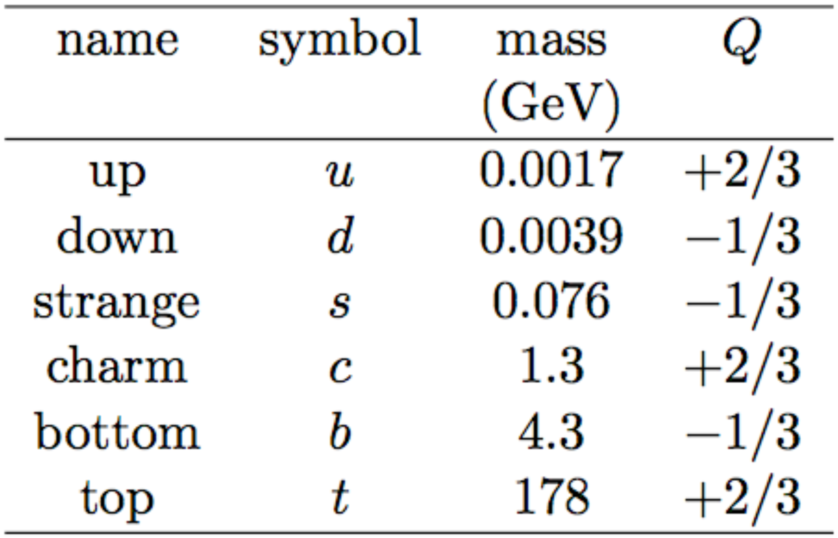
The Standard Model: Quark sector

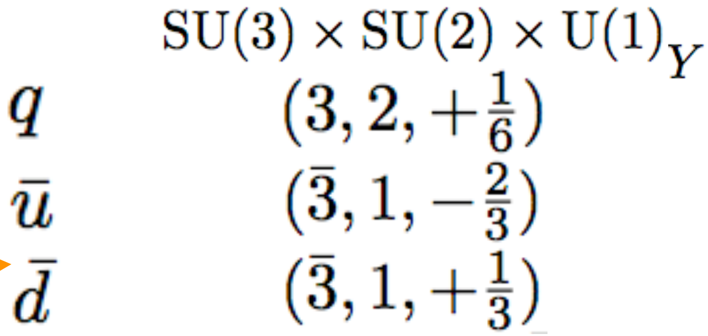
based on S-89

as far as we know there are three generations of quarks:



name	symbol	mass (GeV)	Q
up	u	0.0017	$+2/3$
down	d	0.0039	$-1/3$
strange	s	0.076	$-1/3$
charm	c	1.3	$+2/3$
bottom	b	4.3	$-1/3$
top	t	178	$+2/3$

and they are described by introducing left-handed Weyl fields:



	$\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y$
q	$(3, 2, +\frac{1}{6})$
$\bar{u}$	$(\bar{3}, 1, -\frac{2}{3})$
$\bar{d}$	$(\bar{3}, 1, +\frac{1}{3})$

just a name

the covariant derivatives are:

$q\ (3,2,+\tfrac{1}{6})$

$$(D_\mu q)_{\alpha i} = \partial_\mu q_{\alpha i} - i g_3 A^a_\mu (T^a_3)_\alpha{}^\beta q_{\beta i} - i g_2 A^a_\mu (T^a_2)_i{}^j q_{\beta j} - i g_1 (+\tfrac{1}{6}) B_\mu q_{\alpha i}$$

$\bar{u}\ (\bar{3},1,-\tfrac{2}{3})$

$$(D_\mu \bar{u})^\alpha = \partial_\mu \bar{u}^\alpha - i g_3 A^a_\mu (T^a_{\bar{3}})^\alpha{}_\beta \bar{u}^\beta - i g_1 (-\tfrac{2}{3}) B_\mu \bar{u}^\alpha$$

