The path integral for photons

based on S-57

We will discuss the path integral for photons and the photon propagator more carefully using the Lorentz gauge:

$$Z_0(J) = \int \mathcal{D}A \, e^{iS_0} \, ,$$

$$\mathcal{L} = +\tfrac{1}{2} A_\mu (g^{\mu\nu}\partial^2 - \partial^\mu \partial^\nu) A_\nu + J^\mu A_\mu$$

as in the case of scalar field we Fourier-transform to the momentum space:

$$S_0 = \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\Big[-\widetilde{\varphi}(k)(k^2+m^2)\widetilde{\varphi}(-k) + \widetilde{J}(k)\widetilde{\varphi}(-k) + \widetilde{J}(-k)\widetilde{\varphi}(k)\Big]$$

$$S_0 = \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\Big[-\widetilde{A}_\mu(k)\Big(k^2 g^{\mu\nu} - k^\mu k^\nu\Big)\widetilde{A}_\nu(-k) + \widetilde{J}^\mu(k)\widetilde{A}_\mu(-k) + \widetilde{J}^\mu(-k)\widetilde{A}_\mu(k)\Big]$$

we shift integration variables so that mixed terms disappear...

$$\widetilde{\chi}(k) = \widetilde{\varphi}(k) - \frac{\widetilde{J}(k)}{k^2+m^2}$$

Problem: the matrix ← has zero eigenvalue and cannot be inverted.

To see this, note:

$$k^2 g^{\mu\nu} - k^\mu k^\nu = k^2 P^{\mu\nu}(k)$$

where

$$P^{\mu\nu}(k) \equiv g^{\mu\nu} - k^\mu k^\nu / k^2$$

is a projection matrix

$$P^{\mu\nu}(k) P_\nu{}^\lambda(k) = P^{\mu\lambda}(k)$$

and so the only allowed eigenvalues are 0 and +1

Since

$$P^{\mu\nu}(k) k_\nu = 0$$

$$g_{\mu\nu} P^{\mu\nu}(k) = 3$$

it has one 0 and three +1 eigenvalues.

$$S_0 = \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\Big[-\widetilde{A}_\mu(k)\Big(k^2 g^{\mu\nu} - k^\mu k^\nu\Big)\widetilde{A}_\nu(-k) + \widetilde{J}^\mu(k)\widetilde{A}_\mu(-k) + \widetilde{J}^\mu(-k)\widetilde{A}_\mu(k)\Big]$$

We can decompose the gauge field $\widetilde{A}_\mu(k)$ into components aligned along a set of linearly independent four-vectors, one of which is k_μ and then this component does not contribute to the quadratic term because

$$P^{\mu\nu}(k)k_\nu = 0$$

and it doesn't even contribute to the linear term because

$$\partial^\mu J_\mu(x) = 0 \quad \longrightarrow \quad k^\mu \widetilde{J}_\mu(k) = 0$$

and so there is no reason to integrate over it; we define the path integral as integral over the remaining three basis vector; these are given by

$$k^\mu \widetilde{A}_\mu(k) = 0$$

which is equivalent to

$$\partial^\mu A_\mu(x) = 0$$

Lorentz gauge

$$P^{\mu\nu}(k) \equiv g^{\mu\nu} - k^\mu k^\nu / k^2$$

Within the subspace orthogonal to k_μ the projection matrix is simply the identity matrix and the inverse is straightforward; thus we get:

$$Z_0(J) = \exp\left[\frac{i}{2}\int \frac{d^4k}{(2\pi)^4}\,\widetilde{J}_\mu(k)\,\frac{P^{\mu\nu}(k)}{k^2 - i\epsilon}\,\widetilde{J}_\nu(-k)\right]$$

$$= \exp\left[\frac{i}{2}\int d^4x\, d^4y\, J_\mu(x)\Delta^{\mu\nu}(x-y)J_\nu(y)\right]$$

