The Standard Model: Quark sector

as far as we know there are three generations of quarks:

<table>
<thead>
<tr>
<th>name</th>
<th>symbol</th>
<th>mass (GeV)</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>up</td>
<td>$u$</td>
<td>0.0017</td>
<td>$+\frac{2}{3}$</td>
</tr>
<tr>
<td>down</td>
<td>$d$</td>
<td>0.0039</td>
<td>$-\frac{1}{3}$</td>
</tr>
<tr>
<td>strange</td>
<td>$s$</td>
<td>0.076</td>
<td>$-\frac{1}{3}$</td>
</tr>
<tr>
<td>charm</td>
<td>$c$</td>
<td>1.3</td>
<td>$+\frac{2}{3}$</td>
</tr>
<tr>
<td>bottom</td>
<td>$b$</td>
<td>4.3</td>
<td>$-\frac{1}{3}$</td>
</tr>
<tr>
<td>top</td>
<td>$t$</td>
<td>178</td>
<td>$+\frac{2}{3}$</td>
</tr>
</tbody>
</table>

and they are described by introducing left-handed Weyl fields:

$$q \quad (3, 2, +\frac{1}{6})$$

$$\bar{u} \quad (\bar{3}, 1, -\frac{2}{3})$$

$$\bar{d} \quad (\bar{3}, 1, +\frac{1}{3})$$
the covariant derivatives are:

\[(D_\mu q)_{\alpha i} = \partial_\mu q_{\alpha i} - ig_3 A^a_\mu (T^a_3)_{\alpha \beta} q_{\beta i} - ig_2 A^a_\mu (T^a_2)_{i j} q_{\beta j} - ig_1 (+\frac{1}{6}) B_\mu q_{\alpha i}\]

\[(D_\mu \bar{u})^\alpha = \partial_\mu \bar{u}^\alpha - ig_3 A^a_\mu (T^a_3)_{\alpha \beta} \bar{u}^\beta - ig_1 (-\frac{2}{3}) B_\mu \bar{u}^\alpha\]

\[(D_\mu \bar{d})^\alpha = \partial_\mu \bar{d}^\alpha - ig_3 A^a_\mu (T^a_3)_{\alpha \beta} \bar{d}^\beta - ig_1 (+\frac{1}{3}) B_\mu \bar{d}^\alpha\]

and the kinetic terms are the usual ones for Weyl fields:

\[\mathcal{L}_{\text{kin}} = iq^\dagger_{\alpha i} \bar{\sigma}^\mu (D_\mu q)_{\alpha i} + i\bar{u}^\dagger_{\alpha} \bar{\sigma}^\mu (D_\mu \bar{u})^\alpha + i\bar{d}^\dagger_{\alpha} \bar{\sigma}^\mu (D_\mu \bar{d})^\alpha\]

there is no gauge group singlet contained in any of the products;

it is not possible to write any mass term!
however we can write a Yukawa terms of the form:

$$\mathcal{L}_{\text{Yuk}} = -y' \varepsilon^{ij} \varphi_i q_{\alpha j} \bar{d}^{\alpha} - y'' \varphi^\dagger_i q_{\alpha i} \bar{u}^{\alpha} + \text{h.c.}$$

$$(1, 2, -\frac{1}{2}) \otimes (3, 2, +\frac{1}{6}) \otimes (\bar{3}, 1, +\frac{1}{3}) = (1, 1, 0) \oplus \ldots$$

$$(1, 2, +\frac{1}{2}) \otimes (3, 2, +\frac{1}{6}) \otimes (\bar{3}, 1, -\frac{2}{3}) = (1, 1, 0) \oplus \ldots$$

there are no other terms that have mass dimension four or less!

in the unitary gauge we have:

$$\varphi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

and the Yukawa terms become:

$$\mathcal{L}_{\text{Yuk}} = -\frac{1}{\sqrt{2}} y'(v + H)q_{\alpha 2} \bar{d}^{\alpha} - \frac{1}{\sqrt{2}} y''(v + H)q_{\alpha 1} \bar{u}^{\alpha} + \text{h.c.}$$
\[ \mathcal{L}_{\text{Yuk}} = -\frac{1}{\sqrt{2}} y'(v + H)q_\alpha d^\alpha - \frac{1}{\sqrt{2}} y''(v + H)q_\alpha u^\alpha + \text{h.c.} \]

it is convenient to label the components of the quark doublet as:

\[ q = \begin{pmatrix} u \\ d \end{pmatrix} \]

then we have:

\[
\mathcal{L}_{\text{Yuk}} = -\frac{1}{\sqrt{2}} y'(v + H)(d_\alpha \bar{d}^\alpha + \bar{d}_\alpha^\dagger d^\alpha_\dagger) - \frac{1}{\sqrt{2}} y''(v + H)(u_\alpha \bar{u}^\alpha + \bar{u}_\alpha^\dagger u^\alpha_\dagger)
\]

\[ = -\frac{1}{\sqrt{2}} y'(v + H)\bar{D}^\alpha D_\alpha - \frac{1}{\sqrt{2}} y''(v + H)\bar{U}^\alpha U_\alpha, \]

we define Dirac fields for the down and up quarks

\[ D_\alpha \equiv \begin{pmatrix} d_\alpha \\ \bar{d}_\alpha^\dagger \end{pmatrix}, \quad U_\alpha \equiv \begin{pmatrix} u_\alpha \\ \bar{u}_\alpha^\dagger \end{pmatrix} \]

and we see that the up and down quarks have acquired masses:

\[ m_d = \frac{y'v}{\sqrt{2}}, \quad m_u = \frac{y''v}{\sqrt{2}} \]

...work out interactions with gauge fields, and generalization to three families...