitude measurement spread which dramatically lowers the $\chi^2$ in the betatron amplitude comparisons. The baseline model has always performed well in its beta function prediction. Remember that the LOCO fitted model was discounted because the baseline model’s beta function prediction was superior. However in the beta function prediction, the improved model surpasses the baseline prediction.

II Reproducibility 2 with tunes [3.8002, 2.3826]

The second RingScan reproducibility dataset was collected with the focusing quadrupole power supply BEMP02 set at 522 A and the defocusing quadrupole power supply BEMP03 set at 312.1 A. These quadrupole settings yielded a measured tune of [3.8002, 2.3826]. This places the horizontal tune on the other side of the half integer. The injection offset for this measurement was $[-.05868 \text{ mm}, .05271 \text{ mrad}]$ in the horizontal and $[2.431 \text{ mm}, .9848 \text{ mrad}]$ in the vertical. The tiny horizontal injection offset leads to a larger spread in the horizontal tune measurement. The horizontal CO was also dramatically modified due to the change in the quadrupole power supplies, with COs larger than 10 mm. The comparison of the improved model with the baseline model and measurement is provided in Tab. 4.15.

Again the improved model proves to possess superior predictive capabilities compared to the baseline model in predicting the tune and beta functions. The slight increase in the horizontal phase $\chi^2$ for the improved model compared to the baseline
4.4 Improved Model Verification

Comparison of Model and Measured RingScan data and, $\nu_x = 3.8002$, $\nu_y = 2.3826$

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Improved</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Betatron Tune:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>rms Measurement Spread</td>
<td>$[2.231 \times 10^{-3}, 3.481 \times 10^{-4}]$</td>
<td></td>
</tr>
<tr>
<td>Horizontal Error</td>
<td>.01169</td>
<td>.005775</td>
</tr>
<tr>
<td>Vertical Error</td>
<td>-.04135</td>
<td>-.002603</td>
</tr>
<tr>
<td><strong>Betatron Phase:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean Measurement Spread</td>
<td>$[.185 \text{ mradian}, .021 \text{ mradian}]$</td>
<td></td>
</tr>
<tr>
<td>Total $\chi^2$/DOF</td>
<td>37.870</td>
<td>4.736</td>
</tr>
<tr>
<td>Horizontal $\chi^2$/DOF</td>
<td>1.741</td>
<td>1.931</td>
</tr>
<tr>
<td>Vertical $\chi^2$/DOF</td>
<td>74.000</td>
<td>7.540</td>
</tr>
<tr>
<td><strong>Beta Function:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean Meas. Spread, Large Amp.</td>
<td>$[.2704 \text{ mm}, .2301 \text{ mm}]$</td>
<td></td>
</tr>
<tr>
<td>Mean Meas. Spread, Small Amp.</td>
<td>$[.0673 \text{ mm}, .1198 \text{ mm}]$</td>
<td></td>
</tr>
<tr>
<td>Total $\chi^2$/DOF</td>
<td>3.140</td>
<td>2.244</td>
</tr>
<tr>
<td>Horizontal $\chi^2$/DOF</td>
<td>0.278</td>
<td>0.235</td>
</tr>
<tr>
<td>Vertical $\chi^2$/DOF</td>
<td>6.001</td>
<td>4.252</td>
</tr>
<tr>
<td><strong>Dispersion Function:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SSR/DOF</td>
<td>.002806</td>
<td>.002657</td>
</tr>
</tbody>
</table>

**Table 4.15:** The baseline and improved model predictions compared to the results of the RingScan analysis. The rms measurement spread is applied in the $\chi^2$ calculations for the phase and beta functions.
4.4 Improved Model Verification

model is not worrisome because of the much better horizontal tune prediction.

III Reproducibility 3 with tunes \([2.6539, 3.5829]\)

The last RingScan reproducibility dataset was taken with both quadrupole power supplies (BEMP02 and BEMP03) set to 421.8 A. The measured betatron tunes for these quadrupole set points were found to be \([2.6539, 3.5829]\). Note how this quadrupole setting switched the horizontal and vertical integer tunes. The injection offset was \([0.01013 \text{ mm}, 0.09992 \text{ mrad}]\) in the horizontal and \([-1.600 \text{ mm}, -0.4297 \text{ mrad}]\) in the vertical. Unfortunately, this quadrupole power supply setting resulted in a very small injection offset in both planes, but because the quadrupole strengths were so large, significant amplitudes were still achieved in each direction. The comparison of the improved model with the baseline model and measurement is provided in Tab. 4.16.

This quadrupole setting took the PSR furthest from its production set points. The comparison of the improved model with the baseline model and measurement is provided in Tab. 4.16. Even so, the improved model produces better tune predictions than the baseline model. However, it appears that the baseline model does a better job at predicting the beta functions. This is most likely a result of the RingScan measurement and not because the model prediction is bad. The beta function comparison is shown in Fig. 4.28. The measured vertical amplitudes are erratic in nature. The largest surprises in the vertical amplitude measurement are at BPMs 26 and 28. These are BPMs in defocusing quadrupoles, so the betatron amplitudes should be large. However, the amplitudes at these BPMs are much smaller than expected. Adding the fitted amplitudes to the CO yields positions extremes of the beam at these BPMs to be \(\sim 20 \text{ mm}\). These large positions in the uncalibrated measurement region of the BPM most probably compromised the quality of the amplitude mea-
4.4 Improved Model Verification

measurement at these BPMs. This bad amplitude measurement in the vertical could have been mitigated by centering the CO. The largest contribution to the bad amplitude measurement is the number of turn-by-turn data points in one betatron oscillation. With the fractional vertical tune very close to .5, only two turns of data may be collected per betatron oscillation. Two data points are hardly enough to fit an cosine oscillation.

<table>
<thead>
<tr>
<th>Table 4.16: The baseline and improved model predictions compared to the results of the RingScan analysis. The <em>rms</em> measurement spread is applied in the $\chi^2$ calculations for the phase and beta functions.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Comparison of Model and Measured RingScan data and, $\nu_x = 2.6539$, $\nu_y = 3.5829$</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td><strong>Betatron Tune:</strong></td>
</tr>
<tr>
<td>rms Measurement Spread $[1.178 \times 10^{-3}, 1.050 \times 10^{-3}]$</td>
</tr>
<tr>
<td>Horizontal Error</td>
</tr>
<tr>
<td>Vertical Error</td>
</tr>
<tr>
<td><strong>Betatron Phase:</strong></td>
</tr>
<tr>
<td>Mean Measurement Spread $[.044 \text{ mradian}, .016 \text{ mradian}]$</td>
</tr>
<tr>
<td>Total $\chi^2$/DOF</td>
</tr>
<tr>
<td>Horizontal $\chi^2$/DOF</td>
</tr>
<tr>
<td>Vertical $\chi^2$/DOF</td>
</tr>
<tr>
<td><strong>Beta Function:</strong></td>
</tr>
<tr>
<td>Mean Meas. Spread, Large Amp. $[.2367 \text{ mm}, .1404 \text{ mm}]$</td>
</tr>
<tr>
<td>Mean Meas. Spread, Small Amp. $[.0974 \text{ mm}, .0768 \text{ mm}]$</td>
</tr>
<tr>
<td>Total $\chi^2$/DOF</td>
</tr>
<tr>
<td>Horizontal $\chi^2$/DOF</td>
</tr>
<tr>
<td>Vertical $\chi^2$/DOF</td>
</tr>
<tr>
<td><strong>Dispersion Function:</strong></td>
</tr>
<tr>
<td>SSR/DOF</td>
</tr>
</tbody>
</table>
4.5 Model Improvement Summary

Although the improved model produces a larger $\chi^2$ in the beta function compared to the baseline model, it is believed that this is due to a poor vertical amplitude measurement.

![Figure 4.28: (Color) The measured amplitude from the RingScan data with three $\text{rms}$ measurement spreads (blue circles) and the beta function predictions from the baseline (green squares) and improved (red left pointing triangles) models multiplied by the fit action.](image)

**Figure 4.28:** (Color) The measured amplitude from the RingScan data with three $\text{rms}$ measurement spreads (blue circles) and the beta function predictions from the baseline (green squares) and improved (red left pointing triangles) models multiplied by the fit action.

4.5 Model Improvement Summary

And thus ends the epic of the improved linear PSR model.

The improved model of the PSR is constructed based on the septa characterization measurements and the results of the ray tracing simulations. The improved model was compared against the supporting measurements, measurements of the betatron tune and phase, beta function, and dispersion function, and established as an enhanced model of the PSR. The supporting measurements were thoroughly investigated including a rigorous statistical error analysis. Lastly, the improved model was
experimentally verified at operational set points far away from nominal.

So it is beyond a doubt, and I hope to have convinced you as well, that the improved model of the PSR (with the constrained focal lengths of the edge focusing and the inclusion of the fringe fields of the extraction septum magnets) is really an improved model of the PSR.
Chapter 5

Independent Component Analysis
Applied To Long Bunch Beams In
The Los Alamos Proton Storage Ring

Independent component analysis (ICA) is a multivariate statistical technique particularly adept at dissecting massive amounts of data, also known as data mining. ICA is advertised as being capable of solving the classic cocktail party problem, where there are several people attending the party, which takes place in a single room with many discretely placed microphones to record the conversations, but everyone is talking at the same time, so the microphones record a mixture of all of the conversations. To solve the cocktail party problem, one needs to be able to demix the mixed conversations recorded by the microphones without knowing the location of the people speaking, the placement of the microphones, or what each person is saying.

In essence, ICA diagonalizes an arbitrary mixed signal in frequency space. The
Independent Component Analysis Applied To Long Bunch Beams In The Los Alamos Proton Storage Ring

A mixed signal can then be written as a superposition of the “eigenmodes” of this diagonalization. The eigenmodes are the independent components or sources, which make up the arbitrary signal. The diagonalization of the arbitrary mixed signal in frequency space can be thought of as identifying the frequency peaks of a fast Fourier transform (FFT) of the mixed signal. However, ICA provides additional information than just the frequency. ICA also calculates the strength of the eigenmodes in space, where the conversations at the cocktail party may be understood.

The step-by-step process of ICA is backwards compared to how most physicists think through and organize their experiments. Typically, a physicist will choose a variable or parameter to measure, devise an experiment to measure the quantity, and finally analyze the data from the experiment to obtain the results of the measurement. However, with ICA, the data is collected, analyzed, and then the physicist has to figure out what exactly it was that he measured. As an example, a conversation at the cocktail party is mixed in a distinct and reproducible manner for a particular arrangement of microphones, and ICA is able to pick out this conversation from the mixture, but for ICA to be of any use, the physicist has to know and identify what he is looking for, the distinct and reproducible mixing pattern of the conversation.

ICA has been applied in many fields of data analysis including signal processing, telecommunications, neural networks, economics, image recognition, and biomedical signals, but the extension of ICA is relatively new to accelerator physics[1, 2, 3]. For the application of ICA to accelerator physics, the mixed conversation recorded by the microphones in the cocktail party problem corresponds to the collection of turn-by-turn BPM data collected around the accelerator, and the individual conversations are the types of beam motion: betatron, synchrotron, dispersion, nonlinear betatron motion, coupling, etc. . . .
5.1 Introduction to ICA

ICA models the measured data as a mixture of the independent components. For a system of \( m \) detectors, the data collected at the \( i^{th} \) detector may be written

\[
\mathbf{x}_i = a_{i1} \mathbf{s}_1 + a_{i2} \mathbf{s}_2 + \cdots + a_{in} \mathbf{s}_n,
\]

where there are \( n \) source signals (\( \mathbf{s} \)), and \( a_{ij} \) is an element of the mixing matrix \( \mathbf{A} \) such that

\[
\mathbf{x} = \mathbf{A} \mathbf{s}.
\]

The objective is to measure the independent components (source signals, \( \mathbf{s} \)), but the mixing coefficients are unknown. However, ICA is able to do this.

ICA assumes that the source signals are statistically independent. This is the independent part of independent component analysis. Independence is a more stringent requirement than uncorrelatedness, which is assumed by principle component analysis (PCA). While two random variables, \( y_1 \) and \( y_2 \), are uncorrelated if their covariance is zero, \( y_1 \) and \( y_2 \) are said to be independent if the covariance of any function of \( y_1 \) and \( y_2 \) is zero.

There are many methods for applying ICA to a data set. These include maximization of non-gaussianity, consideration of time structures, and non-stationary source signals. Since the measured BPM data is a time signal, time-correlated ICA is best for the application to accelerator physics. The ICA assumption of statistical independence of the source signals is converted to an assumption that the auto-covariance of the independent components are different. This is the same as assuming that the source signals are narrow band in frequency space with non-overlapping power spectra. In time structured ICA, the source signals are only a function of time, while the elements of the mixing matrix are a function of position. The measured data is collected as a function of both time and position.
The auto-covariance of a source signal is the covariance between the signal and itself delayed by some time lag $\tau$. The time lagged covariance matrix is defined as $C_s(\tau) \equiv \langle s(t)s(t+\tau)^T \rangle$ and is diagonal because different source signals are temporally uncorrelated by assumption and by definition. Applying the definition of the time-lagged correlation matrix to Eq. (5.2),

$$C_x(\tau) = AC_s(\tau)A^T,$$

(5.3)

where $C_x(\tau) \equiv \langle x(t)x(t+\tau)^T \rangle$ is the time-lagged covariance matrix of the measured data. Since $C_s(\tau)$ is diagonal because it is the time-lagged covariance matrix of the source signals, the mixing matrix, $A$, must diagonalize the time-lagged covariance matrix of the measured data, $C_x(\tau)$.

If $C_x(\tau)$ is non-degenerate, the source separation may be achieved by diagonalizing the time-lagged covariance matrix of the measured data with any time lag. The ICA algorithm for a single time lag is called Algorithm for Multiple Unknown Signals (AMUSE)[39]. However, the source separation is more reliable and robust if $C_x(\tau)$ is diagonalized for many time lags. This means that the ICA algorithm will need to be capable of diagonalizing many matrices simultaneously. Such a technique with Jacobi angles is discussed in Ref. [40]. In practice, the simultaneous diagonalization of several matrices is only approximate. The ICA algorithm which accommodates multiple time lags is called the Second Order Blind Identification (SOBI)[41]. The SOBI algorithm will be applied in this analysis.

The SOBI algorithm requires a few inputs by the user including: the measured data matrix ($x(t)$), the number of ICA modes or source signals ($n$) to be included in the analysis, and the time lags, $\{\tau\}$. The first operation is to obtain whitened data (take out any constant offset in the signal), reduce the dimension of the data space, reduce the noise of the measured data, uncorrelate, and normalize the data. This is done by computing the zero time-lag covariance matrix of the measured data
5.1 Introduction to ICA

\( C_x(0) = \langle x(t)x(t)^T \rangle \) and performing eigenvalue decomposition,

\[
C_x(0) = (U_1, U_2) \begin{pmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{pmatrix} \begin{pmatrix} U_1^T \\ U_2^T \end{pmatrix},
\]

where \( U_1 \) and \( U_2 \) are collections of eigenvectors oriented vertically corresponding to the diagonal matrices of eigenvalues \( \Lambda_1 \) and \( \Lambda_2 \). The separation between the two sets of eigenvalues and eigenvectors is determined by an eigenvalue cutoff value, \( \lambda_c \). \( \lambda_c \) may be determined in a few ways including a threshold cutoff. However for this analysis, the user inputted number of ICA modes to be analyzed determines \( \lambda_c \).