going back to the position space

$$\Delta^{\mu\nu}(x-y) = \int \frac{d^4k}{(2\pi)^4}\, e^{ik(x-y)}\,\frac{P^{\mu\nu}(k)}{k^2 - i\epsilon}$$

propagator in the Lorentz gauge (Landau gauge)

we can again neglect the term with momenta because the current is conserved and we obtain the propagator in the Feynman gauge:

$$\Delta^{\mu\nu}(x-y) \equiv \int \frac{d^4k}{(2\pi)^4}\, e^{ik(x-y)}\,\tilde{\Delta}^{\mu\nu}(k) \qquad \tilde{\Delta}^{\mu\nu}(k) = \frac{g^{\mu\nu}}{k^2 - i\epsilon}$$

Quantum electrodynamics (QED)

based on S-58

Quantum electrodynamics is a theory of photons interacting with the electrons and positrons of a Dirac field:

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\overline{\Psi}\partial\!\!\!/\Psi - m\overline{\Psi}\Psi + e\overline{\Psi}\gamma^\mu\Psi A_\mu$$

$e = -0.302822$

$\alpha = e^2/4\pi = 1/137.036$

Noether current of the lagrangian for a free Dirac field $j^\mu(x) = e\overline{\Psi}(x)\gamma^\mu\Psi(x)$

$$\partial_\mu j^\mu(x) = \delta\mathcal{L}(x) - \frac{\delta S}{\delta\varphi_a(x)}\delta\varphi_a(x)$$

we want the interaction term to be gauge invariant and so we need to enlarge the gauge transformation also to the Dirac field:

$$A^\mu(x) \to A^\mu(x) - \partial^\mu\Gamma(x) \,,$$

$$\Psi(x) \to \exp[-ie\Gamma(x)]\Psi(x) \,,$$

$$\overline{\Psi}(x) \to \exp[+ie\Gamma(x)]\overline{\Psi}(x) \,.$$

global symmetry is promoted into local

$\Psi \to e^{-i\alpha}\Psi$

$\overline{\Psi} \to e^{+i\alpha}\overline{\Psi}$

symmetry of the lagrangian no matter if equations of motion are satisfied

We can write the QED lagrangian as:

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\overline{\Psi}\not{D}\Psi - m\overline{\Psi}\Psi$$

$$D_\mu \equiv \partial_\mu - ieA_\mu$$

covariant derivative

(the covariant derivative of a field transforms as the field itself)

$$\Psi(x) \to \exp[-ie\Gamma(x)]\Psi(x)$$

$$D_\mu\Psi(x) \to \exp[-ie\Gamma(x)]D_\mu\Psi(x)$$

and so the lagrangian is manifestly gauge invariant!

Proof:

$$\begin{aligned} D_\mu\Psi &\to \Big(\partial_\mu - ie[A_\mu - \partial_\mu\Gamma]\Big)\Big(\exp[-ie\Gamma]\Psi\Big) \\ &= \exp[-ie\Gamma]\Big(\partial_\mu\Psi - ie(\partial_\mu\Gamma)\Psi - ie[A_\mu - \partial_\mu\Gamma]\Psi\Big) \\ &= \exp[-ie\Gamma]\Big(\partial_\mu - ieA_\mu\Big)\Psi \\ &= \exp[-ie\Gamma]D_\mu\Psi\ . \end{aligned}$$

$$\Psi(x) \to \exp[-ie\Gamma(x)]\Psi(x)$$
$$D_\mu\Psi(x) \to \exp[-ie\Gamma(x)]D_\mu\Psi(x)$$

We can also define the transformation rule for D:

$$D_\mu \to e^{-ie\Gamma} D_\mu \, e^{+ie\Gamma}$$

then

$$D_\mu\Psi \to \left(e^{-ie\Gamma} D_\mu e^{+ie\Gamma}\right)\left(e^{-ie\Gamma}\Psi\right)$$
$$= e^{-ie\Gamma} D_\mu \Psi \; ,$$

as required.