$\bar{d}\ (\bar{3},1,+\tfrac{1}{3})$

$$(D_\mu \bar{d})^\alpha = \partial_\mu \bar{d}^\alpha - i g_3 A^a_\mu (T^a_{\bar{3}})^\alpha{}_\beta \bar{d}^\beta - i g_1 (+\tfrac{1}{3}) B_\mu \bar{d}^\alpha$$

and the kinetic terms are the usual ones for Weyl fields:

$$\mathcal{L}_{\text{kin}} = i q^{\dagger \alpha i} \bar{\sigma}^\mu (D_\mu q)_{\alpha i} + i \bar{u}^\dagger_\alpha \bar{\sigma}^\mu (D_\mu \bar{u})^\alpha + i \bar{d}^\dagger_\alpha \bar{\sigma}^\mu (D_\mu \bar{d})^\alpha$$

there is no gauge group singlet contained in any of the products;

it is not possible to write any mass term!

however we can write a Yukawa terms of the form:

$$\mathcal{L}_{\text{Yuk}} = -y'\varepsilon^{ij}\varphi_i q_{\alpha j}\bar{d}^\alpha - y''\varphi^{\dagger i}q_{\alpha i}\bar{u}^\alpha + \text{h.c.}$$

$$(1,2,-\tfrac{1}{2}) \otimes (3,2,+\tfrac{1}{6}) \otimes (\bar{3},1,+\tfrac{1}{3}) = \underline{(1,1,0)} \oplus \ldots$$

$$(1,2,+\tfrac{1}{2}) \otimes (3,2,+\tfrac{1}{6}) \otimes (\bar{3},1,-\tfrac{2}{3}) = \underline{(1,1,0)} \oplus \ldots$$

there are no other terms that have mass dimension four or less!

in the unitary gauge we have:

$$\varphi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

and the Yukawa terms become:

$$\mathcal{L}_{\text{Yuk}} = -\tfrac{1}{\sqrt{2}}y'(v+H)q_{\alpha 2}\bar{d}^\alpha - \tfrac{1}{\sqrt{2}}y''(v+H)q_{\alpha 1}\bar{u}^\alpha + \text{h.c.}$$

$$\mathcal{L}_{\rm Yuk} = -\tfrac{1}{\sqrt{2}} y'(v+H)\, q_{\alpha 2}\bar{d}^{\alpha} - \tfrac{1}{\sqrt{2}} y''(v+H)\, q_{\alpha 1}\bar{u}^{\alpha} + \text{h.c.}$$

it is convenient to label the components of the quark doublet as:

$$q = \begin{pmatrix} u \\ d \end{pmatrix}$$

then we have:

$$\begin{aligned}\mathcal{L}_{\rm Yuk} &= -\tfrac{1}{\sqrt{2}} y'(v+H)(d_\alpha \bar{d}^{\alpha} + \bar{d}^{\dagger}_{\alpha} d^{\dagger\alpha}) - \tfrac{1}{\sqrt{2}} y''(v+H)(u_\alpha \bar{u}^{\alpha} + \bar{u}^{\dagger}_{\alpha} u^{\dagger\alpha}) \\ &= -\tfrac{1}{\sqrt{2}} y'(v+H)\overline{\mathcal{D}}^{\alpha}\mathcal{D}_\alpha - \tfrac{1}{\sqrt{2}} y''(v+H)\overline{\mathcal{U}}^{\alpha}\mathcal{U}_\alpha \,,\end{aligned}$$

we define Dirac fields for the down and up quarks

$$\mathcal{D}_\alpha \equiv \begin{pmatrix} d_\alpha \\ \bar{d}^{\dagger}_{\alpha} \end{pmatrix}, \qquad \mathcal{U}_\alpha \equiv \begin{pmatrix} u_\alpha \\ \bar{u}^{\dagger}_{\alpha} \end{pmatrix}$$

and we see that the up and down quarks have acquired masses:

$$m_d = \frac{y' v}{\sqrt{2}}\,, \qquad m_u = \frac{y'' v}{\sqrt{2}}$$

...work out interactions with gauge fields, and generalization to three families...