\( C_x(0) \) is an \( m \times m \) matrix, where there are \( m \) detectors in the system. Thus, there are \( m \) eigenvalues in the eigenvalue decomposition of \( C_x(0) \). Each ICA mode corresponds to an eigenvalue and eigenvector in the eigenvalue decomposition of \( C_x(0) \). So the number of eigenvalues and eigenvectors contained in \( \Lambda_1 \) and \( U_1 \) respectively is equal to the user inputted number of modes to include in the ICA (\( n \)) such that

\[
\lambda_1 \geq \lambda_2 \geq \ldots \lambda_n = \lambda_c.
\]

Since the eigenvalue decomposition orders the eigenvalues and their corresponding eigenvectors in order of strength, the \( n \) strongest modes are kept for the ICA, while the rest of the eigenvalues and eigenvectors are dropped. The whitened data matrix in the smaller \( n \times n \) data space is

\[
\xi = Vx(t), \quad \text{where} \quad V \equiv \Lambda_1^{-1/2}U_1^T.
\]

Here \( \Lambda_1^{-1/2} \) indicates the inverse square root to be applied to the eigenvalues on the diagonal of \( \Lambda_1 \) individually.

The second step in the ICA algorithm is to compute the time-lagged covariance matrices of the whitened data (\( \xi \)) for the set of user inputted time lags (\( \{\tau\} \)),

\[
C_\xi(\tau_k) = \langle \xi(t)\xi(t + \tau_k)^T \rangle.
\]

Because of the discrete nature of the digitized BPM signals to be analyzed by ICA, the time lags are constrained to be integers, \( \{\tau\} = \)
5.1 Introduction to ICA

\[ \{1, 2, 3, \ldots, k\} \]. Symmetric time-lagged covariance matrices are formed by
\[ \overline{C}_{\xi}(\tau_k) = \frac{1}{2} \left( C_x(\tau_k) + C_\xi(\tau_k)^T \right) \]. The \( k \) symmetric time-lagged covariance matrices \( \overline{C}_{\xi}(\tau_k) \) are simultaneously diagonalized to obtain the unitary diagonalizer matrix \( W \).

Last, the ICA computes the source signals and the mixing matrix
\[ s(t) = W^T V x(t) \quad \text{and} \quad A = V^{-1} W. \] (5.6)

So for each of the eigenvalues retained from the eigenvalue decomposition of \( C_x(0) \) there is an associated source signal (a row of \( s(t) \)) and several mixing elements, a column of \( A \). The source signals, rows of \( s(t) \), are called ICA temporal modes because they describe the strength of the source signal in time. The columns of \( A \) are also called ICA spatial modes because they describe the strength (the mixing) of the source signal in space.

Thus, for each eigenvalue from the eigenvalue decomposition, there are two ICA modes: a spatial mode and a temporal mode. The spatial mode holds information about the strength of a particular source signal in space and is what makes ICA superior to a straight fft of the measured data.

ICA can be thought of as an extension of principle component analysis (PCA). the general form of PCA is basically a singular value decomposition (SVD). Given a \( m \times n \) measured data matrix \( (x(t)) \) from a system with \( m \) detectors collecting data over \( n \) units of time, the SVD decomposition of \( x(t) \) is
\[ x(t) = U D V^T, \] (5.7)
where \( U \) is an \( m \times m \) matrix of “eigenvectors” in \( M \)-space, \( V \) is an \( n \times n \) matrix of “eigenvectors in \( N \)-space, and \( D \) is an \( M \times N \) matrix with singular values (“eigenvalues”) on the diagonal connecting the corresponding basis vectors in \( N \) and \( M \)-space. Since there are \( m \) detectors in the system, the columns of \( U \) describe a set of basis vectors in space and may be likened to the columns of the mixing matrix \( A \), the ICA
spatial modes. Likewise, since the data was collected over the course of $n$ units of time, the columns of $V$ describe a basis in time and may be likened to the source signals, the ICA temporal modes. Note that PCA is applied to whiten the data in the ICA algorithm.

\subsection*{5.2 ICA and Betatron Motion}

Typically, ICA has been applied to turn-by-turn BPM data collected from many BPMs around the accelerator. Such analysis yields spatial modes describing the lattice functions (such as the betatron amplitude and dispersion functions) and temporal mode describing the frequency of such lattice function motions, the betatron and synchrotron tunes.

As an example, ICA will be applied to calculate the betatron amplitude functions at the BPMs and the betatron phase advance between the BPMs. The turn-by-turn BPM data needs to be arranged in a matrix where each row corresponds to data from a single BPM and each column contains data collected at each BPM for a single turn,

\[ \mathbf{x}(t) = \begin{pmatrix} x_1(1) & x_1(2) & \ldots & x_1(n) \\ x_2(1) & x_2(2) & \ldots & x_2(n) \\ \vdots & \vdots & \ddots & \vdots \\ x_m(1) & x_m(2) & \ldots & x_m(n) \end{pmatrix}, \quad (5.8) \]

where there are $m$ BPMs and $n$ turns of BPM data. To prevent the system of equations in Eq. (5.2) from being under determined, there must be more turns of data than BPMs in $\mathbf{x}(t)$, Eq. (5.8). Actually, because the time lags cause certain turns not to be included in the data set, the number of BPMs must be less than the number of turns minus the maximum time lag, $m < n - \tau_{\text{max}}$. This is because the covariance of the BPM data with the maximum time lag is $C_x(\tau_{\text{max}}) = \langle \mathbf{x}(1 \leq t \leq}$
5.2 ICA and Betatron Motion

\( n - \tau_{\text{max}} \leq t \leq n \). So only the first (or last depending on plus or minus time lags) \( n - \tau_{\text{max}} \) turns of BPM data are included in the whitened data matrix during the ICA.

Now consider only betatron motion that is recorded at \( m \) BPMs for \( n \) turns. From Eq. (1.38), the element of the measured BPM data matrix for the \( i^{th} \) BPM and \( j^{th} \) turn is

\[
x_{ij} = \sqrt{2J_i} \cos(2\pi \nu n + \phi_i),
\]

where \( \beta \) and \( \phi \) are the betatron amplitude and phase functions at the \( i^{th} \) BPM, \( \nu \) is the betatron tune, \( n \) is the turn number, and \( J \) is the action and initial betatron phase respectively determined by the initial conditions.

Applying PCA to the matrix of betatron motion BPM data yields

\[
U = \begin{pmatrix}
P \sqrt{2} \beta_1 \sin \phi_1 & P \sqrt{2} \beta_1 \cos \phi_1 & 0 & \ldots \\
P \sqrt{2} \beta_2 \sin \phi_2 & P \sqrt{2} \beta_2 \cos \phi_2 & 0 & \ldots \\
\vdots & \vdots & \vdots & \ddots \\
P \sqrt{2} \beta_m \sin \phi_m & P \sqrt{2} \beta_m \cos \phi_m & 0 & \ldots \\
\end{pmatrix}
\]

\[
\Lambda = \begin{pmatrix}
\sqrt{2J_{nn}} & 0 & 0 & \ldots \\
0 & \sqrt{2J_{nn}} & 0 & \ldots \\
0 & 0 & 0 & \ldots \\
\vdots & \vdots & \vdots & \ddots \\
\end{pmatrix}
\]

\[
\mathbf{V}^T = \begin{pmatrix}
\sqrt{2} \cos(2\pi \nu \cdot 1) & \sqrt{2} \cos(2\pi \nu \cdot 2) & \ldots & \sqrt{2} \cos(2\pi \nu \cdot n) \\
\sqrt{2} \sin(2\pi \nu \cdot 1) & \sqrt{2} \sin(2\pi \nu \cdot 2) & \ldots & \sqrt{2} \sin(2\pi \nu \cdot n) \\
0 & 0 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
\end{pmatrix}
\]

where \( P \) is a normalization coefficient for \( U \). As expected from the SVD of the BPM data, all of the position information relating to the location of the BPMs (betatron
5.2 ICA and Betatron Motion

amplitude functions and betatron phase advances) are contained in $U$. Likewise, all of the time dependence (betatron tune) is in $V$.

Apparently a single motion such as betatron motion can have two ICA modes. This is the case for all motions that can be written as Eq. (5.9) with both a time part and position part inside the trigonometric function. By the angle addition rule, the cosine of Eq. (5.9) may be rewritten as a cosine of the spatial part multiplied by the sine of the temporal part plus the sine of the spatial part multiplied by the cosine of the temporal part. PCA also decomposes the betatron motion into two different modes. The first mode has a sine spatial mode and cosine temporal mode, and the second mode has a cosine spatial mode and sine temporal mode.

It becomes apparent from Eq. (5.10) that the betatron amplitude functions and betatron phase advances may be solved by manipulating the first two columns of $U$,

$$U_{11}^2 + U_{12}^2 = \frac{2\beta_i P^2}{m} \propto \beta_i \quad \text{and} \quad \phi_i = \tan^{-1} \left( \frac{U_{11}}{U_{12}} \right). \quad (5.11)$$

In a similar manner, the first two spatial modes from the ICA corresponding to betatron motion may be combined to solve for the beta function and phase advance,

$$\beta_i = c \left( A_{11}^2 + A_{12}^2 \right) \quad \text{and} \quad \phi_i = \tan^{-1} \left( \frac{A_{11}}{A_{12}} \right), \quad (5.12)$$

where $c$ is a constant related to the action, but may be fit when comparing ICA and PCA results with measurement.

This method of applying ICA may be employed to the turn-by-turn RingScan data. ICA was applied to the RingScan reproducibility dataset of Chap. 2. To construct the measured data matrix for input to ICA ($X(t)$), each turn was averaged across all 101 scans at a BPM. The horizontal and vertical BPM data was analyzed separately by ICA. The first two ICA modes were identified to represent the betatron motion because the frequency of the temporal modes equaled the betatron tune. The first two spatial modes were applied in Eq. (5.12) to calculate ICA measured beta
functions and betatron phase advances. The beta functions and phase advances from the ICA analysis are compared with the measured beta functions from the quadrupole perturbation method for Sec. 3.4 in Fig. 5.1 and the measured phase advances from the RingScan reproducibility dataset in Fig. 5.2.

![Figure 5.1](image-url)

Figure 5.1: (Color) The horizontal (blue circles) and vertical (green squares) beta functions from the quadrupole perturbation measurement with a systematic error and the horizontal (red left pointing triangles) and vertical (black right pointing triangles) beta functions from the ICA of the RingScan reproducibility dataset.

The beta functions from ICA do very well in reproducing the small beta functions measured with the quadrupole perturbation method. However, larger beta functions calculated from the ICA do not match. The difference between the two measurements may be quantified by a $\chi^2$/DOF with the systematic error from the quadrupole perturbation method acting as the measurement error. Such quantification leads to a $\chi^2$/DOF of 7.59 between the horizontal measurements and a $\chi^2$/DOF of 11.51 for the vertical. Since the beta functions from the ICA do not overall reproduce the beta functions measured via the quadrupole perturbation method, ICA should not be applied for measurement of the betatron amplitude functions in the PSR.
ICA does a much better job at producing the measured phase advances between the BPMs. Figure 5.2 plots the difference between the betatron phase advances measured by fitting the turn-by-turn BPM data to a cosine wave and those calculated from ICA. All but four of the ICA calculated phase advances agree with the cosine fitted phases to within three errors on the average of the measured phase distribution. Quantifying the difference between the two measurement methods yields a $\chi^2/\text{DOF}$ of 4.32 for the horizontal measurements and a $\chi^2/\text{DOF}$ of 1.96 for the vertical. The BPMs with the largest phase discrepancies between cosine wave fitting and ICA are not x-y symmetric or BPM type dependent.

It is interesting that ICA should do so well in reproducing the phase advances when it is not able to match the beta functions measured by the quadrupole perturbation method. Actually, it is surprising that ICA is even able to produce any results worthwhile because of the limited number of turns of BPM data. Traditionally, this method of ICA is applied to 1000 or more turns of turn-by-turn BPM data. That the

![Figure 5.2:](Image)
5.3 ICA Applied to Long Bunch Beams

RingScan data, which is collected on several different beam pulses, may be inputted to ICA is a good result. However, the ICA result with the RingScan data is limited because of the low number of turns that the BPMs are able to collect. This is most likely why the ICA calculated beta functions did not match the beta functions measured by the quadrupole perturbation method. Since the traditional method for applying ICA does not yield fruitful results in the PSR, a new method of applying ICA is considered in the next section.

5.3 ICA Applied to Long Bunch Beams

Because of the limited capabilities of the PSR BPM system (beam measurement at only one BPM per beam pulse, only tens of turns of data may be collected, ...) thoroughly discussed in Sec. 2.1, it is of interest to apply ICA in a new way. Since data from several BPMs around the ring does not suffice, perhaps data at a single BPM digitized over an entire macropulse will.

For this analysis data is collected from two different diagnostics: SRWC41 and SRWM41. SRWC41 is the fast current monitor in section 4 of the PSR. It is the device employed to correct the energy and measure the TOF delay for the beam momentum measurements. SRWM41 is a BPM also in the straight section of section 4. SRWM41, like the other BPMs in the PSR, is a stripline BPM, but the stripline electrode length of SRWM41 is half as long, so the peak frequency response is 400 MHz. The electrode signals of SRWM41 are not processed in the same manner as the other PSR BPMs, i.e. there is no MUX or AM to PM processing. In the past, SRWM41 has been employed has the pickup for the active damping system in the PSR[42]. SRWM41 produces four signals which may be applied in ICA: the horizontal and vertical sum and difference signals (hd, hs, vd, and vs).

While the current signal from SRWC41 is obviously proportional to the longi-
tudinal beam intensity, the horizontal and vertical sum signals from SRWM41 are proportional to the time derivative of the intensity, and the horizontal and vertical difference signals are proportional to the time derivative of the intensity signal multiplied by the transverse position of the beam centroid with respect to the electronic center of SRWM41. Because all of the signals from SRWM41 are proportional to the derivative of the current (intensity), it is also of interest to integrate the BPM signals. The integrated sum signals are proportional to the beam intensity and the integrated difference signals can be thought of as proportional to the beam current multiplied by the transverse position. The integration and differentiation of the BPM signals emphasizes low and high frequencies respectively.

The signals from SRWC41 and SRWM41 for an entire machine cycle may be digitized. The digitized signal can be divided into time slices of equal length but no smaller than the bin size of the digitization. The long vector of digitized signal is then stacked into a matrix such that each column is the beam signal from a different turn and each time slice is at a fixed location in the revolution period. Note that this arrangement is the same as that for the ICA input data described in Eq. (5.8). This indicates that the number of “BPMs” in the ICA is equal to the number of time slices in a turn. So the ICA “BPMs” of the input data matrix are no longer located around the ring as in typical ICA, but the “BPMs” are located along the phase axis of longitudinal phase space.

Since the application of the source signals have not changed with this new approach of ICA, the interpretations of the ICA temporal modes should remain unaffected. But since this new method of data collection for ICA has positioned the “BPMs” along the beam profile instead of around the machine as in the tradition method for applying ICA, the ICA spatial modes will no longer describe the lattice functions of the machine but the strength of the source signals along the beam. Thus, the ICA modes produced in this new method of applying ICA should describe motion
within the accumulated beam, specifically longitudinal motion.

5.3.1 ICA measurement setup

The PSR setup for collection of ICA data follows typical production-type beams as described in Sec. 1.2. The only deviation from production is the addition of storage time after the end of accumulation. The store is normally set to 200 $\mu$s, so the sequence of a machine cycle is as follows: beam is injected into the PSR for 625 $\mu$s and then stored for 200 $\mu$s before extraction to the 1L target.

The additional store gives the beam more time to evolve, specifically rotating in longitudinal phase space. However, the beam is generally unstable under long stores (unless the buncher voltage is increased) and losses may be high, so the rep rate is reduced to a few hertz for this exercise. This reduction in rep rate does not affect the data acquisition times for the signals from SRWC41 and SRWM41 because data from only a single macropulse is digitized for ICA.

The five signals from SRWC41 and SRWM41 are piped up from the PSR to the Ring Equipment Building (REB) where they are connected to two oscilloscopes at the development station. The oscilloscopes are Lecroy models LC584AXL and LC684DXL. These oscilloscopes are capable of capturing and digitizing the signals from SRWC41 and SRWM41 with .5 ns bin widths. The digitized signals are saved to the development computer with a Lecroy oscilloscope reader program for offline analysis. The process to transport the oscilloscope traces to the development computer takes several minutes which is why the low PSR repetition rate does not affect the data acquisition time.