Now we can express the field strength in terms of D's:

$$D_\mu \equiv \partial_\mu - ieA_\mu$$

$$[D^\mu, D^\nu]\Psi(x) = -ieF^{\mu\nu}(x)\Psi(x)$$

$$F^{\mu\nu} = \frac{i}{e}[D^\mu, D^\nu]$$

$$F^{\mu\nu} = \frac{i}{e}[D^\mu, D^\nu]$$

$$D_\mu \to e^{-ie\Gamma} D_\mu \, e^{+ie\Gamma}$$

Then we simply see:

$$\begin{aligned} F^{\mu\nu} &\to \frac{i}{e}\left[e^{-ie\Gamma} D^\mu \, e^{+ie\Gamma}, e^{-ie\Gamma} D^\nu \, e^{+ie\Gamma}\right] \\ &= e^{-ie\Gamma}\left(\frac{i}{e}[D^\mu, D^\nu]\right)e^{+ie\Gamma} \\ &= e^{-ie\Gamma} F^{\mu\nu} e^{+ie\Gamma} \\ &= F^{\mu\nu} \, . \end{aligned}$$

no derivatives act on exponentials

the field strength is gauge invariant as we already knew

To get Feynman rules we follow the usual procedure of writing the interacting lagrangian as a function of functional derivatives, ...

We have to make more precise statement over which field configurations we integrate because now also the Dirac fields transform under the gauge transformation (next semester).

$$Z(\overline{\eta},\eta,J) \propto \exp\left[ie\int d^4x\left(\frac{1}{i}\frac{\delta}{\delta J^\mu(x)}\right)\left(i\frac{\delta}{\delta\eta_\alpha(x)}\right)(\gamma^\mu)_{\alpha\beta}\left(\frac{1}{i}\frac{\delta}{\delta\overline{\eta}_\beta(x)}\right)\right] \; Z_0(\overline{\eta},\eta,J)$$

$$Z_0(\overline{\eta},\eta,J) = \exp\left[i\int d^4x\, d^4y\; \overline{\eta}(x)S(x-y)\eta(y)\right]$$

$$\times\exp\left[\frac{i}{2}\int d^4x\, d^4y\; J^\mu(x)\Delta_{\mu\nu}(x-y)J^\nu(y)\right]$$

$$S(x-y) = \int\frac{d^4p}{(2\pi)^4}\,\frac{(-\not{p}+m)}{p^2+m^2-i\epsilon}\,e^{ip(x-y)}$$

$$\Delta_{\mu\nu}(x-y) = \int\frac{d^4k}{(2\pi)^4}\,\frac{g_{\mu\nu}}{k^2-i\epsilon}\,e^{ik(x-y)}$$

Imposing $Z(0,0,0)=1$ we can write it as:

$$Z(\overline{\eta},\eta,J) = \exp[iW(\overline{\eta},\eta,J)]$$

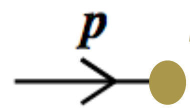
sum of connected Feynman diagrams with sources!

(no tadpoles)

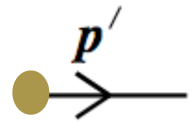
Feynman rules to calculate $i\mathcal{T}$:

- external lines:

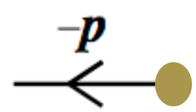
vertex and the rest of the diagram

incoming electron 

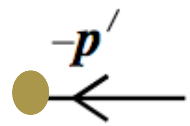
$u_{s_i}(\mathbf{p}_i)$

outgoing electron 

$\overline{u}_{s'_i}(\mathbf{p}'_i)$

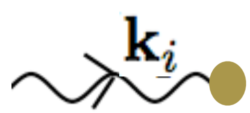
incoming positron 

$\overline{v}_{s_i}(\mathbf{p}_i)$

outgoing positron 

$v_{s'_i}(\mathbf{p}'_i)$

$$a^{\dagger}_{\lambda}(\mathbf{k})_{\text{in}} \to i\,\varepsilon^{\mu*}_{\lambda}(\mathbf{k}) \int d^4x\; e^{+ikx}(-\partial^2)A_\mu(x)$$

