

P511 Quantum Mechanics
Exam II
Tuesday, November 25, 2008

Spherical harmonics:

$$\begin{aligned} Y_0^0 &= \frac{1}{\sqrt{4\pi}}, & Y_1^0 &= \sqrt{\frac{3}{4\pi}} \cos \theta, \\ Y_1^{\pm 1} &= \mp \sqrt{\frac{3}{8\pi}} \sin \theta e^{\pm i\phi}, \\ Y_2^0 &= \sqrt{\frac{5}{16\pi}} (3 \cos^2 \theta - 1), \\ Y_2^{\pm 1} &= \mp \sqrt{\frac{15}{8\pi}} \sin \theta \cos \theta e^{\pm i\phi}, \\ Y_2^{\pm 2} &= \sqrt{\frac{15}{32\pi}} \sin^2 \theta e^{\pm 2i\phi}. \end{aligned}$$

Rotation operators:

$$d^{(1/2)}(\beta) = \begin{pmatrix} \cos(\frac{\beta}{2}) & -\sin(\frac{\beta}{2}) \\ \sin(\frac{\beta}{2}) & \cos(\frac{\beta}{2}) \end{pmatrix},$$

$$d^{(1)}(\beta) = \begin{pmatrix} \frac{1}{2}(1 + \cos \beta) & -\frac{1}{\sqrt{2}} \sin \beta & \frac{1}{2}(1 - \cos \beta) \\ \frac{1}{\sqrt{2}} \sin \beta & \cos \beta & -\frac{1}{\sqrt{2}} \sin \beta \\ \frac{1}{2}(1 - \cos \beta) & \frac{1}{\sqrt{2}} \sin \beta & \frac{1}{2}(1 + \cos \beta) \end{pmatrix}.$$

Raising and lowering operators:

$$\begin{aligned} J_{\pm} |j, m\rangle &= \sqrt{(j \mp m)(j \pm m + 1)\hbar} |j, m \pm 1\rangle, \\ &= \sqrt{j(j+1) - m(m \pm 1)\hbar} |j, m \pm 1\rangle. \end{aligned}$$