The time slice size is chosen to be equal to the digitization bin size, .5 ns. Since the revolution period of the PSR is about 358 ns, this leads to 716 “BPMs” or time slices per turn. The revolution period is very important for dividing the digitized
signal vector into turns. The revolution period is rarely a multiple of .5 ns, so a time slice may have to be added or taken away from each turn. This is a tricky process, but is very important for the ICA because the time slices ("BPMs") need to be locked onto the phase axis of the longitudinal phase space.

The revolution period is found by finding the average distance in between peaks of the current profile. If the longitudinal injection into the PSR is off, this will cause the peak current to oscillate about the center of the rf bucket, but this oscillation should average out after 1000 turns. The revolution period is measured from the digitized signals because the revolution frequency is not always known a priori. An interesting example of this case will be discussed later in Sec. 5.3.4 III.

A better way to measure the revolution period would be to digitize the "moving" 2.8 MHz signal, which dictates the synchronous frequency in the PSR. The time delay between the moving 2.8 MHz and the SRWC41 signals would be required for a concrete time slice coordinate system along the phase axis. This method allows for the time slices to be locked to the phase axis of the longitudinal phase space. As it was, the moving 2.8 MHz signal was not digitized for this analysis, so there is a unknown relative offset between the time slices and the rf bucket.

5.3.2 ICA and simulated data

Simulated data was created and input into ICA to aid in the identification of the spatial and temporal modes output by ICA. Each set of simulated data is simple and contrived and allows for one or two particular motions to be resolved by ICA. Several sets of simulated data are discussed in this subsection including constant wave forms (Sec. 5.3.2 I), time slice jumps to represent mis-stacking of the digitized signal vector (Sec. 5.3.2 II), modulating wave form heights and widths (Sec. 5.3.2 III), and wave forms oscillating in phase, Sec. 5.3.2 IV.
I Constant wave forms

The most basic wave form for ICA to analyze is a wave form that does not change from turn-to-turn. In this case 1000 turns of a constant triangle wave was input into ICA. ICA was asked to apply 50 time lags in the diagonalization and to find the 5 strongest modes. The strongest ICA mode possessed a constant time mode of $\sim$0.04 and a spatial mode on the order of $10^{-12}$. This mode is a null mode describing the zero whitened data matrix. The non-zero spatial and temporal modes indicate round off error on the order of machine precision. As expected, ICA does not find any significant motion from the constant wave form. This is because one of the first steps in the ICA is to whiten the input data which takes out the constant offset at each time slice.

If ICA does not find significant motion in a constant wave form (as it should not), how does ICA handle a continuous uniformly growing wave form? For this case of simulated data, a triangle wave with constant base width grows from zeros for 1000 turns with peak height as a linear function of turn number. This case may be likened to a constant rate of beam accumulation, which varies longitudinally along the bunch.

ICA uncovers one independent motion for this case, the growth of the wave form shown in Fig. 5.3. The spatial mode for this motion is proportional to the shape of the wave form, a triangle where the growth occurs and zeros outside. The spatial mode indicates that the growth of the wave form is largest at the peak and decreases linearly away from the center of the wave form as dictated by the triangular shape.

The temporal mode describing the wave form growth is, as expected, a linear function of turn number. The temporal mode indicates that the wave form growth rate is constant. But what is most surprising about the temporal mode result is that it is negative for the first 1000 turns. This is because in the case of linear growth, turn 1000 is the average value for every time slice. So the whitened dataset is the
input dataset minus the value of every time slice at turn 1000. Thus the whitened data is antisymmetric about turn 1000.

The negative temporal mode can lead to confusing interpretations. To recover the motion described by this ICA mode, the spatial mode is multiplied by the transpose of the temporal mode. This would indicate a negative waveform in the first 1000 turns, which is of course not describing the real wave form inputted into ICA but is describing the whitened data.

II Time slice jumps

It is very important for this method of applying ICA that the turn-by-turn stacking of the digitized signal vector is correct, but would ICA be able to identify a sudden shift in the wave form by one time slice due to mis-stacking the turn-by-turn data matrix? The discrete nature of the digitization becomes an inconvenience when stacking turns with a non-integer number of time slices. Round off error in the stacking algorithm could cause the wave form to slip by a time slice in either direction.
A data matrix with a square wave form for each turn was created to test the response from ICA. The square wave form was constant for the first 500 turns and then was shifted forward by one time slice at turn 501. The square wave form for turns 501 to 1000 were then misaligned with the square wave form at turn zero by one time slice.

ICA recovers only one significant mode, which is shown in Fig. 5.4. The significant temporal mode is a step function with the step as expected between turns 500 and 501 where the time slice slip occurred. The initial value of the temporal mode is equal and opposite the final value, antisymmetric about turn 500.5. The initial value of the temporal mode is negative.

**Figure 5.4:** The spatial mode (blue) with last turn beam profile (green) (top left), the fft of the spatial mode (bottom left) and integral of the spatial mode (blue), the last turn beam profile (green) (top middle), the correlation with time lagged data (bottom center), the temporal mode (top right), and the fft of the temporal mode, bottom right.

The spatial mode for this motion is a set of delta-type functions. The first delta function is positive and is located at the time slice immediately before the beginning of the wave form at turn 1, the time slice the wave form slips into. The second delta function is the same height as the first but negative and at the location of the last time slice in the wave form at turn 1, the time slice the wave form slips away from
at turn 501. The ICA spatial mode of this motion is antisymmetric about the center of the wave form and seems to indicate motion to early time slices by adding to the front of the wave form and taking away from the end. The integral of the spatial mode recovers the final square wave form. Interestingly, the spatial mode is in the shape of the whitened data.

The square wave may be an overly contrived dataset because the whitened data of a square wave form with a single time slice jump is a set of delta functions at the edges of the wave form just like the spatial modes that ICA was able to recover in the previous example. To add a bit of complexity, the time slice jump at turn 501 is repeated for a triangle wave.

ICA discovers two modes with significant frequency components with the time-lagged data, the first of which is shown in Fig. 5.5. However, only one of these modes possesses a significant singular value. Actually, both modes seem to describe the same motion. Both temporal modes are step functions with the step occurring between turns 500 and 501. However, the initial and final values of the temporal modes are not equal and opposite as in the previous case of the square wave form. The initial value of the first temporal mode is equal to the final value of the second temporal mode, while the final value of the first temporal mode is equal and opposite the first value of the second time mode. Because the data is whitened, ICA should recover temporal modes with zero offset. This would seem to indicate that these two source signals are really the same source signal, which ICA has separated.

The spatial modes from the two modes only differ in height. As might be expected by the results of the square wave form with the single time slice jump, the spatial modes are in the same shape as the whitened data. In this case the spatial mode is a positive square wave for the first half of the triangular wave form and a negative square wave for the second half of the triangular wave form. This can be thought of as a positive delta-type function for every time slice where the wave form increases
height due to the time slice jump between 500 and 501 and a negative type-delta function where the amplitude of the wave form decreases due to the time slice jump. As for the square wave form example, the integral of the spatial mode recovers the triangle wave at turn 1000.

Adding the spatial and temporal mode products of both ICA modes recovers the whitened data.

### III Wave forms with modulating heights and widths

Wave forms with modulating heights and or widths can be likened to many examples in accelerator physics including betatron motion and longitudinal motion in the rf bucket indicating injection mismatch. Several simulated wave forms will be created and explored to increase understanding of ICA of such motions.

The first example is that of a square wave with top that modulates in height with a constant fractional tune of .002. The amplitude of the oscillation varies across the top as a symmetric triangle. The middle of the wave form grows initially. The area
under the wave form is kept constant, so the sides of the wave form grow as the middle shrinks and vice versa.

ICA identifies one mode for this amplitude modulation. This is the first example where the motion in the simulated data has had a distinct frequency component; the motion is sinusoidal as shown in Fig. 5.6. The temporal motion is a perfect sine wave with zero offset and equal valued positive and negative peaks. An fft of the temporal mode yields the fractional “tune”. The peak frequency of the fft was \(1.957 \times 10^{-3}\). The accuracy of the fft is controlled by the number of turns, most specifically one over the number of turns minus the number of time lags. Often times the sine wave fit is more accurate at calculating the fractional tune of the source signal. The temporal mode was also fit to a sine wave. The fractional “tune” was fit as .002 which was the frequency employed to create the data.

**Figure 5.6:** The spatial mode (blue) with last turn beam profile (green) (top left), the fft of the spatial mode (bottom left) and integral of the spatial mode (blue), the last turn beam profile (green) (top middle), the correlation with time laged data (bottom center), the temporal mode (top right), and the fft of the temporal mode, bottom right.

The spatial mode for this motion is an upside down triangle with edge corners of height 3.732 and center height of \(-7.464\). The corners are half the height and opposite sign as the peak of the triangle. This shows the constraint in the simulated
data that the area under the wave form remained constant. The spatial wave form points negatively which is acceptable because the temporal mode is initially negative as well indicating initial growth of the peak which checks with the simulated data.

In this example the amplitude modulation of the top of the square wave starts at zero phase. The wave form at turn 1 is initially flat topped with no modulation. However, if only turns 100 through 1000 are ran through ICA, the first ICA temporal mode does not have nonzero offset. There is a second ICA mode with insignificant singular value but good correlation with the time lagged data which matches the first ICA mode in describing this motion. Like for the triangle wave time slice example, this second ICA modes possesses a spatial mode that matches the first, and a temporal mode with the same frequency and phase as the first. Shifting starting turn of the input data shifts the phase of the amplitude oscillation to a trigonometric function with a frequency and phase argument like Eq. (5.9), which was shown to split into two modes in the betatron motion example, Sec. 5.2. So this division of one motion into ICA modes is expected.

Now if modulating width is added in concert with modulating height as in the case of a triangle wave with varying height and width such that the area of the triangle is held constant, ICA identifies many modes of significant motion, the first of which is shown in Fig. 5.7. The fractional “tune” of the modulation in the simulated data was again .002. The first ICA mode is perfect sine wave with fractional tune of .002. All of the other ICA modes for this data are harmonics of this mode possessing temporal modes with fractional tunes of .004, .006, .008, .01, . . . .

The spatial mode for the strongest ICA mode is negative for the middle half of the triangle wave form at turn 1 and positive for the outer quarters of the triangular wave form at turn 1. This spatial mode extends past the edges of the first turn wave profile to cover the maximum width of the wave form later in the oscillation. The spatial mode describes motion from the center of the wave form to a growing base of the
wave form. The higher order spatial modes are higher harmonics of the first spatial mode with more wiggles in the outside portions of the spatial mode. No wiggles are added to the triangle at the center of spatial modes.

**IV Wave forms oscillating in phase**

The last set of simulated data discussed in this section is the triangle wave form with modulating height and width and preserved area with an additional motion. This additional motion is an oscillation of the wave form center along the time slices. This type of motion may be representative of beam oscillation in the rf bucket. The modulation of the triangle wave’s height and width was created with a fractional tune of .002, while the phase oscillation along the time slices was created with a fractional tune of $5.5 \times 10^{-4}$.

ICA was able to uncover multiple modes of motion, the first of which is shown in Fig. 5.8. The first ICA mode was represented the modulation of the height and widths. The spatial and temporal modes for the strongest ICA mode matched the
first ICA mode from the previous case of the triangle wave with the height and width modulation but not the oscillation in phase. The first temporal mode indicates a fractional tune of .002.

Figure 5.8: The spatial mode (blue) with last turn beam profile (green) (top left), the fft of the spatial mode (bottom left) and integral of the spatial mode (blue), the last turn beam profile (green) (top middle), the correlation with time lagged data (bottom center), the temporal mode (top right), and the fft of the temporal mode, bottom right.

The second ICA mode describes the oscillation in phase of the wave form center along the time slices. The second temporal mode is a sine wave with a fractional tune of $5.5 \times 10^{-4}$. The second spatial mode is similar to the spatial mode describing the triangle wave and the single time slice jump, however the tops of the square waves are slightly tilted and the outsides are curved indicating a continuing shift of the wave form center.

As in the previous example, ICA also recovers the higher order modes of the height-width modulation.
5.3 ICA Applied to Long Bunch Beams

5.3.3 ICA and Continuums

The base assumption of ICA is that the source signals are independent. The independence of the source signals means that the power spectra of the source signals are non-overlapping. So how does ICA handle data with overlapping source signals? One difficulty when applying ICA to data from “BPMs” along the beam pulse is that many frequency continuums may be included in the dataset. The most obvious of these continuums is due to the space charge coherent tune shift across the pulse.

One of the first experimental datasets applied to ICA was that of a single turn kick experiment collected by R.J. Macek and R.C. McCrady during the October 9, 2006 accelerator development. The PSR was set up for 200 µs of additional accumulation and 200 µs of additional store. The vertical pinger in section 3 of the PSR was fired with 8 kV for 340 ns after the end of accumulation. The pinger supplied single turn kick induced coherent vertical betatron motion which is necessary for ICA of betatron motion. The signals from SRWC41 and SRWM41 were digitized on the oscilloscope and captured by the development computer. ICA was performed on the vd signal from SRWM41 offline.

Because the vd signal is proportional to the time derivative of the vertical position it is sensitive to this type of vertical coherent motion. Thus, many of the ICA modes output from the vd signal represent the coherent betatron motion due to the single turn kick. The temporal modes for these ICA modes are zero for all but the last 500 turns or so after the single turn kick. The last 500 turns of the temporal modes are sinusoidal waves of appreciable amplitude. The fractional tune for these temporal modes range between .169 and .2.

The betatron motion due to the single turn kick is not an independent motion. It is really a continuum in frequency space with overlapping power spectra due to the space charge coherent tune shift. Figure 5.9 graphs a contour plot of the fft of
the turn-by-turn stacked vd signal along turn number for each time slice. The last
turn beam profile extends between time slices 57 and 647. As expected the space
charge coherent tune shift is strongest at the center of the bunch were the intensity
is greatest. The asymmetry in the coherent tune shift was not expected. The small
island around time slice 500 represents a small resonance at the 5th order resonance,
a fractional tune of .2.

![Figure 5.9: FFT of the turn-
by-turn stacked vd signal along
turn number for each time slice
(contour plot) and the ICA spa-
tial mode peaks versus the fft
peaks from the temporal ICA
modes (red circles) for the first
21 betatron ICA modes.](image)

Also plotted in Fig. 5.9 are the locations of the peak strengths in space and in
fractional tune for the all of the betatron modes in the first 30 ICA modes. Note the
almost random arrangement of the peak strengths of the ICA modes along the fft
contour plot. The order of the ICA modes seems haphazard. Also very few of these
betatron modes are doubles as expected from the previous betatron motion example
in Sec. 5.2.

Additional simulated data was created to examine ICA results to a continuum
of source signals in frequency space. In this case the simulated data started as a
rectangular wave of length 500 time slices. The rectangular wave was generated with
an amplitude modulation. The frequency of the modulation is varied across the wave
with fractional tunes from .2 to .1. So like the space charge coherent tune shift, each time slice in the simulated data represents motion from a different source signal with just slightly different frequency from the neighboring time slices. Thus there are 500 different source singles in this simulated data. ICA confirms that it is able to recover any number of significant modes of motion.

When ICA is asked to analyze a continuum of source signals with limited mode number, it diagonalizes the dataset as best as possible. However, the output ICA modes for such a diagonalization do not represent independent motions as normal ICA modes do. Thus, it is not hopeful that ICA should recover useful information about transverse motion. Since ICA cannot recover useful information from the coherent betatron motion due to the space charge coherent tune shift, the coherent betatron motion is no longer something that may be studied with ICA. This means that pinging the beam to induce coherent betatron motion is no longer necessary during data collection for ICA, so ICA data may be collected during production.