incoming photon 

$\varepsilon^{\mu*}_{\lambda_i}(\mathbf{k}_i)$

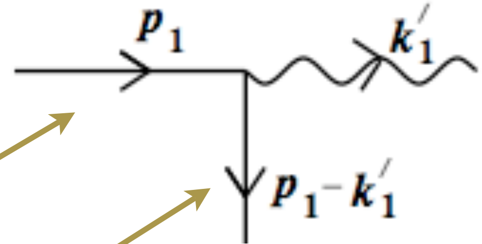
$$a_{\lambda}(\mathbf{k})_{\text{out}} \to i\,\varepsilon^{\mu}_{\lambda}(\mathbf{k}) \int d^4x\; e^{-ikx}(-\partial^2)A_\mu(x)$$

outgoing photon 

$\varepsilon^{\mu}_{\lambda'_i}(\mathbf{k}'_i)$

- vertex

the arrow for the photon can point both ways



one arrow in and one out

$$ie\gamma^\mu$$

- draw all topologically inequivalent diagrams
- for internal lines assign momenta so that momentum is conserved in each vertex (the four-momentum is flowing along the arrows)
- propagators

for each internal photon $\quad -ig^{\mu\nu}/(k^2 - i\epsilon)$

for each internal fermion $\quad -i(-\not{p} + m)/(p^2 + m^2 - i\epsilon)$

- spinor indices are contracted by starting at the end of the fermion line that has the arrow pointing away from the vertex, write $\overline{u}_{s_i'}(\mathbf{p}_i')$ or $\overline{v}_{s_i}(\mathbf{p}_i)$; follow the fermion line, write factors associated with vertices and propagators and end up with spinors $u_{s_i}(\mathbf{p}_i)$ or $v_{s_i'}(\mathbf{p}_i')$.

  follow arrows backwards!

  The vector index on each vertex is contracted with the vector index on either the photon propagator or the photon polarization vector.

- assign proper relative signs to different diagrams

  draw all fermion lines horizontally with arrows from left to right; with left end points labeled in the same way for all diagrams; if the ordering of the labels on the right endpoints is an even (odd) permutation of an arbitrarily chosen ordering then the sign of that diagram is positive (negative).

- sum over all the diagrams and get $i\mathcal{T}$

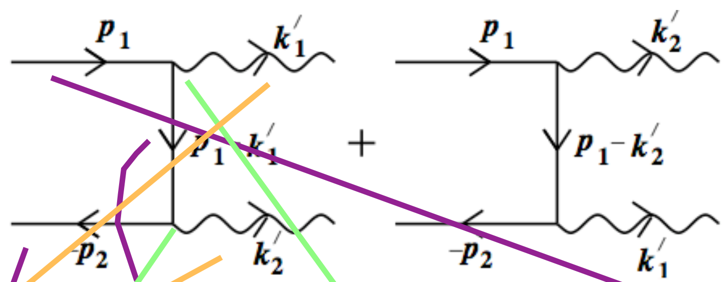
additional rules for counterterms and loops

Scattering in QED

based on S-59

Let's calculate the scattering amplitude for a simple process:

$$e^+e^- \to \gamma\gamma$$



$$\mathcal{T} = e^2\, \varepsilon^{\mu}_{1'}\varepsilon^{\nu}_{2'}\, \bar{v}_2 \left[\gamma_\nu \left(\frac{-\not{p}_1 + \not{k}'_1 + m}{-t+m^2} \right) \gamma_\mu + \gamma_\mu \left(\frac{-\not{p}_1 + \not{k}'_2 + m}{-u+m^2} \right) \gamma_\nu \right] u_1$$

$$s = -(p_1+p_2)^2 = -(k_1'+k_2')^2$$

$$t = -(p_1-k_1')^2 = -(p_2-k_2')^2$$

$$u = -(p_1-k_2')^2 = -(p_2-k_1')^2$$

$$s+t+u = 2m^2$$

$$\mathcal{T} = e^2\, \varepsilon^{\mu}_{1'}\varepsilon^{\nu}_{2'}\, \bar{v}_2 \left[\gamma_\nu \left(\frac{-\not{p}_1 + \not{k}'_1 + m}{-t+m^2}\right)\gamma_\mu + \gamma_\mu \left(\frac{-\not{p}_1 + \not{k}'_2 + m}{-u+m^2}\right)\gamma_\nu\right] u_1$$

