

P221, Spring 2001, Chapter 8 Summary

Work

- Work done by any force, \vec{F} , on a system of mass, m , is $W = \int_i^f \vec{F} \cdot d\vec{r}$ where the integral is along the path of the system from the initial to final point.
- The units of work are Nm or Joules (J).
- If the force is constant (in both direction and magnitude) along the path, then $W = \vec{F} \cdot \Delta\vec{r}$ where $\Delta\vec{r}$ is the change in position vector from the initial to the final point $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$.
- The work done by a force can be >0 , <0 , or $=0$ depending upon the orientation of the \vec{F} and $\Delta\vec{r}, d\vec{r}$ vectors.
- The work done by a *conservative* force (e.g. gravity, force from a spring) is independent of path.
- The work done by a *non-conservative* force (e.g. kinetic friction) depends upon the path.
- A zero-work force (e.g. static friction, normal force) always does zero-work regardless of path.

Potential and Kinetic Energy

- The work done by a conservative force is equal to the negative of the change of the potential energy function : $W_{conserv} \equiv -\Delta PE$.
- The potential energy function depends on a location in space and upon a choice for a zero-point of PE . The physics depends only upon the *change* in PE .
- Potential Energy Functions:
 - gravity (near earth surface): $PE_{local} = mgy$ (\hat{j} pointing up)
 - gravity (general form): $PE_{grav} = \frac{-GMm}{r}$
 - Hooke's Law Spring: $PE_{spring} = \frac{1}{2} kx^2$
- Kinetic Energy: $KE \equiv \frac{1}{2} m v^2$

CWE Theorem (Conservation of Energy)

– CWE Theorem: Work done by total force on system is equal to change in kinetic energy of system: $W_{total} = \Delta KE$

– The work done by non-conservative (other) forces on a system is equal to the change in total mechanical energy of the system:

$$W_{other} = \Delta E = \Delta (KE + PE)$$

– If all forces acting on system are conservative (or zero-work forces), energy is "conserved": $\Delta E = \Delta (KE + PE) = 0$ or $(KE + PE)_{initial} = (KE + PE)_{final}$

– The *escape speed* is the speed that gives a system sufficient energy to

overcome the gravitational *binding energy*: $v_{escape} = \left(\frac{2GM}{R} \right)^{\frac{1}{2}}$

– A simple harmonic oscillator has total mechanical energy:

$E = \frac{1}{2} k A^2 = \frac{1}{2} k x^2 + \frac{1}{2} m v^2$. The energy "oscillates" from potential to kinetic.

Power

– Power is the amount of work done in a given time and has units of J/s or Watts (W).

– The average power of a force on a system is defined: $P_{ave} = \frac{W}{\Delta t}$

– The instantaneous power is defined: $P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$

Energy Diagrams

– A plot of PE as a function of position for a system with $W_{other} = 0$ is an *energy diagram*. The total energy E is a constant and the KE may be read as the difference between PE and KE.

– The force associated with the PE is the negative of the slope of the PE on the

energy diagram: $F_x = -\frac{d(PE)}{dx}$