5.3.4 ICA modes

After the ICA is performed on the turn-by-turn stacked data signal, the ICA modes must be identified in order for the analysis to have qualitative or quantitative application. One needs to be careful in assigning meaning to the ICA modes because as shown above in the continuum example ICA applied to data with broadband power spectra may not always represent the independent source signals. In this subsection, many of the ICA modes are identified.

I Noise mode

Some ICA modes do not represent beam motion but noise in the measurement of the digitized signal. Some noise modes possess significant singular values meaning they
appear as strong ICA modes. The noise modes need to be properly identified in order to obtain the proper interpretation of the ICA. An example ICA noise mode from the integral of the vs signal is shown in Fig. 5.10. The data was collected during the September 25, 2010 accelerator development. The PSR was set up with a pattern width of 290 ns, 625 µs of injection, 200 µs of additional store, a countdown of 6 (only every sixth turn was injected into the PSR), 2 Hz rep rate, and buncher voltage of 6.96 kV. All other operational parameters were the same as production.

Figure 5.10: The spatial mode (blue) with last turn beam profile (green) (top left) and the fft of the spatial mode (bottom left), the correlation of this ICA mode with time-lagged data (bottom center), and the temporal mode (top right) and the fft of the temporal mode, bottom right.

There several indications that an ICA mode may be noise, but the best two pointers are the correlation of the ICA mode with the time-lagged data (shown in the bottom center of Fig. 5.10) and the fft of the temporal mode, shown in the bottom right of Fig. 5.10. If the ICA mode describes motion which oscillates in time, then one expects that the correlation of a particular ICA mode with the time-lagged data should also be oscillatory as the ICA mode and time-lagged data shift in and out of phase due to the time lags. ICA modes that do not represent the oscillatory motion will only possess one significant correlation with the time-lagged data. The large correlation coincides the zero time-lagged dataset. The rest of the ICA mode’s
correlations with the time-lagged data are very small. As observed in the bottom center plot of Fig. 5.10, the ICA mode shown is a noise mode because it possesses only one large correlation with the time-lagged data at $\tau = 0$.

The second indication that the ICA mode presented in Fig. 5.10 is a noise mode is in the fft of the temporal mode. The bottom right plot of Fig. 5.10 shows a noisy frequency spectrum with no distinct frequency peaks. This is contrary to what is expected as the output of ICA, where the ICA modes are independent with non-overlapping power spectra in frequency space. Also the frequency peaks of the fft are about an order of magnitude smaller than the fft peaks of ICA modes that describe independent components of the beam motion.

There are two more secondary characteristics of ICA noise modes that should be examined before a mode is discarded as noise. The temporal mode should have constant amplitude across the entire dataset. This signifies that this ICA mode is oblivious to the end of injection or increasing transverse motion and perhaps instability during the storage time after injection but before accumulation. The other secondary characteristic lies in the spatial mode. Noise modes will generally possess large spatial modes for time slices where beam is not present. This is clearly observed in the top left of Fig. 5.10 where, although the spatial mode is zero at the beginning and end of the turn, the spatial mode grows steadily before and decreases quickly after the last turn beam profile. The strong spatial mode strength for “BPMs” where the beam is not located indicates that there is measurement noise in every digitization bin.

ICA noise modes are most common in the analysis of integral signals of SRWM41 and the current signal of SRWC41. This is most likely because the integration, which emphasizes lower frequencies, is unable to retain the higher frequency modes of the independent components.
II Scope and device modes

Some ICA modes possess unusually sharp FFT peaks from both their spatial and temporal modes. Such ICA modes often contain sinusoid spatial modes of equal amplitude across the entire turn and constant amplitude temporal modes. One might incorrectly identify these modes as ICA noise modes because of the significant spatial mode for time slices without beam present and because of the constant amplitude in the time mode, which is unaffected by the end of accumulation or growing instabilities during the storage time before extraction. However, this ICA mode possesses oscillatory correlation values with the time-lagged data suggesting that this mode describes an oscillatory frequency component in the signal. This fact is verified by the sharp FFT peak of the temporal mode, indicating an independent component.

ICA scope/device modes come in pairs of two just like the betatron modes of Sec. 5.2. This is because the ICA scope/device mode represents an inherent frequency in the diagnostic device or in the clocking of the oscilloscope, which is most likely not an integer harmonic of the revolution frequency such that there is a phase difference between each turn. The turn-to-turn phase difference in the signal produces the double ICA mode as explained in the case of betatron motion in Sec. 5.2. This motion, just like the betatron motion, may be decomposed into a cosine spatial mode with a sine temporal mode and a sine spatial mode with a cosine temporal mode. The fact that ICA scope/device modes come in pairs is another indication that these modes are not noise. The final evidence that these ICA scope/device modes are real independent components of some type is that higher harmonics of these modes are sometimes observed.

The first example of a scope/device ICA mode is presented in Fig. 5.11. This scope/device ICA mode is an independent component in the signal, but it seems unaffected by the presence of the beam because this ICA has a constant amplitude...
sinusoidal spatial mode even where beam is not. Thus, this mode must describe a
clocking in the oscilloscope or the device which captured the data.

Figure 5.11: The spatial mode (blue) with last turn beam profile (green) (top left), the fft of
the spatial mode (bottom left) and integral of the spatial mode (blue), the last turn beam profile
(green) (top middle), the correlation with time lagged data (bottom center), the temporal mode
(top right), and the fft of the temporal mode, bottom right.

It is easy to experimentally verify whether a mode is a scope or device mode by
switching which oscilloscope the signal is plugged into for data collection. If after
ICA is performed on the new captured signal the ICA scope/device mode stays with
the signal, the ICA mode must be a device mode, and if the mode is not uncovered
by ICA, the ICA mode must be a scope mode.

The ICA mode shown in Fig. 5.11 appears in all four signals of SRWM41 but
not in the ICA of the SRWC41 signal. However, this alone is not evidence for either
scope or device mode because all of the SRWM41 signals are captured on the same
oscilloscope. To test whether the mode in Fig. 5.11 is a SRWM41 device mode or
an LC684DXL Lecroy oscilloscope mode, the signals from SRWM41 were switched
to the other oscilloscope at the development station. ICA of SRWM41 signal data
captured at the other oscilloscope (Lecroy LC584AXL) was unable to recover the
mode in Fig. 5.11. This suggests that the ICA mode presented Fig. 5.11 represents
something internal to the Lecroy LC684DXL oscilloscope.

The oscilloscope clocking results helps to explain the precise nature of the sinusoidal spatial mode shown for this ICA mode in Fig. 5.11. The spatial mode performs just under 11.5 oscillations in a signal turn. The same result for the number of oscillations per turn may also be recovered by adding the fft peaks of the spatial and temporal modes to get 11.4599. In this case, when the spatial mode is a sinusoidal oscillation, the fft peak of the spatial mode can be thought of as the integer tune number, and the fft peak of the temporal mode can be thought of as the fractional tune. Thus, the total tune is simply the addition of the integer and fractional tunes.

A higher order mode is needed to resolve the ambiguity of the fractional tune about the half integer, .5. In the case of the LC684DXL scope mode a second harmonic was observed with tune 34.38 indicating that the fractional tune of the LC684DXL scope mode is really .4599 and not .5401.

Since the tune is defined as the number of oscillations per revolution, the frequency of the Lecroy LC684DXL clock mode may be found by multiplying the “moving” 2.8 MHz imposed revolution frequency of the PSR by 11.4599. ICA calculates an exact 32 MHz for the LC684DXL clock, which is an awfully round number for a random beam motion, indicating precision expected of an electronic timing system.

A second scope/device mode also observed by ICA is presented in Fig. 5.12. This mode was originally observed in ICA of signals from the fast current monitor SRWC41. To test if the ICA mode in Fig. 5.12 is a scope mode or a device mode, data was captured with SRWC41 plugged into the LC684DXL Lecroy scope. It was found that ICA was still able to recover the ICA mode presented in Fig. 5.12 proving that this mode describes a frequency resonance of the current monitor. ICA of the SRWC41 signal at the new oscilloscope also recovered the LC684DXL scope mode presented in Fig. 5.11.

There are a few characteristics that separate the SRWC41 ICA device mode from
the LC684DXL ICA scope mode. First, note that there is some sort of amplitude variation between the first 200 turns and the rest in the temporal mode of the SRWC41 device mode. This could possibly indicate a sort of threshold in beam current for the device resonance to persist. Second, observe that the spatial mode seems to decrease while beam is present. Both of these characteristics indicate that this device mode is affected by the presence of the beam, which is a characteristic not observed in the scope clocking mode.

The first harmonic of the SRWC41 device mode is also recovered in the ICA of the digitized current signal. The tune for this higher order mode comes out to 179.05865. This indicates that the fractional tune of the fundamental SRWC41 device mode is not .4712 but is really .5288. This results in a tune of 89.5288 for the device mode. Multiplying the tune of the device mode by the PSR revolution frequency yields another exact frequency of 250 MHz. The response of the wall current monitor, SRWC41, is described in Ref. [43].
III 201.25 MHz mode

One of the first modes identified from the ICA analysis was the 201.25 MHz beam structure mode of the newly injected beam. This mode was identified by the character of its temporal mode. Notice that the temporal mode in Fig. 5.13 has constant amplitude for the first 1400 turns of the analysis and then the amplitude suddenly reduces to noise for the last 400 turns. This reduction in the amplitude of the temporal mode coincides with the end of accumulation and the beginning of storage. Thus, this most describes the newly injected beam.

**Figure 5.13:** The spatial mode (blue) with last turn beam profile (green) (top left), the fft of the spatial mode (bottom left) and integral of the spatial mode (blue), the last turn beam profile (green) (top middle), the correlation with time lagged data (bottom center), the temporal mode (top right), and the fft of the temporal mode, bottom right.

The power of ICA, which lies in the spatial mode, is most prominent in this example. The spatial mode of the 201.25 MHz mode describes where in space (the rf bucket) the beam is injected. The spatial mode also describes the length of injection for each turn.

The 201.25 MHz mode comes in pairs with one ICA mode containing a sine spatial mode and a cosine temporal mode and the other ICA mode a cosine spatial mode
and a sine temporal mode. ICA also recovers the first and second harmonics of the 201.25 MHz mode.

The spatial mode shown in Fig. 5.13 is sinusoidal with 10 time slices between peaks or 5 ns, the period of the 201.25 MHz frequency. There are 58 peaks in the spatial mode represent each micropulse injected into the PSR each turn for a pattern width of 290 ns. As expected the spatial mode is constant amplitude across the region of injection and zero outside. This indicates that the charge in each micropulse does not vary significantly across the minipulse, a turn. One might expect the 201.25 MHz spatial mode to be positive definite mirroring the current of the newly injected beam. However due to the whitened dataset, this is not the case. The whitening constrains the 201.25 MHz spatial mode to have an average of zero.

The higher order mode of the 201.25 MHz mode yields a tune of 144.14272 indicating a fractional tune of .07136 for the 201.25 MHz mode. Thus, the tune of the 201.25 MHz mode is 72.07136. This is exactly as expected because the PSR revolution frequency is operated at the 72.07 subharmonic of 201.25 MHz. Multiplying the tune for the ICA 201.25 MHz mode by the revolution frequency of the PSR yields exactly 201.25 MHz.

The following gives an example of the power of ICA and the 201.25 MHz mode. It was observed at the beginning of the 2010 LANSCE production run cycle that there was an unusual amount of hash on the current profile of the beam in the PSR. ICA data was taken to verify the imposed “moving” 2.8 MHz revolution frequency of the PSR. ICA uncovered the 201.25 MHz mode presented in Fig. 5.14. The ICA mode in Fig. 5.14 is reminiscent of the 201.25 MHz mode shown in Fig. 5.13. The most obvious difference between the two modes lies in the temporal mode. The 201.25 MHz mode presented in Fig. 5.14 possesses a much smaller fractional tune such that the actual sinusoidal behavior may be observed. A fractional tune of .07 describes an oscillation that repeats every 14 turns, however a fractional tune of .009 describes an
oscillation that repeats every 111 turns, which is why the oscillation is observable in the temporal mode of Fig. 5.14.

**Figure 5.14:** The spatial mode (blue) with last turn beam profile (green) (top left), the fft of the spatial mode (bottom left) and integral of the spatial mode (blue), the last turn beam profile (green) (top middle), the correlation with time lagged data (bottom center), the temporal mode (top right), and the fft of the temporal mode, bottom right.

The spatial mode of the 201.25 MHz in Fig. 5.14 has a notch in the middle and different constant amplitudes on either side. The slow rastering of the injected beam indicated by the .009 fractional tune causes the beam to be stacked more longitudinally than in normal operations when the fractional tune for the 201.25 MHz mode is .07. The notch in the middle of the injection pattern may be due to higher synchrotron tune in the center of the rf bucket and a quick decoherence of the 201.25 MHz stacked beam structure. Along similar lines of thought, the amplitude of the spatial mode may indicate whether the beam motion in phase in the rf bucket is aligned or anti-aligned with the rastering of the injection pattern.

Identifying the mode presented in Fig. 5.14 as the 201.25 MHz mode, which is an unchanging frequency, the revolution frequency of the PSR and thus the frequency of the “moving” 2.8 MHz may be easily calculated by dividing 201.25 MHz by the tune of the ICA 201.25 MHz mode shown in Fig. 5.14. This results in a “moving” 2.8
MHz frequency of 2.7948 MHz. However, the “moving” 2.8 MHz frequency should be 2.7924 MHz. So the “moving” 2.8 MHz frequency was off by 2.4 kHz.

The “moving” 2.8 MHz frequency is created by a frequency generator at the front end of the machine. A frequency number is typed into a computer to assign the frequency of the “moving” 2.8 MHz signal. The number inputted into the frequency generator for the “moving” 2.8 MHz signal was found to be 2.7948 MHz, exactly the number predicted by ICA.

5.4 Summary

In this chapter, I have introduced the concepts of ICA and applied ICA in the traditionally manner by analyzing turn-by-turn BPM data. Although this method yields somewhat accurate results for the betatron amplitude function and betatron phase compared to direct measurement of these quantities, it was found that the diagnostics in the PSR were not optimum for this method for applying ICA.

I have applied ICA in a new manner. The analysis of ICA along the bunch yields new information not observed previously in other methods of applying ICA. ICA was first applied to simulated data in order to understand the output modes.

Applying the knowledge gained from the analysis of simulated data, ICA was applied to measured beam signals from a wall current monitor (SRWC41) and a short stripline BPM (SRWM41) in the PSR.

I have identified, experimentally verified, and presented several of the ICA modes from real beam signal.
Chapter 6

Conclusions

In this thesis, I relate two tales. The first story tells of the extensive effort to improve the model of the PSR starting with a fundamental and statistically rigorous understanding of the beam position measurement in the PSR and extending this measurement to other experiments directed toward model improvement. The second story addressed in this thesis involves the application of independent component analysis (ICA) along the long bunch of the PSR.

In Sec. 1.3, at the end of Chap. 1, I introduce what I call the baseline model for the PSR. This is basically the model as I found it when I started the exercise to improve the model. The baseline model is an extension of F. Neri’s PSR DIMAD deck. F. Neri’s model was expanded to include the vertical correctors and the beam position monitors (BPMs). Although the real PSR does not possess any horizontal corrector magnets, thin lens horizontal correctors at the centers of the benders are included in the baseline model for application in the orbit response matrix (ORM) analysis. The magnet positions are constrained by alignment data from 2006, and the edge focusing of the rectangular dipoles are adjusted with a fringe field integral parameter.

After introducing the baseline model, I compare the baseline model’s predictions
of the betatron tunes, betatron phases, betatron amplitude functions, and dispersion function with measurement. These quantities were chosen because they are the values of interest from a linear model. While the baseline model was able to predict the horizontal betatron tune, the baseline model’s vertical tune prediction was off by $\sim 0.05$ compared with measurement. I compare the baseline model’s betatron phase prediction with measurement for a clearer picture of this phenomenon. The vertical betatron phase predictions of the baseline model produce an increasing difference compared to measurement. This indicates a systematic mistreatment of the vertical focusing in the baseline model. This type of behavior in the model-measured phase residual also suggests that the model mishandles the vertical focusing in multiple elements of the same type and that this error is not due to a single element. I should also note, if only to complicate the situation, the baseline model is able to predict the measured betatron amplitude and dispersion functions within measurement error.