We follow the same procedure as before:

$$\mathcal{T} = \varepsilon^{\mu}_{1'}\varepsilon^{\nu}_{2'}\, \bar{v}_2 A_{\mu\nu} u_1$$

$$A_{\mu\nu} \equiv e^2 \left[\gamma_\nu \left(\frac{-\not{p}_1 + \not{k}'_1 + m}{-t+m^2}\right)\gamma_\mu + \gamma_\mu \left(\frac{-\not{p}_1 + \not{k}'_2 + m}{-u+m^2}\right)\gamma_\nu\right]$$

$$\mathcal{T}^* = \overline{\mathcal{T}} = \varepsilon^{\rho *}_{1'}\, \varepsilon^{\sigma *}_{2'}\, \bar{u}_1\, \overline{A_{\rho\sigma}}\, v_2$$

$$\overline{\not{a}\not{b}\ldots} = \ldots\not{b}\not{a}$$

$$\overline{A_{\rho\sigma}} = A_{\sigma\rho}$$

and the amplitude squared is:

$$|\mathcal{T}|^2 = \varepsilon^{\mu}_{1'}\varepsilon^{\nu}_{2'}\, \varepsilon^{\rho *}_{1'}\, \varepsilon^{\sigma *}_{2'}\, (\bar{v}_2 A_{\mu\nu} u_1)(\bar{u}_1 A_{\sigma\rho} v_2)$$

averaging over the initial electron and positron spins we get:

$$\frac{1}{4}\sum_{s_1,s_2} |\mathcal{T}|^2 = \frac{1}{4}\varepsilon^{\mu}_{1'}\varepsilon^{\nu}_{2'}\varepsilon^{\rho *}_{1'}\, \varepsilon^{\sigma *}_{2'}\, \mathrm{Tr}\Big[A_{\mu\nu}(-\not{p}_1 + m) A_{\sigma\rho}(-\not{p}_2 - m)\Big]$$

$$\frac{1}{4}\sum_{s_1,s_2}|\mathcal{T}|^2 = \frac{1}{4}\varepsilon^{\mu}_{1'}\varepsilon^{\nu}_{2'}\varepsilon^{\rho*}_{1'}\varepsilon^{\sigma*}_{2'}\,\mathrm{Tr}\Big[A_{\mu\nu}(-\not{p}_1+m)A_{\sigma\rho}(-\not{p}_2-m)\Big]$$

we also want to sum over the final photon polarizations:

$$\sum_{\lambda=\pm}\varepsilon^{\mu}_{\lambda}(\mathbf{k})\varepsilon^{\rho*}_{\lambda}(\mathbf{k})$$

in the Coulomb gauge we found the polarization sum to be:

$$\sum_{\lambda=\pm}\varepsilon^{\mu}_{\lambda}(\mathbf{k})\varepsilon^{\rho*}_{\lambda}(\mathbf{k}) = g^{\mu\rho} + \hat{t}^{\mu}\hat{t}^{\rho} - \hat{z}^{\mu}\hat{z}^{\rho}$$

$$\hat{t}^{\mu} = (1,\mathbf{0})$$

$$\hat{z}^{\mu} = \frac{k^{\mu} + (\hat{t}\cdot k)\hat{t}^{\mu}}{[k^2 + (\hat{t}\cdot k)^2]^{1/2}}$$