Now that I have shown the main deficiency of the baseline model in the vertical tune prediction and motivated the study to improve the baseline model, I need to describe the game plan for finding the additional vertical focusing in the baseline model. There are three possible sources of the additional vertical focusing in the PSR: the quadrupoles, the fringe fields of the PSR extraction septa, and in the edge focusing of the rectangular dipoles. While the fringe fields of the extraction septa are not believed to be the cause of the systematic mistreatment of the vertical betatron phase in the baseline model, it is known that the beam feels some amount of quadrupole focusing from these fringe fields and should be included in the improved model. A different experimental method will be applied to test the model’s treatment of each of these three types of accelerator elements. An ORM analysis will be employed to investigate the quadrupoles, beam-based response to the septa fringe fields will allow for characterization of the magnet multipole components of the fringe fields, and ray tracing through simulated 3D magnetic fields of the rectangular PSR dipoles will yield
a focal length for this focusing for constraining the fringe field integral parameter in the improved model.

As well as the model improvement experiments listed above, I will need to be able to measure the quantities of interest predicted by the model: the betatron tune, phase, and amplitude function and the dispersion function. While these measurements do not directly improve some aspect of the model, they allow for a means of comparing model and measurement. The predictions of the baseline model compared to measurement may then be compared with the improved model’s predictions of the same measured quantities. This method will not only be applied to establish the improved model, but will also be employed to verify the improved model at many different PSR operational set points. All of the measurements listed up to this point rely heavily on the beam position measurement in the PSR. Thus, it becomes a necessity that I completely understand this measurement.

In Chap. 2, I examine the beam position measurement in the PSR. The preferred method for taking beam position monitor (BPM) data is by executing the RingScan program, which records turn-by-turn BPM data at each BPM in the PSR. Because a matrix switch is employed in the diagnostic setup in the PSR, data at only one BPM may be collected, analyzed, and stored during each macropulse. This greatly complicates the BPM measurement because the CO measurement takes place over 20 different macropulses, where each macropulse is a slightly different beam with different central momentum and travels on a slightly different CO. Thus, large variations may be encountered in the BPM measurement. In order to measure the model predicted quantities of interest and the most demanding ORM, I need to completely understand this BPM measurement.

After detailing the signal processing of the beam signal in the BPM measurement in Sec. 2.1, I describe the PSR setup and data analysis for the RingScan measurement. I motivate the fitting of a cosine wave to the turn-by-turn position data as
a means to extract the amplitude, tune, phase, and CO of the betatron oscillation at the location of the BPM. However, not all scans of turn-by-turn data result in cosine waves. I identified 9 different types of data acquisition errors in the RingScan measurement, Sec. 2.4. Each data acquisition error was investigated and understood. Analysis scripts were constructed to identify these data acquisition errors and remove them from the dataset. Understanding how the data acquisition errors manifest in the data acquisition has led to mitigation efforts. The most successful of these involved a new method for arming the analog to digital converters (ADCs) in the BPM measurement, Sec. 2.4.3. This mitigation has resulted in all but completely eliminating the occurrence of two of the data acquisition errors, the BPM selection error and flat line error.

I found that 10.23% of scans of turn-by-turn data possessed a data acquisition error or an outlier in one of the fitting parameters and 82.18% of the RingScan measurements (40 scans or a complete CO measurement) possessed a data acquisition error or an outlier in one of the fitting parameters. These statistics are not very convincing for an experimenter, but I have developed a complete suite of analysis scripts to identify and take out data acquisition errors. I also show that if many RingScans measurements are made, more than enough good data remains after the removal of the data acquisition errors for proper statistics.

I further investigate the cosine wave fit to the turn-by-turn BPM data through an analysis of the residuals along the scan. This study reveals the quality of the BPM measurement as a function of turn number. I found that the residual between measured and fit is directly related to the distance of the beam position from the center of the BPM. I also encountered a constant offset drift across the scan. This drift is the main contributor to the single turn BPM measurement error from the maximum likelihood (ML) error analysis of the cosine wave fit. Additionally, I observed that the first turn of vertical data was suspect because the residual for this turn is so
large. Lastly, I noted that the rms spread and maximum residual for BPMs, which also suffer from missed turn errors, are largest in the region where these BPMs miss turns. This result is most interesting since this analysis was performed after the removal of data acquisition errors from the dataset. Could this be a possible footprint of the missing turn data acquisition error?

Finally, I present the results of the RingScan measurement in Sec. 2.6. The average, rms measurement spread, and fitting error for each of the fitting parameters (amplitude, tune, phase, and offset) are discussed in detail. The statistically rigorous study of the BPM measurement produces measurement limits and quantifies the precision and accuracy. It was in this analysis that it was found that the measurement spread of the CO tracked the dispersion function with some nonlinear scaling factor, a fact of most importance in the measurement of the pulse-to-pulse momentum variations. I also discuss the single turn BPM measurement error from the ML error analysis of the cosine wave fitting and the sum of squares of residuals (SSR) which is applied as a quality factor in the fitting. Lastly, I examined the correlations in the fitting. The only correlation of significance was between the tune and the phase parameters, -.87.

Now I apply the results of the BPM measurement for calculation of the injection offset in Sec. 2.7. I study the results of three different methods for calculating the injection offset and compare with the result of the program used during operations and tuning. I found that the injection offset calculation arising from data in a single scan produced a result with uncertainty much smaller than the distribution of the total injection offset calculation. This indicates that the injection offset is not constant and does change from pulse-to-pulse. This effect is mostly due to the changing CO at the point of injection due to the pulse-to-pulse momentum variations although changes in the injection steering with momentum may also contribute. The suspect first turn of vertical data was also an outcome of the injection offset calculation. While the
first turn of vertical data produced a injection offset that agreed with all of the other
turns, the SSR for the first turn fits calculating the injection offset were about twice
as large as the other turns. The injection offset was calculated including the scans
with data acquisition errors to simulate what an operator might observe during beam
tuning. I found that 10% of the injection offset calculations led to faulty injection
offsets. Sometimes the position or angle of these outlying injection offset calculations
changed sign.

In Sec. 2.8, I make a further digression and relate the CO measurement spread with
the dispersion function as a means of measuring the magnitude of the pulse-to-pulse
momentum variations. I found that the magnitude of the pulse-to-pulse fractional
momentum variations is \( \sigma_\delta = 4.56 \times 10^{-5} \). This is about 10% of the momentum
spread in the injected beam.

Finally, with the BPM measurement well understood, I am ready to measure the
model predicted quantities of interest and other supporting experiments in Chap. 3.
The first set of experiments I discuss in Chap. 3 involve magnet hysteresis. I applied
measurements of the beam’s CO as a test to the effect of dipole hysteresis in Sec. 3.1.
The horizontal benders are employed as horizontal correctors in the measurement
of the ORM. It is important to know whether hysteresis in the dipoles effects the
reproducibility of the baseline CO in the ORM measurement. If the COs before and
after the horizontal orbit bump are not identical to within CO measurement spread,
then the dipole hysteresis is a factor, and the main dipole power supply should be
cycled in between each ORM kick, which is a very time consuming process. Luckily,
the baseline CO was not observed to change within measurement spread from an orbit
bump similar to those that will be applied in the ORM measurement. I conclude that
a \( \pm 4 \) A and back to the original set point change in the current of the PSR benders
does not measurably affect the CO.

The dipole hysteresis measurement illuminated a problem with the stability of the
horizontal CO at BPM 20 (SRPM92x). This prompted more detailed investigation in to the beam position measurement, the results of which are detailed in Chap. 2. The CO measurement at BPM 20 fluctuated so much that this BPM could not be applied in the ORM analysis.

The second hysteresis experiment investigated the affects of hysteresis in the quadrupole magnets and is reported in Sec. 3.2. This measurement was prompted by preliminary results from an ORM analysis, which suggested a $\sim$5% change in the strength of the defocusing quadrupoles. The difference in the quadrupole pole-tip magnetic field between the top and bottom of the hysteresis curves at the operations set point was found to be .53% in the defocusing quadrupoles and .39% in the focusing quadrupoles. Not observing the suggested value from the ORM analysis helped to discredit the model produced by the ORM analysis.

After the hysteresis experiments, the discussion of Chap. 3 moves to measurements of model predicted quantities not described yet, the betatron amplitude and dispersion functions. Remember, the betatron tune and phase measurements are direct results of the beam position measurement described in Chap. 2. I first report the results of the dispersion function measurement in Sec. 3.3. The dispersion function is defined as the change in the CO as a function of the fractional beam momentum. The CO measurement is well understood from Chap. 2, so I only needed to investigate the momentum measurement.

I compared two different methods for measurement of the beam momentum in the PSR. At the time, both methods were competing procedures, so I employed a rigorous statistical analysis to compare the two methods. Both methods make use of a time of flight (TOF) measurement but employ different conditions to convert the $\frac{\Delta T}{T_0}$ to a fractional momentum deviation, $\delta$. One method centers and un-centers the CO for each TOF measurement such that approximately the same orbit and path length is traveled for all beam momentum settings. This method is time consuming and much
Conclusions

Care needs to be taken so as to minimize the systematic errors, which are introduced in almost every step of the procedure. The method that won out at the end of the day makes use of the model momentum compaction factor, $\alpha_c$. I also introduce in this section the idea to use a calibrated BPM in the high dispersion region of the transport (LDPM03) for quick measurements of the beam momentum via the beam position at LDPM03. This last method will be applied in the characterization of the fringe fields of the extraction septa. Despite the differences in the momentum measurement methods, both methods resulted in the same dispersion function. I also found that the baseline model was able to predict this dispersion function within the measurement error.

I formulated a new fitting scheme, which takes into account both the dependent and independent variable uncertainties, in order to fit two sets of measured quantities. I employed this new fitting scheme in the dispersion function measurement because both the beam CO and fractional momentum deviation have known measurement spreads. I discuss the quality of this fitting scheme in Appendix A.2.

The last supporting measurement is the beta function measurement, Sec. 3.4. I employ the quadrupole perturbation method in the measurement of the beta functions. The beta function measurement requires a good betatron tune measurement, which I am confident in from Chap. 2. However, an interesting surprise awaited me in the analysis of the beta function measurement data. I observed that the baseline tune measurement (when the portable shunt was set to 0 A) drifted systematically during the measurement. I observed a $\sim 0.004$ change in the horizontal tune and a $\sim 0.002$ change in the vertical tune. This shift was not due to energy drift through chromaticity because the baseline CO was constant for the entire measurement. I concluded that this drift was due to the hysteresis in the quadrupoles. The baseline model was able to reproduce this effect in the tunes with a $-0.11\%$ and $-0.125\%$ change in the focusing and defocusing quadrupole strengths respectively. These values are
within the limits of the quadrupole hysteresis measurement discussed in Sec. 3.2 and thus plausible for hysteresis affects.

I found that the baseline model predicted beta functions agreed with measurement. It was found that the dominating error in the beta function measurement is the systematic error from the uncertainty in the current to gradient length fits employed in the model and in the analysis.

The beta function measurement dataset was perfect for two additional calculations, and I was unable to resist following through in their applications. The first of these additional calculations is a comparison of the action at each BPM, Sec. 3.5. Since the beta function was measured independent of the amplitude of the betatron oscillation, I could divide the two and obtain the action as each BPM. Now if the BPMs possessed different gains, which only affects the amplitude measurement, the relative difference in the gain of two BPMs is apparent in the difference in the action calculated at each BPM. I also compared the action suggested by the baseline model Courant-Snyder parameters at the foil and the measured injection offset. I found that the action at all BPMs agreed within measurement error. I also noted that the action calculated from the injection offset was on the low side of the distribution of actions at the BPMs. I believe this measurement should be redone at larger injection offsets so that the difference in the BPM gains is more easily observed.

The second additional calculation is a beam-based alignment analysis, discussed in Sec. 3.6. The beta function measurement was not performed under ideal circumstances for the beam-based alignment analysis, but I was still able to obtain limits for a future beam-based alignment study. Starting from the minimum measurable change in the CO, I was able to deduce the necessary minimum dipole kick and thus the smallest initial CO offset in the quadrupole for a good measurement of the BPM offset in the quadrupole. The results of this analysis suggested that some off the BPMs were misaligned with the center of the quadrupoles by as much as 3 mm. This
cannot be a physical misalignment because of there is not enough room, but could easily be an electronic error due to bad terminations resulting from radiation damage.

I discuss the model improvement experiments in Chap. 4. The first model improvement experiment is the ORM measurement, Sec. 4.1. I employed Linear Optics from Closed Orbits (LOCO) as the ORM analysis code. I first studied LOCO to see how far away from the solution the baseline model could be to obtain the correct result. Next I reanalyze the results of previous ORM measurement attempts in the PSR. These measurements suffered most from a drifting dipole power supply. I formulated an experimental procedure to minimize the problems from the previous measurements. As it turns out, implementing this new procedure was a very good idea because I misdialed the return to baseline set point of the one of the vertical correctors, which would have destroyed ORM measurement with the old procedure. Because I was not confident in the given corrector gains, I chose to fit the ORM per ampere of the corrector kick in LOCO. So LOCO fits for the corrector gain instead of the corrector kick. I ran the LOCO analysis without coupling. LOCO was only allowed to modify the BPM gains, corrector gains, and the quadrupole strengths. LOCO was able to reduce the difference in the measured and model ORM to the noise of the measurement and a $\chi^2$/DOF of 5.4.

The LOCO results indicated that most of the BPMs had gains within ±5%. BPMs 17 and 19 (the horizontal diamond-type BPMs, SRPM81x and 91x) had the largest adjusted gains of 10%. LOCO fit a systematically lower corrector kick gain for all of the correctors. The results of the horizontal corrector gains could be from hysteresis, but LOCO was unable identify the difference between the 7″ and 11″ vertical correctors. LOCO indicated a ~2.5% systematic decrease in the strength of the defocusing quadrupoles. The 2.5% was not found in the power supply output or read back in the controls system.

The results of the LOCO fit maybe applied to construct a LOCO fitted model.
The LOCO fitted model may be the improved model of the PSR that I am looking for, so I should compare it with measurement and with the baseline model. Such comparison shows that the LOCO fitted model improves the vertical tune prediction; however, it does so at the expense of the beta function prediction. This result is unsatisfactory because the LOCO fitted model did not retain all of the predictive capabilities of the baseline model.

I report the results of the characterization of the PSR extraction septa fringe fields in Sec. 4.2. I measured the integrated dipole, quadrupole, and sextupole components of the fringe fields observed by the circulating beam as a function of septa on and off and of trim coil current. The results of this measurement will be included in the improved model of the PSR as a thin lens multipole. Measuring the magnetic components as a function of trim coil current allows for an operational model. The measurements of the dipole and quadrupole components were successful, but the conditions of the PSR were led too far away from the tuned set points during the measurement of the sextupole components for reliable sextupole results for all trim coil currents. Lastly, I compared the results of the second extraction septum (RODM02) with the results of a 3D magnetic field simulation and found the quadrupole component to agree.

The last model improvement experiment is parallel ray tracing through 3D magnetic fields from simulation of the PSR dipoles as discussed Sec. 4.3. The ray tracing results may be employed to measure the focal length of the edge focusing. The focal length from the ray tracing may be compared to the focal length of the edge focusing employed in the baseline model. I found a systematic increase in the focal length from the ray tracing data for all PSR benders. A longer focal length indicates less focusing and a smaller tune. The focal length difference of the common 36° benders is only .5 m, but has the most effect on the vertical tune.

The focal length of the edge focusing from the ray tracing may be constrained in
the model by the fringe field integral parameter. This changed the fringe field integral parameter for the common 36° benders from .54 to .90. Constraining the focal length of the edge focusing in the model reduces the systematic mistreatment of the vertical betatron phase.

Now I can construct an improved model of the PSR by including the quadrupole component of the septa fringe fields and by constraining the focal length of the edge focusing. This improved model was compared with measurement and the baseline and LOCO fitted models in Sec. 4.4.2. I found that the improved model made a better tune prediction while preserving or slightly enhancing the predictive capabilities of the baseline model. Now that I have established the improved model at one PSR set point, I need to verify the improved model at several different PSR set points far away from nominal operations (Sec. 4.4.3) to be sure that it really is an improved model. I did this at three different set points, each with a betatron tune much different than typical operations. Comparison of the improved model with measurement and the baseline model at these other set points shows that the improved model does better at predicting the quantities of interest in a linear model (the betatron tune, phase, beta function, and dispersion function) than the baseline model.

Thus, I was able to experimentally improve, establish, and verify an improved model of the PSR.

The last discussion that I pursue in this thesis is application of ICA to the PSR, Chap. 5. ICA is one of the hot topics in accelerator physics nowadays because of its capability to analyze massive amounts of data at one time. The typical method to apply ICA in an accelerator employs turn-by-turn BPM data at each BPM around the ring. Typically 1000 turns or more are used in the analysis. The PSR is not optimal for ICA in this manner because from Chap. 2, I can only collect 40 turns of the beam position data. So I needed to formulate a new method to apply ICA in the PSR. One thing that the PSR has that these other accelerators do not is a long
bunch. The PSR bunch fills about \( \frac{5}{6} \) of the circumference, \( \sim 72 \) m. So I can apply ICA along the beam pulse. Now each time slice along the beam becomes a BPM in the ICA, and the ICA modes describe motion within the beam.

Because ICA has never been applied in this manner and this whole venture is a fishing expedition, I analyze simulated data with ICA to gain understanding in how ICA handles certain types of signal motion. Most of the beam motion within the beam forms a continuum in frequency space which means that the motions are not really independent, thus it is important to understand how ICA deals with a continuum of motion.

Lastly, I end the thesis with a highlight reel, if you will, of the experimentally identified ICA modes. The most notable ICA mode is the 201.25 MHZ linac frequency mode, discussed in Sec. 5.3.4 III. ICA was able to diagnose a misdialed frequency input into the timing generator at the front end of the accelerator. This caused the PSR to be operated at the 72.009 subharmonic of the linac frequency instead of the 72.07. ICA was able to observe the difference in the subharmonic operation and exactly predict the number input to the timing generator.

The ability to properly model the linear lattice functions is very important to operations and future studies of the PSR. The model of the PSR is employed for ring setup and tuning. Many of the parameters of interest, like the injection offset, are measured via a model. It is common for the operators to execute orbit bumps in certain sections of the PSR, which depend only on a model. Perhaps the improved model will help us to understand why the minimum beam loss tune dialed in by the operators after days of optimizing is so different from the basic physics tune, which comes directly from the model.

I have improved the understanding of the beam position measurement in the PSR. Prior to my study, the quality of the beam position measurement was unknown. I was the first to study this measurement in detail. And I found that I could obtain
quality results for fundamental accelerator physics measurements after identifying and removing the data acquisition errors. I have applied this improved understanding of the beam position measurement to measure the lattice functions of the PSR. I was also the first to measure and document the magnitude of the pulse-to-pulse momentum variations in the PSR.

An improved model of the PSR has increased our understanding of the machine. Now with a solid foundation, LANSCE physicists are able to track particles in loss studies, electron cloud studies, and space charge studies more accurately. This improved understanding of the PSR is crucial for future upgrades, especially for increasing beam current to the 1L target for increased neutron production at the Lujan Center.
Fitting data, whether to affirm theoretical predictions or for data analysis, is an important part of any study, which is why it is employed frequently in this thesis. The analysis in this thesis makes use of several different fitting methods. The type of fitting method employed is dictated by the circumstances. For completeness of my thesis story, I include brief descriptions on the standard linear regression fitting methods (Appx. A.1) and a more detailed discussion for an improved fitting scheme for both dependent and independent variable measurement errors in Appx. A.2. There is an apparent asymmetry of the dependent and independent variables in the $\chi^2$ for the fitting scheme involving both dependent and independent variable uncertainties. This asymmetry is studied in Appx. A.3. Lastly, in Appx. A.4, I compare different fitting schemes to convert a measured horizontal beam position at a calibrated LDPM03 to a fractional momentum deviation.
A.1 Linear Regression

Linear regression is a fitting method which yields an analytic solution for the fitting parameters. Given a fitting function, the solution is found by minimizing a quality factor, usually the sum of squares of residuals (SSR) or the $\chi^2$, with respect to each fitting parameter and solving the resulting system of equations. In order to obtain a system of linear equations, the fitting equation must be linear in the fitting parameters. The most common type of linear fitting function is the polynomial. In this case, the fitting parameters are the coefficients of the polynomial. The line fit is invoked most often in this thesis to solve for the slope of the line fit to the data.

Two forms of linear regression appear in this thesis. The first form of linear regression fits the fitting function to a dependent variable without uncertainty as a function of a known, errorless independent variable (Appx. A.1.1), and the second form fits the fitting function to a dependent variable with an known uncertainty as a function of a known, errorless independent variable, Appx. A.1.2.

A.1.1 Linear regression without uncertainty

Linear regression without uncertainty in the dependent variable is the most basic form of fitting. Given a set of observed dependent variables $\{y_i\}$ as a function of independent variables $\{x_i\}$ and a fitting function $f(x_i; \vec{a})$ dependent on the independent variables and linearly dependent on the fitting parameters $\vec{a}$, the quality factor may be defined as the SSR,

$$SSR = \sum_{i}^{N} (y_i - f(x_i; \vec{a}))^2,$$  \hspace{1cm} (A.1)
A.1 Linear Regression

where \( N \) is the number of data points. A system of equations is formed by minimizing the SSR with respect to each fitting parameter,

\[
\frac{\partial \text{SSR}}{\partial a_j} = 0 \quad \text{where} \quad j = 1, 2, 3, \ldots, J
\]

where \( J \) is the number of fitting parameters. The system of equations in Eq. (A.2) will be linear in the fitting parameters and may be solved analytically with Cramer’s rule. A maximum likelihood (ML) error analysis may be applied to calculate the fitting error on the fitting parameters.

A fit of a line to a set of data may be applied as an example of basic linear regression. In this case the fitting function is

\[
f(x_i; \vec{a}) = a + bx_i
\]

where \( a \) (y-intercept) and \( b \) (slope) are the fitting parameters. The quality factor for this fit is defined

\[
\text{SSR} = \sum_i^N \left[ y_i - (a + bx_i) \right]^2
\]

and whose minimization with respect to each fitting parameter yields the following system of equations

\[
\frac{\partial \text{SSR}}{\partial a} = 0 = \sum_i^N y_i - aN - b \sum_i^N x_i
\]

\[
\frac{\partial \text{SSR}}{\partial b} = 0 = \sum_i^N x_iy_i - a \sum_i^N x_i - b \sum_i^N x_i^2
\]

or

\[
\begin{pmatrix}
\sum_i^N y_i \\
\sum_i^N x_iy_i
\end{pmatrix} = \begin{pmatrix}
N & \sum_i^N x_i \\
\sum_i^N x_i & \sum_i^N x_i^2
\end{pmatrix} \begin{pmatrix}
a \\
b
\end{pmatrix}.
\]

Applying Cramer’s rule to the system of linear equations in Eq. (A.6) results in the analytical solutions for the fitting parameters

\[
a = \frac{\sum_i^N y_i \sum_i^N x_i^2 - \sum_i^N x_i \sum_i^N x_iy_i}{N \sum_i^N x_i^2 - \left( \sum_i^N x_i \right)^2}
\]
and
\[ b = \frac{N \sum_{i}^{N} x_i y_i - \sum_{i}^{n} x_i \sum_{i}^{N} y_i}{N \sum_{i}^{N} x_i^2 - \left( \sum_{i}^{N} x_i \right)^2}. \] (A.8)

And as stated previously, the fitting error on the y-intercept and the slope may be found by a ML error analysis.

**A.1.2 Linear regression with uncertainty**

The second form of linear regression is very similar to the first except now the set of observed dependent variables \( \{y_i\} \) possesses some known \textit{rms} uncertainty \( \sigma_{y_i} \). The \( \chi^2 \) is defined as the quality factor for the fit of a fitting function \( f(x_i; \vec{a}) \) linearly dependent on the fitting parameters \( \vec{a} \),

\[ \chi^2 = \sum_{i}^{N} \left( \frac{y_i - f(x_i; \vec{a})}{\sigma_{y_i}} \right)^2. \] (A.9)

Minimizing the \( \chi^2 \) with respect to each fitting parameter yields a system of \( J \) equations

\[ \frac{\partial \chi^2}{\partial a_j} = 0 \quad \text{where} \quad j = 1, 2, 3, \ldots, J \] (A.10)

where \( J \) is the number of fitting parameters. Since the fitting function \( f(x_i; \vec{a}) \) is linear in the fitting parameters \( \vec{a} \), the system of equations defined in Eq. (A.10) is also linear in the fitting parameters. Thus, the analytic solution for the fitting parameters may be found by Cramer’s rule. Because the uncertainty on the dependent variable is known, the fitting error may be found directly from the analytic solution for the fitting parameters. Applying the ML error analysis yields the same fitting errors.

The line fit may again be applied as an example. The fitting function described in Eq. (A.3) yields a quality factor

\[ \chi^2 = \sum_{i}^{N} \left[ \frac{y_i - (a + bx_i)}{\sigma_{y_i}} \right]^2. \] (A.11)
A.1 Linear Regression

Minimizing the $\chi^2$ with respect to each fitting parameter generate a system of equations

$$\frac{\partial \chi^2}{\partial a} = 0 = \sum_i^N \frac{y_i}{\sigma_{y_i}^2} - a \sum_i^N \frac{1}{\sigma_{y_i}^2} - b \sum_i^N \frac{x_i}{\sigma_{y_i}^2}$$

$$\frac{\partial \chi^2}{\partial b} = 0 = \sum_i^N \frac{x_i y_i}{\sigma_{y_i}^2} - a \sum_i^N \frac{x_i}{\sigma_{y_i}^2} - b \sum_i^N \frac{x_i^2}{\sigma_{y_i}^2}$$

(A.12)

or

$$\begin{pmatrix}
\sum_i^N \frac{y_i}{\sigma_{y_i}^2} \\
\sum_i^N \frac{x_i y_i}{\sigma_{y_i}^2}
\end{pmatrix} =
\begin{pmatrix}
\sum_i^N \frac{1}{\sigma_{y_i}^2} & \sum_i^N \frac{x_i}{\sigma_{y_i}^2} \\
\sum_i^N \frac{x_i}{\sigma_{y_i}^2} & \sum_i^N \frac{x_i^2}{\sigma_{y_i}^2}
\end{pmatrix}
\begin{pmatrix}
a \\
b
\end{pmatrix}.$$  

(A.13)

The solutions for the fitting parameters may be solved with Cramer’s rule

$$a = \frac{\sum_i^N \frac{1}{\sigma_{y_i}^2} \sum_i^N \frac{x_i^2}{\sigma_{y_i}^2} - \sum_i^N \frac{x_i y_i}{\sigma_{y_i}^2} \sum_i^N \frac{x_i}{\sigma_{y_i}^2}}{\sum_i^N \frac{1}{\sigma_{y_i}^2} \sum_i^N \frac{x_i^2}{\sigma_{y_i}^2} - \left( \sum_i^N \frac{x_i}{\sigma_{y_i}^2} \right)^2}$$

(A.14)

and

$$b = \frac{\sum_i^N \frac{1}{\sigma_{y_i}^2} \sum_i^N \frac{x_i y_i}{\sigma_{y_i}^2} - \sum_i^N \frac{x_i y_i}{\sigma_{y_i}^2} \sum_i^N \frac{x_i}{\sigma_{y_i}^2}}{\sum_i^N \frac{1}{\sigma_{y_i}^2} \sum_i^N \frac{x_i^2}{\sigma_{y_i}^2} - \left( \sum_i^N \frac{x_i}{\sigma_{y_i}^2} \right)^2}.$$  

(A.15)

Propagating the uncertainty of $y_i$ to the fitting parameters or applying a ML error analysis to the fit yields the same fitting errors on the fitting parameters

$$\sigma_{a}^2 = \frac{\sum_i^N \frac{x_i^2}{\sigma_{y_i}^2}}{\sum_i^N \frac{1}{\sigma_{y_i}^2} \sum_i^N \frac{x_i^2}{\sigma_{y_i}^2} - \left( \sum_i^N \frac{x_i}{\sigma_{y_i}^2} \right)^2}$$

(A.16)

and

$$\sigma_{b}^2 = \frac{\sum_i^N \frac{x_i y_i}{\sigma_{y_i}^2}}{\sum_i^N \frac{1}{\sigma_{y_i}^2} \sum_i^N \frac{x_i^2}{\sigma_{y_i}^2} - \left( \sum_i^N \frac{x_i}{\sigma_{y_i}^2} \right)^2}.$$  

(A.17)

Thus, the linear regression fit for observed dependent variables $\{y_i\}$ with corresponding uncertainty $\sigma_{y_i}$ weights the fit according to the error associated with each dependent data point.
A.2 Fitting with both Dependent and Independent Variable Errors

Something interesting occurs when applying Eqs. (1.67) or (3.6) to fit for the dispersion function and on-momentum CO. Notice that there are two measured quantities; in Eq. (1.67), the measured off-momentum CO and $\delta$ have measurement errors, and both position measurements at LDPM03 and the PSR BPM have measurement errors in Eq. (3.6). It is well known how to fit for random, mean zero, Gaussian measurement errors in the dependent variable with a maximum likelihood or least squares method. Here is derived a fitting scheme for fitting a set of data with measurement errors in both dependent and independent variables.

1 Starting with the standard errors-in-variables model assumption, say there is a set of observed variables $\{x_i, y_i\}$ with $rms$ standard deviations or measurement spreads $\{\xi_i, \epsilon_i\}$ and that there exists a set of variables with no errors $\{\eta_i, f(\eta_i; \vec{a})\}$, where $i$ is an index over measurement number and $\vec{a}$ is a vector of fitting parameters. In order to maximize the probability that $f(\eta_i; \vec{a})$ fits $\{x_i, y_i\}$ the $\hat{x}$ and $\hat{y}$ components of $\vec{d}_i$ relative to $\{\xi_i, \epsilon_i\}$ need to be minimized, where $\vec{d}_i$ is the residual vector between the fitted point $(\eta_i, f(\eta_i; \vec{a}))$ and the measured point $(x_i, y_i)$. Thus a goodness of fit quality factor for this fitting scheme can be written as

$$\chi^2 = \sum_i \left[ \left( \frac{x_i - \eta_i}{\xi_i} \right)^2 + \left( \frac{y_i - f(\eta_i; \vec{a})}{\epsilon_i} \right)^2 \right]. \quad (A.18)$$

The least squares problem is solved by minimizing the $\chi^2$ with respect to the

---

1Refer to Fig. A.1 for an illustration of the fitting scheme for fitting data with both horizontal and vertical measurement errors.
fitting parameters ($\vec{a}$) and the assumed error-less independent variable, $\eta_i$.

\[
\frac{\partial \chi^2}{\partial a_j} = 0 = \sum_i \left[ \frac{y_i - f(\eta_i; \vec{a})}{\epsilon_i^2} \right] \frac{\partial f(\eta_i; \vec{a})}{\partial a_j}
\]

\[
\frac{\partial \chi^2}{\partial \eta_i} = 0 = \frac{x_i - \eta_i}{\epsilon_i^2} + \frac{y_i - f(\eta_i; \vec{a})}{\epsilon_i^2} \frac{\partial f(\eta_i; \vec{a})}{\partial \eta_i}.
\]

Notice that the assumed error-less independent variable of the fitting function is now also a fitting parameter. This means that for all fitting functions save the trivial fitting functions $f(\eta; \vec{a}) = a_1$ and $f(\eta; \vec{a}) = a_1 + \eta$, Eq. (A.19) is nonlinear and a nonlinear least squares routine must be applied to solve for the $\eta_i$’s and $\vec{a}$. Including the $\eta_i$’s as fitting parameters greatly increases the number of fitting parameters, but in doing so, the measured independent variables ($x_i$’s) are counted in the size of the sample set because each measured point, $(x_i, y_i)$, is really two different measurements. This directly cancels the affect of including the $\eta_i$’s as fitting parameters in calculating the number of degrees of freedom for the fit.