$$A^{\mu}(x) \to A^{\mu}(x) - \partial^{\mu}\Gamma(x)\ ,$$

$$\mathbf{A}(x) = \sum_{\lambda=\pm}\int \widetilde{dk}\,\Big[\boldsymbol{\varepsilon}^{*}_{\lambda}(\mathbf{k})a_{\lambda}(\mathbf{k})e^{ikx} + \boldsymbol{\varepsilon}_{\lambda}(\mathbf{k})a^{\dagger}_{\lambda}(\mathbf{k})e^{-ikx}\Big]$$

$$\varepsilon^{\mu}_{\lambda}(\mathbf{k}) \to \varepsilon^{\mu}_{\lambda}(\mathbf{k}) - i\tilde{\Gamma}(k)k^{\mu}$$

$$\mathcal{T} = \varepsilon^{\mu}_{\lambda}(\mathbf{k})\mathcal{M}_{\mu}$$

$$\mathcal{T} = \varepsilon^{\mu*}_{\lambda}(\mathbf{k})\mathcal{M}_{\mu}$$

doesn't contribute: the scattering amplitude should be invariant under a gauge transformation and so we should have: $k^{\mu}\mathcal{M}_{\mu} = 0$

Ward identity

in addition, $k^2 = 0$ and so we find:

$$\boxed{\sum_{\lambda=\pm}\varepsilon^{\mu}_{\lambda}(\mathbf{k})\varepsilon^{\rho*}_{\lambda}(\mathbf{k}) \to g^{\mu\rho}}$$

$$\frac{1}{4}\sum_{s_1,s_2}|\mathcal{T}|^2 = \frac{1}{4}\varepsilon^{\mu}_{1'}\varepsilon^{\nu}_{2'}\varepsilon^{\rho *}_{1'}\varepsilon^{\sigma *}_{2'}\,\mathrm{Tr}\Big[A_{\mu\nu}(-\not{p}_1+m)A_{\sigma\rho}(-\not{p}_2-m)\Big]$$

$$\sum_{\lambda=\pm}\varepsilon^{\mu}_{\lambda}(\mathbf{k})\varepsilon^{\rho *}_{\lambda}(\mathbf{k}) \to g^{\mu\rho}$$

for the spin averaged/summed amplitude we get:

$$\langle|\mathcal{T}|^2\rangle \equiv \frac{1}{4}\sum_{\lambda'_1,\lambda'_2}\sum_{s_1,s_2}|\mathcal{T}|^2$$

$$= \frac{1}{4}\mathrm{Tr}\Big[A_{\mu\nu}(-\not{p}_1+m)A^{\nu\mu}(-\not{p}_2-m)\Big]$$

$$A_{\mu\nu} \equiv e^2\left[\gamma_\nu\left(\frac{-\not{p}_1+\not{k}'_1+m}{-t+m^2}\right)\gamma_\mu + \gamma_\mu\left(\frac{-\not{p}_1+\not{k}'_2+m}{-u+m^2}\right)\gamma_\nu\right]$$

$$= e^4\left[\frac{\langle\Phi_{tt}\rangle}{(m^2-t)^2} + \frac{\langle\Phi_{tu}\rangle+\langle\Phi_{ut}\rangle}{(m^2-t)(m^2-u)} + \frac{\langle\Phi_{uu}\rangle}{(m^2-u)^2}\right]$$

$$\langle\Phi_{tt}\rangle = \frac{1}{4}\mathrm{Tr}\Big[\gamma_\nu(-\not{p}_1+\not{k}'_1+m)\gamma_\mu(-\not{p}_1+m)\gamma^\mu(-\not{p}_1+\not{k}'_1+m)\gamma^\nu(-\not{p}_2-m)\Big]$$

$$\langle\Phi_{uu}\rangle = \frac{1}{4}\mathrm{Tr}\Big[\gamma_\mu(-\not{p}_1+\not{k}'_2+m)\gamma_\nu(-\not{p}_1+m)\gamma^\nu(-\not{p}_1+\not{k}'_2+m)\gamma^\mu(-\not{p}_2-m)\Big]$$