In the case of the dispersion function measurements, the fitting function is a line,

\[
f(\eta; \vec{a}) = a + b\eta_i,
\]

(A.20)
A.3 Fitting Independent Variable as a Function of Dependent Variable

and Eq. (A.19) reduces to

\[
\frac{\partial \chi^2}{\partial a} = 0 = \sum_i \frac{y_i - a - b \eta_i}{\epsilon_i^2}.
\]

\[
\frac{\partial \chi^2}{\partial b} = 0 = \sum_i \frac{(y_i - a - b \eta_i) \eta_i}{\epsilon_i^2}.
\]

\[
\frac{\partial \chi^2}{\partial \eta_i} = 0 = \frac{x_i - \eta_i}{\xi_i^2} + \frac{(y_i - a - b \eta_i) b}{\epsilon_i^2}.
\]

One might worry about the apparent asymmetry between the measured independent and dependent data points \((x_i, y_i)\) and their errors \((\xi_i, \epsilon_i)\) in Eqs. (A.18), (A.19), and (A.21). This was taken into consideration while fitting the dispersion function and tested by switching the \(x_i\)’s and \(y_i\)’s and fitting \(\{y_i, x_i\}\). The results of this variation are discussed in Appx. A.3.

A.3 Fitting Independent Variable as a Function of Dependent Variable

To study whether the apparent asymmetry in the \(\chi^2\) of Eq. (A.18) will affect the fit, a second fitting function was also used to fit the BPM and TOF data. The form of this second fitting function is the inverse of Eq. (3.8) written such that the independent variable is a function of the dependent variable, \(\delta_i(x_i)\),

\[
\delta_i = \frac{a}{b} + \frac{x_i}{b}, \quad \text{or}
\]

\[
\delta_i = c + dx_i, \quad \text{where}
\]

\[
c = \frac{a}{b} = \frac{a_{BPM}}{D_{BPM}}, \quad \text{and}
\]

\[
d = \frac{1}{b} = \frac{1}{D_{BPM}}.
\]

and \(c\) is the fractional momentum deviation when the CO or position equals zeros and \(d\) is the inverse of the dispersion function at the BPM. The error on the fitted
A.3 Fitting Independent Variable as a Function of Dependent Variable

parameters $a$, $b$, $c$, and $d$ are derived by a maximum likelihood (ML) error analysis. In the case of fitting $\delta_i(x_i)$ with Eq. (A.23), the fitted values and fitting errors calculated for $c$ and $d$ are propagated to the more important quantities $a_{BPM}$ and $D_{BPM}$.

| Results of Switching Dependent and Independent Variables in Fitting Scheme |
|-----------------------------|-----------------------------|-----------------------------|
|                             | Fitting $x_i(\delta_i)$    | Fitting $\delta_i(x_i)$    |
| $a_{BPM}$ [mm]              | .02                        | .02                        |
| $\sigma_{a_{BPM}}$ [mm]     | .11                        | .11                        |
| $D$ [m]                     | $-2.767$                   | $-2.769$                   |
| $\sigma_D$ [m]              | .081                       | .081                       |
| $\chi^2$/DOF               | .15002                     | .14954                     |

**Table A.1:** The dispersion fitting results at BPM 1 (SRPM01) with the center method. The data was fit with dependent variable as a function of independent variable ($x_i(\delta_i)$) and independent variable as a function of dependent variable, $\delta_i(x_i)$. Reported in this table are the resulting values and fitting errors for the on-momentum CO ($a_{BPM}$) and the dispersion function, $D$. The actual fitting values and fitting errors in the $\delta_i(x_i)$ version of the fit needed to be propagated to the on-momentum CO and dispersion function for comparison.

As a means to test the apparent asymmetry in the $\chi^2$ of Eq. (A.18) and the quality of the fitting scheme, the independent variable in the fits for the dispersion function was fit as a function of the dependent variable ($\delta_i(x_i)$) with the fitting function of the form in Eq. (A.23). The results were compared to the fits with the standard fitting scheme, $x_i(\delta_i)$. Although the fitting parameters themselves ($a$, $b$, $c$, and $d$) represent different quantities, the fitted values and fitting errors can be propagated to the dispersion function and on-momentum CO. A comparison of the results of a dispersion measurement from both versions of the fitting schemes for BPM 1 applying the center method of momentum measurement from Sec. 3.3 are presented in Tab.
A.4 Comparison of Fitting Scheme with a Calibrated LDPM03

A.1.

As shown in Tab. A.1, the fitted values and fitting errors agree well for both versions of the fitting scheme: \(x_i(\delta_i)\) and \(\delta_i(x_i)\). This study lends confidence to the fitting scheme introduced in Sec. A.2 and lays to rest any worries about problems caused by the apparent asymmetry in the \(\chi^2\) for this fitting scheme.

A.4 Comparison of Fitting Scheme with a Calibrated LDPM03

There are many different methods that may be applied to employ a calibrated LDPM03 in experiments requiring measurements of the beam momentum. In this section, six of these methods are derived including the fitting scheme of Eq. (4.7) applied to the momentum measurements in Sec. 4.2. All six methods are compared with each other in order to prove a quality line fit for measurements of the dispersion function, natural chromaticity, and chromaticity. These six methods are variants of the fitting scheme in Eq. (A.18). While the derivations and fittings schemes presented in this section are for the dispersion function measurements, they can easily be converted for application to chromaticity measurements by substituting \(\nu\) and \(\sigma_{\nu}\) for \(x_{PSR}\) and \(\sigma_{PSR}\) and substituting \(C\) for \(D_{PSR}\).

All of the fitting scheme variants can be derived from the following relationships between the measured position and the momentum,

\[
x_{LD} = a_{LD} + D_{LD}\delta
\]

\[
x_{PSR} = a_{PSR} + D_{PSR}\delta,
\]

(A.26)

where \(x\) is the measured off-momentum CO or position, \(a\) is the on-momentum CO in the ring or calibrated on-momentum position at LDPM03, \(D\) is the dispersion function known at LDPM03 though calibration and fit at the PSR BPM, \(\delta\) is the fractional
momentum deviation, and indices \( LD \) and \( PSR \) indicate values at LDPM03 and a
PSR BPM respectively.

Only horizontal data at LDPM03 is applied in these fits for the measurements
involving the beam momentum. This is true even for fits of vertical data in the
PSR. The horizontal position at LDPM03 is employed, since the dispersion function
is largest at LDPM03, it has the best position to momentum resolution. For mea-
surements of the dispersion function, natural chromaticity, and chromaticity, the CO
or tune in the PSR are fit to a line versus the fractional momentum deviation. But
in this case, the momentum is not directly measured. So instead of inferring a \( \delta \)
from the measured horizontal position at LDPM03, propagating measurement errors,
and then fitting the CO or tune with respect to an inferred quantity, it is better to
fit only measured quantities. Each of the six methods presented in this section will
accomplish this means.

The first variant is named in this thesis as fitting scheme ‘xy’ because the depen-
dent variable is written as a function of the independent variable (in the order). This
fitting scheme fits a line to \( x_{PSR} \) as a function of \( x_{LD} \). Solving Eq. (A.26) for \( x_{PSR} \)
results in

\[
x_{PSR} = a_{PSR} - \frac{D_{PSR}}{D_{LD}} a_{LD} + \frac{D_{PSR}}{D_{LD}} x_{LD},
\]

such that the slope of the line \( x_{PSR}(x_{LD}) \) is equal to the ratio of the dispersion
functions at LDPM03 and the PSR BPM,

\[
b_{xy} = \frac{D_{PSR}}{D_{LD}}
\]

and the y-intercept of \( x_{PSR}(x_{LD}) \) is

\[
a_{xy} = a_{PSR} - \frac{D_{PSR}}{D_{LD}} a_{LD}.
\]
Comparison of Fitting Scheme with a Calibrated LDPM03

The values of $a_{LD}$ and $D_{LD}$ are known via the calibration of LDPM03.

The manipulations of the relationships in Eq. (A.26) effectively write $\delta$ in terms of the horizontal position at LDPM03 and then substitute back in such that the CO in the PSR is a function of the measured quantity ($x_{LD}$) instead of an inferred quantity, $\delta$. Following the model of Eq. (A.18), the $\chi^2$ for this fit is

$$\chi^2_{xy} = \sum_i \left[ \left( \frac{x_{LD,i} - \eta_i}{\sigma_{LD,i}} \right)^2 + \left( \frac{x_{PSR,i} - a_{xy} - b_{xy}\eta_i}{\sigma_{PSR,i}} \right)^2 \right],$$

(A.30)

where $\sigma_{LD,i}$ and $\sigma_{PSR,i}$ are the measurement error on the average position at LDPM03 and CO at the ring BPM respectively. The fitting parameters in Eq. (A.30) are $a_{xy}$ and $b_{xy}$, which are defined above in Eqs. (A.28) and (A.29), and the $\eta$’s, which fit for the errorless horizontal position at LDPM03.

Note the asymmetry in the $\chi^2$ of Eq. (A.30) between the dependent and independent variables. As is done in Appx. A.3 to test the fitting scheme, the fitting scheme may be inverted such that the independent variable becomes a function of the dependent variable. This may be done by solving Eq. (A.26) for $x_{LD}$,

$$x_{LD} = a_{LD} - \frac{D_{LD}}{D_{PSR}}a_{PSR} + \frac{D_{LD}}{D_{PSR}}x_{PSR},$$

(A.31)

$$= a_{yx} + b_{yx}x_{PSR}$$

The $\chi^2$ for this fitting scheme is the effectively the same as the $\chi^2$ in Eq. (A.30) with the PSR and LD indices switched,

$$\chi^2_{yx} = \sum_i \left[ \left( \frac{x_{PSR,i} - \eta_i}{\sigma_{PSR,i}} \right)^2 + \left( \frac{x_{LD,i} - a_{yx} - b_{yx}\eta_i}{\sigma_{LD,i}} \right)^2 \right].$$

(A.32)

Here, $\eta$ is the fitted errorless CO at the PSR BPM. This fitting scheme is named ‘yx’ in this thesis because the independent variable is written as a function of the dependent variable (not in the order or way expected). The slope and y-intercept in the ‘yx’ fitting scheme are

$$b_{yx} = \frac{D_{LD}}{D_{PSR}} \quad \text{and} \quad a_{yx} = a_{LD} - \frac{D_{LD}}{D_{PSR}}a_{PSR}.$$
Notice that the slope of the ‘yx’ fitting scheme is inversely proportional to the dispersion function in the PSR, where as the slope of the ‘xy’ fitting scheme is directly proportional to the dispersion function in the PSR. This relationship between the ‘xy’ and ‘yx’ fitting schemes and $D_{PSR}$ is expected, but it will have interesting consequences when propagating the fitting errors from $b_{yx}$ to the dispersion function in the PSR.

The third variant of Eq. (A.30) applied to measurements with data from a calibrated LDPM03 involves simultaneously fitting lines to the positions at LDPM03 and at the ring BPM with respect to the fractional momentum deviation, as in Eq. (A.26). Here, the measured quantities $x_{LD}$ and $x_{PSR}$ are applied to explicitly fit for the fractional momentum deviation. This fitting scheme also has the advantage that the on-momentum position and the dispersion function at the PSR BPM are fitting parameters.

The $\chi^2$ for this fit is the same as applying the $\chi^2$ in Eq. (A.11) for two different fits of lines at the different locations of LDPM03 and a PSR BPM,

$$\chi^2_D = \sum_i \left[ \left( \frac{x_{LD,i} - a_{LD} - D_{LD} \delta_i}{\sigma_{LD,i}} \right)^2 + \left( \frac{x_{PSR,i} - a_{PSR} - D_{PSR} \delta_i}{\sigma_{PSR,i}} \right)^2 \right], \quad (A.34)$$

where $a_{LD}$ and $D_{LD}$ are known via the calibration of LDPM03 and $\delta$, the fractional momentum deviation, is also fit. This fitting scheme is referred to as ‘D’ for dispersion in this thesis because it directly fits for the dispersion function at the PSR BPM.

A fourth fitting scheme is a direct variant on the ‘D’ method. The fitted $\delta$’s for each momentum setting, $i$ in the summation of Eq. (A.34), should be the same because the PSR data was taken at the same time and momentum setting. A fitting scheme can constrain the fitted $\delta$’s to be the same for each measurement if the data is simultaneously fit at LDPM03 and all of the PSR BPMs in a grand fit. Indeed this fitting scheme is referred to in this thesis as ‘GF’ for grand fit.

The $\chi^2$ for the ‘GF’ fitting scheme is an extension of the $\chi^2$ from the ‘D’ fitting
The comparison of fitting scheme with a calibrated LDPM03 as in Eq. (A.34). The $\chi^2$ for the ‘GF’ fitting scheme has additional terms for each BPM in the PSR,

$$\chi^2_{GF} = \sum_i \left( \left( \frac{x_{LD,i} - a_{LD} - D_{LD}\delta_i}{\sigma_{LD,i}} \right)^2 + \sum_j \left( \frac{x_{PSR,j} - a_{PSR,j} - D_{PSR,j}\delta_i}{\sigma_{PSR,j}} \right)^2 \right),$$  

where the index $j$ sums over each PSR BPM. The other variables in Eq. (A.35) are the same as Eq. (A.34).

The last two fitting variants to be discussed in this section are the linear regression versions of the ‘xy’ and ‘yx’ fitting schemes entitled here as ‘x’ and ‘y’ respectively. Including comparison with linear fits will help to discern the benefits of fitting for the errorless independent variable which is what makes the line fit nonlinear when taking into account both dependent and independent variable uncertainties.

The ‘x’ fitting scheme fits a line to $x_{PSR}(x_{LD})$ with

$$\chi^2_x = \sum_i \left( \frac{x_{PSR,i} - a_x - b_xx_{LD}}{\sigma_{PSR,i}} \right)^2.$$

For this fit, the measurement error on the independent variable is assumed to be zero. The y-intercept and slope for the ‘x’ fitting scheme are the same as the y-intercept and slope for the ‘xy’ fitting scheme in Eqs. (A.28) and (A.29),

$$a_x = a_{xy} = a_{PSR} - \frac{D_{PSR}}{D_{LD}}a_{LD} \quad \text{and} \quad b_x = b_{xy} = \frac{D_{PSR}}{D_{LD}}.$$  

Likewise, the ‘y’ fitting scheme fits a line to $x_{LD}(x_{PSR})$ assuming that CO measurement at the PSR BPM is errorless with

$$\chi^2_y = \sum_i \left( \frac{x_{LD,i} - a_y - b_yx_{PSR}}{\sigma_{LD,i}} \right)^2,$$

where for the ‘x’ fitting scheme, $a_y$ and $b_y$ are the same as $a_{yx}$ and $b_{yx}$ from Eq. (A.33),

$$a_y = a_{yx} = a_{LD} - \frac{D_{LD}}{D_{PSR}}a_{PSR} \quad \text{and} \quad b_y = b_{yx} = \frac{D_{LD}}{D_{PSR}}.$$
Now, finally with all of the six methods for applying a calibrated LDPM03 to measure the dispersion function, natural chromaticity, and chromaticity derived, it is time to perform the comparisons. The first comparison will be to compare the actual fit in $x_{PSR}(x_{LD})$ space. Figure A.2 shows the fit for the dispersion function at BPM1 for each of the six fitting schemes. It is very apparent that all of the fitting schemes yield very similar solutions for the fit of the dispersion function at BPM 1. All of the fitting schemes yield good fits with $\chi^2$/DOF around one and $R^2$’s very close to one. Note that the fitting schemes with the largest $\chi^2$/DOF for the dispersion fit at BPM 1 are the two linear fitting schemes ‘x’ and ‘y’.