$$\langle\Phi_{tu}\rangle = \frac{1}{4}\mathrm{Tr}\Big[\gamma_\nu(-\not{p}_1+\not{k}'_1+m)\gamma_\mu(-\not{p}_1+m)\gamma^\nu(-\not{p}_1+\not{k}'_2+m)\gamma^\mu(-\not{p}_2-m)\Big]$$

$$\langle\Phi_{ut}\rangle = \frac{1}{4}\mathrm{Tr}\Big[\gamma_\mu(-\not{p}_1+\not{k}'_2+m)\gamma_\nu(-\not{p}_1+m)\gamma^\mu(-\not{p}_1+\not{k}'_1+m)\gamma^\nu(-\not{p}_2-m)\Big]$$

$$k'_1 \leftrightarrow k'_2$$
$$t \leftrightarrow u$$

$$\langle\Phi_{tt}\rangle = \tfrac{1}{4}\mathrm{Tr}\Big[\gamma_\nu(-\not{p}_1+\not{k}'_1+m)\gamma_\mu(-\not{p}_1+m)\gamma^\mu(-\not{p}_1+\not{k}'_1+m)\gamma^\nu(-\not{p}_2-m)\Big]$$

$$\langle\Phi_{uu}\rangle = \tfrac{1}{4}\mathrm{Tr}\Big[\gamma_\mu(-\not{p}_1+\not{k}'_2+m)\gamma_\nu(-\not{p}_1+m)\gamma^\nu(-\not{p}_1+\not{k}'_2+m)\gamma^\mu(-\not{p}_2-m)\Big]$$

$$\langle\Phi_{tu}\rangle = \tfrac{1}{4}\mathrm{Tr}\Big[\gamma_\nu(-\not{p}_1+\not{k}'_1+m)\gamma_\mu(-\not{p}_1+m)\gamma^\nu(-\not{p}_1+\not{k}'_2+m)\gamma^\mu(-\not{p}_2-m)\Big]$$

$$\langle\Phi_{ut}\rangle = \tfrac{1}{4}\mathrm{Tr}\Big[\gamma_\mu(-\not{p}_1+\not{k}'_2+m)\gamma_\nu(-\not{p}_1+m)\gamma^\mu(-\not{p}_1+\not{k}'_1+m)\gamma^\nu(-\not{p}_2-m)\Big]$$

$$\begin{aligned}\gamma^\mu\gamma_\mu &= -4\ ,\\ \gamma^\mu\not{a}\gamma_\mu &= 2\not{a}\ ,\\ \gamma^\mu\not{a}\not{b}\gamma_\mu &= 4(ab)\ ,\\ \gamma^\mu\not{a}\not{b}\not{c}\gamma_\mu &= 2\not{c}\not{b}\not{a}\ ,\end{aligned}$$

$$\begin{aligned}p_1p_2 &= -\tfrac{1}{2}(s-2m^2)\ ,\\ k'_1k'_2 &= -\tfrac{1}{2}s\ ,\\ p_1k'_1 = p_2k'_2 &= +\tfrac{1}{2}(t-m^2)\ ,\\ p_1k'_2 = p_2k'_1 &= +\tfrac{1}{2}(u-m^2)\ .\end{aligned}$$

$$p_1^2 = p_2^2 = -m^2$$

$$k_1'^2 = k_2'^2 = 0$$

$$\begin{aligned}\langle\Phi_{tt}\rangle &= 2[tu - m^2(3t+u) - m^4]\ ,\\ \langle\Phi_{tu}\rangle &= 2m^2(s-4m^2)\ ,\\ \langle\Phi_{uu}\rangle &= 2[tu - m^2(3u+t) - m^4]\ ,\\ \langle\Phi_{ut}\rangle &= 2m^2(s-4m^2)\ .\end{aligned}$$

we can plug the result to the formulae for differential cross section...