Since the fits in Fig. A.2 are all very similar it should not be surprising that the fitted dispersion function and on-momentum position for each BPM in the PSR from
each fitting scheme also agree. However, what is surprising is that the measured
dispersion function for fitting schemes ‘xy’, ‘yx’, and ‘D’ agree exactly. Even if they
do not match exactly, the other fitting schemes yield fitted dispersion functions that
agree within 2 cm. The baseline dispersion function measured by each fitting scheme
with both PSR extraction septa off and the trim coils set to zero is shown in Fig. A.3.

Figure A.3: (Color) The measured dispersion function for each of the six fitting schemes. The fit-
tting schemes are shown solid lines and blue circles for ‘xy’, green squares for ‘yx’, red left point-
ing triangles for ‘D’, black right pointing triangles for ‘GF’, magenta up pointing triangles for
‘x’, and cyan down pointing triangles for ‘y’. The vertical line separates horizontal (left) and
vertical (right) BPMs.

The fitted on-momentum positions from each fitting scheme agree within 7 µm. Again, the ‘xy’, ‘yx’, and ‘D’ fitting schemes yield the exact same results. This is
amazing since most of the fitting schemes do not fit for the dispersion or on-momentum position directly.

However, the fitting errors on the dispersion and on-momentum fitting parameters are more important when comparing the different fitting schemes. Figure A.4 plots the fitting error (or the propagation of the fitting error in the case of the ‘yx’ and ‘y’ fitting schemes) on the dispersion function fitted at each PSR BPM. Because most
of the fitting schemes do not fit directly for the dispersion function the fitted slope parameter \( b \) needs to be manipulated and the fitting errors on \( b \) \( (\sigma_b) \) propagated to the error on the dispersion function plotted in Fig. A.4.

![Figure A.4](image-url)

**Figure A.4:** (Color) The fitting error on the dispersion function for each of the six fitting schemes. The fitting schemes are shown as blue circles for ‘xy’, green squares for ‘yx’, red left pointing triangles for ‘D’, black right pointing triangles for ‘GF’, magenta up pointing triangles for ‘x’, and cyan down pointing triangles for ‘y’. The vertical line separates horizontal (left) and vertical (right) BPMs.

In the horizontal, all of the nonlinear fitting schemes yield the same result for the fitting error on the measured dispersion function. The linear fitting schemes (‘x’ and ‘y’) yield smaller fitting errors than their nonlinear counterparts because the linear fits only take into account one of the two measurement errors in the fitted data. The ‘y’ fitting scheme yields a smaller fitting error on the horizontal dispersion function than the ‘x’ fitting scheme. This is because the measurement error on the CO in the PSR was smaller than the measurement error on the beam position at LDPM03.

It is also interesting how the fitting error on the horizontal dispersion function shifts greater and smaller from one BPM to the next. Actually, the error on the fitted dispersion function is very well correlated with the measured dispersion func-
A.4 Comparison of Fitting Scheme with a Calibrated LDPM03

This is because the horizontal CO measurement in the PSR is not completely precision limited by the digitization of the BPM signal. The measurement spread in the horizontal CO is directly related to the dispersion function through the pulse-to-pulse momentum variations. This fact is what allowed for the measurement of the magnitude of the pulse-to-pulse momentum variations in Sec. 2.8.

The fitting error on the vertical dispersion function is interesting when separated into BPM type. This is because the vertical CO measurement in the PSR is precision limited and not dominated by the pulse-to-pulse momentum variations as the horizontal CO measurement. The fitting error on the vertical dispersion function qualitatively traces out the intrinsic position resolution of each BPM. Also because the vertical CO measurement is precision limited, the ‘x’ fitting scheme produces the same results as the ‘xy’, ‘D’, ‘GF’, and most of the ‘yx’ fitting schemes.

Interestingly for some BPMs, the ‘yx’ fitting scheme produces fitting errors on the vertical dispersion function that do not agree with the other nonlinear fitting schemes as for the horizontal dispersion fitting error. This should seem unusual since the ‘yx’ fits identical values for the dispersion function and on-momentum position as the other nonlinear schemes. So what is different about these vertical BPMs to cause the fitting error on the dispersion be much different? The vertical dispersion function is very small. It is easy to imagine a zero slope fit to the data in $x_{PSR}(x_{LD})$ space. But the slope becomes nearly infinite in the inverted $x_{LD}(x_{PSR})$ space. Dealing with nearly infinite slopes is definitely a reason for the fitting error from the ‘yx’ fitting scheme to diverge from the other nonlinear fitting schemes.

The other fitting error that is of interest is the fitting error on the on-momentum position, Fig. A.5. Like the dispersion function, most of the fitting schemes do not directly fit for the on-momentum position, so the fitting error of the y-intercept ($a$), and sometimes even the slope ($b$), must be propagated to the error on the on-momentum position. Only the fitting errors on the on-momentum horizontal positions
A.4 Comparison of Fitting Scheme with a Calibrated LDPM03

are shown in Fig. A.5 because the vertical axis is dominated by the fitting error from
the ‘yx’ fitting scheme from the fitting error on the on-momentum vertical positions,
which can be as large as 4.5 mm. This extreme of a scale on the vertical axis dwarfs
the differences in the horizontal fitting error.

Figure A.5: (Color) The fitting error on the on-momentum position at the horizontal BPMs
in the PSR from each of the six fitting schemes. The fitting schemes are shown as blue circles
for ‘xy’, green squares for ‘yx’, red left pointing triangles for ‘D’, black right pointing triangles for
‘GF’, magenta up pointing triangles for ‘x’, and cyan down pointing triangles for ‘y’.

Once again, the ‘xy’, ‘D’, and ‘GF’ fitting schemes yield the same results in the
fitting error of the on-momentum position. Interestingly the other nonlinear fitting
scheme (‘yx’) does not come close to producing the same results. The ‘y’ fitting
scheme, like the ‘yx’ fitting scheme, suffers from small slopes at horizontal BPMs and
nearly infinite slopes at the vertical BPMs. Since the slope of the horizontal data in
the inverted $x_{LD}(x_{PSR})$ space is small, the slope is not well constrained, leading to a
more variable y-intercept. This effect is compounded at the vertical BPMs because
the y-intercept may be outside and far away from the data that is fit. This is why
the fitting errors on the on-momentum position from the ‘yx’ fitting schemes can be as large as 4 mm.

The ‘x’ fitting scheme produces the smallest fitting error on the on-momentum position. This is because the slope from the ‘x’ fitting scheme is better constrained than the slope from the ‘xy’ fitting scheme since the ‘xy’ fitting scheme also includes independent variable measurement error in the fit.

Finally the goodness of fit quality factor is compared for each fitting scheme. The $\chi^2$/DOF for the dispersion function fit at each horizontal BPM is shown in Fig. A.6. Only the horizontal BPMs are shown in Fig. A.6 because some of the $\chi^2$/DOF at the vertical BPMs from the ‘y’ fitting scheme are as large as several thousand. The ‘y’ fitting scheme suffers from fitting a nearly infinite slope in an inverted space with the independent variable as a function of the dependent variable with small measurement errors.

Figure A.6: (Color) The $\chi^2$/DOF of the dispersion fit at the horizontal BPMs for each of the six fitting schemes. The fitting schemes are shown as blue circles for ‘xy’, green squares for ‘yx’, red left pointing triangles for ‘D’, black right pointing triangles for ‘GF’, magenta up pointing triangles for ‘x’, and cyan down pointing triangles for ‘y’.
The $\chi^2$/DOF for the ‘xy’, ‘yx’, and ‘D’ fitting schemes are the same. This is not surprising since these three fitting schemes yielded identical results for the fitted dispersion function and on-momentum position. These fitting schemes also agree in the vertical, where the ‘yx’ fitting scheme results in different fitting errors on the fitting parameters.

The linear fitting schemes, ‘x’ and ‘y’, generally have larger $\chi^2$/DOF than the nonlinear fitting schemes because they fit less parameters. A fit with more fitting parameters will in general have a smaller $\chi^2$. So in that sense, the nonlinear fits are better because they fit two plus the measurement number of fitting parameters while the linear fitting schemes only fit two fitting parameters.

The $\chi^2$/DOF for the ‘GF’ fitting schemes is constant for all BPMs. This is because the ‘GF’ fitting scheme is only one fit whereas the other fitting schemes are employed individually at each BPM. The one $\chi^2$/DOF result from the ‘GF’ fitting scheme is plotted at each BPM for direct comparison. The $\chi^2$/DOF from the ‘GF’ fitting scheme can be thought of as an average value of the $\chi^2$/DOF from the ‘D’ fitting scheme.

There is one final quantity that should be compared between fitting schemes; this is the fitted fractional momentum deviation values ($\delta$’s) from the ‘D’ and ‘GF’ fitting schemes. The main reason for including the ‘GF’ fitting scheme in this comparison was to observe how the fitting results change when the fitted $\delta$’s were constrained to be the same for all BPMs. The fitted $\delta$’s at each BPM in the ‘D’ fitting scheme and the fitted $\delta$’s from the ‘GF’ fitting scheme are plotted in Fig. A.7.

As the previous analysis suggests, the individual fits at each BPM from the ‘D’ fitting scheme are not variable. The rms standard deviation of the fitted $\delta$ distribution for all BPMs from the ‘D’ fitting scheme is several times $10^{-6}$. The largest deviation between the fitted $\delta$’s from the ‘D’ fitting scheme and the ‘GF’ fitting scheme is $2.34 \times 10^{-5}$, which is about half of the magnitude of the pulse-to-pulse momentum
A.4 Comparison of Fitting Scheme with a Calibrated LDPM03

Figure A.7: (Color) The fitted δ’s from the ‘GF’ fitting scheme (solid lines) and the fitted δ’s from the ‘D’ fitting scheme at each BPM, shapes. The blue circles are for the baseline momentum setting, green squares are the return to baseline momentum setting, red right pointing triangles (−2.2 mm), black left pointing triangles (−2.6 mm), magenta up pointing triangles (.6 mm), and cyan down pointing triangles, average horizontal CO 2.3 mm with both septa is off.

variations. It is rather satisfying that even when the δ’s are individually fit at each BPM in the ‘D’ fitting scheme, the δ results for all BPMs agree within the spread due to the pulse-to-pulse momentum variations.
Appendix B

Maximum Likelihood (ML) Error Analysis

Maximum likelihood (ML) error analysis is an extension of the maximum likelihood fitting schemes. ML error analysis is employed in this thesis as a means to estimate the fitting errors on a set of fitting parameters as well the fitting error, covariance matrix, and measurement error of the data to be fit if not known a priori. The ML error analysis is particularly beneficial for calculating the fitting errors of the fitting parameters in a nonlinear fit where there is no analytic solution and numeric methods must be employed.

Both the ML error analysis and ML fitting scheme start with what is called the ML function. The ML function is the product of normalized probability functions describing the likelihood that each data point is fit well by the fitting function. For the case where the measurement error of the dependent data to be fit is not known
and can be assumed to be the same for all data points, the ML function is

\[ L = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi \sigma}} \exp \left[ -\frac{(y_i - f(x_i; \vec{a}))^2}{2\sigma^2} \right] \]

\[ = \left( \frac{1}{\sqrt{2\pi \sigma}} \right)^N \exp \left[ -\sum_{i=1}^{N} \frac{(y_i - f(x_i; \vec{a}))^2}{2\sigma^2} \right], \quad (B.1) \]

where \( \sigma \) is the \textit{rms} measurement uncertainty, \( y \) is the observed dependent variable fit to a fitting function \( f(x; \vec{a}) \) which is dependent on the independent variable \( x \) and the fitting parameters \( \vec{a} \), and the index \( i \) run from 1 to the number of data points to be fit, \( N \).

This ML analysis assumes that the residuals \( (y_i - f(x_i; \vec{a})) \) in the fit are random, gaussian, and mean-zero. Thus in order for the ML error analysis to yield the proper results, the fitting function cannot introduce a systematic error.

Obtaining a good fit in the ML fitting scheme is the same as maximizing the probability that the fitting functions fits the data or maximizing the ML function with respect to each of the fitting parameters. To simplify this process, it is convenient to introduce the log-likelihood function,

\[ w = \ln L = -\frac{N}{2} \ln(2\pi) - N \ln \sigma - \sum_{i=1}^{N} \frac{(y_i - f(x_i; \vec{a}))^2}{2\sigma^2}. \quad (B.2) \]

A system of equations may be generated by maximizing the log-likelihood function with respect to each parameter,

\[ \frac{\partial w}{\sigma} = 0 = -\frac{N}{\sigma} + \sum_{i=1}^{N} \frac{(y_i - f(x_i; \vec{a}))^2}{\sigma^3} \]

\[ \frac{\partial w}{a_j} = 0 = \frac{1}{\sigma^2} \sum_{i=1}^{N} \left[ (y_i - f(x_i; \vec{a})) \frac{\partial f(x_i; \vec{a})}{\partial a_j} \right], \quad (B.3) \]

where \( j \) is an index ran along the length of the fitting parameter vector \( \vec{a} \). One of the great advantages of ML is that if the measurement error is not known before the fit, the measurement error may be included as one of the fitting parameters at no extra
Maximum Likelihood (ML) Error Analysis

Equation (B.3) yields a system of \(J + 1\) equations where \(J\) is the number of fitting parameters. The \(\sigma\) equation of Eq. (B.3) may be solved for the well known \(\sigma\) estimator,

\[
\sigma = \sqrt{\frac{\sum_i^N (y_i - f(x_i; \vec{a}))^2}{N}}. \tag{B.4}
\]

If the system of equations described by Eq. (B.3) is linear in the fitting parameters, the fitting parameters may be solved with Cramer’s rule as in the case of linear regression. If the system of equations generated by Eq. (B.3) is nonlinear in the fitting parameters, a numeric solver must be employed to solve for the fitting parameters. The nonlinear solver of choice for the analysis in this thesis is the \textit{lsqnonlin} Matlab routine.

After the fitting parameters are obtained, it is important to consider the fitting error on such fitting parameters. The Hessian matrix must be found in order to calculate the fitting errors. The Hessian matrix is defined as the second derivative matrix of the log-likelihood function,

\[
H_{jk} = -\frac{\partial^2 w}{\partial a_j \partial a_k}. \tag{B.5}
\]

The second derivatives of the log-likelihood matrix are

\[
\frac{\partial^2 w}{\partial \sigma^2} = \frac{N}{\sigma^2} - \frac{3 \sum_i^N (y_i - f(x_i; \vec{a}))^2}{\sigma^4}
\]

\[
\frac{\partial^2 w}{\partial a_j \partial \sigma} = -\frac{2}{\sigma^2} \sum_i^N \left( (y_i - f(x_i; \vec{a})) \frac{\partial f(x_i; \vec{a})}{\partial a_j} \right)
\]

\[
\frac{\partial^2 w}{\partial a_j \partial a_k} = \frac{1}{\sigma^2} \sum_i^N \left[ \frac{\partial f(x_i; \vec{a})}{\partial a_j} \frac{\partial f(x_i; \vec{a})}{\partial a_k} - (y_i - f(x_i; \vec{a})) \frac{\partial^2 f(x_i; \vec{a})}{\partial a_j \partial a_k} \right], \tag{B.6}
\]

where both \(j\) and \(k\) are indices which run along the fitting parameters and may be equal to produce the diagonal elements of the Hessian matrix.

The covariance matrix of the fitted parameters is estimated as the inverse of the Hessian matrix. The fitting errors and their correlations may be calculated from the covariance matrix.
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Education

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• Peer Reviewed
  

• Not Peer Reviewed

- **Invited Talks**


- **Talks and Posters Presentations**

