Laboratory #20A: Electrical Resonance

Goal: Study the phenomenon of electrical resonance; know some of the conditions under which resonance occurs; quality factor Q

Equipment: non-polar capacitor (0.001 μF), inductance, several resistors, sine-wave generator, and oscilloscope.

(A) Introduction

A dramatic feature of ac circuits is *resonance*. Inductive reactance $X_L = wL$ increases as the frequency is increased, but capacitive reactance $X_C = -1/wC$ decreases with higher frequencies. Because of these opposite characteristics, for any *LC* combination there is a frequency at which X_L equals X_C, as one increases and the other decreases. This case of equal is called *resonance*, and the circuit is then a *resonant circuit*.

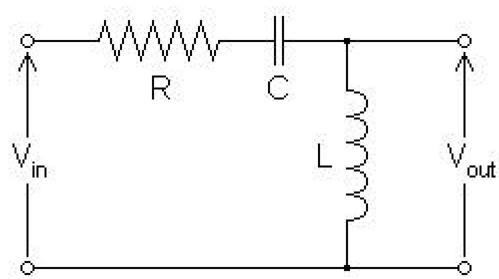

Figure 1: Series RLC circuit.

For a series *RLC* circuit, as shown in Fig. 1, the total impedance Z, the phase angle j, the current i, the output voltage V_{out} and the gain G are:

$$Z = R + jwL - \frac{j}{wC} \qquad |Z| = \sqrt{R^2 + \left(w^2L^2 + \frac{1}{w^2C^2} - 2\frac{L}{C}\right)}$$

$$f = \arctan\frac{wL - \frac{1}{wC}}{R}$$

$$i = \frac{V}{Z} = \frac{V}{R + jwL - \frac{j}{wC}} \qquad V_{out} = iZ = \frac{-\frac{jV}{wC}}{R + jwL - \frac{j}{wC}}$$

$$G = \left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{wC|Z|}$$

The frequency, at which the reactances just cancel and the impedance is a pure resistance, is the resonant frequency w_0.

$$w_0 = \sqrt{\frac{1}{LC}} \quad \text{or} \quad f_0 = \frac{1}{2p\sqrt{LC}}$$

The *quality factor, Q*, of the circuit is defined as

$$Q = \frac{X_L}{R} = \frac{X_C}{R} = \frac{w_0 L}{R} = \frac{1}{w_0 CR}.$$

It is a measure of the energy-storage property (*L and C*) in relation to the energy dissipation property (*R*) of the circuit. At resonance, the output voltage (across the inductor *L*, for example) is equal to *Q* times the input voltage. In addition to indicating the magnitude of the resonance rise, *Q* is also a measure of the frequency selectivity of the circuit. A circuit with *high Q* has a sharp resonance; a *low-Q* circuit is less selective. Another quantitative measure of this selectivity is the *bandwidth*. The bandwidth of a resonating circuit is the width of the resonant band of frequencies around f_0 with a response of 70.7% or more of the maximum response (half-power points). The bandwidth *Df* is given by

$$\Delta f = f_1 - f_2 = \frac{f_0}{Q}$$

(B) Experiments

1. Connect the inductor and capacitor in series to the output of the sine wave generator. Measure beforehand the ohmic resistance of the inductor. Set the input voltage for the resonant circuit to 1 V peak-to-peak. Measure the voltage V_{out} across the inductor and the phase angle j between V_{in} and V_{out} as a function of the frequency around the resonance frequency. Use a low-capacitance oscilloscope probe set to *´10*. Trigger the oscilloscope with the TTL output of the sine wave generator. Plot V_{out}/V_{in} and j as a function of the frequency. Determine f_0, *Df*, and *Q*.

***Note**, the resonance is very sharp! Proceed as follows: First find the resonant frequency, write down $V_{out}^{\max}$, then decrease the frequency till $V_{out} \approx \frac{1}{10} V_{out}^{\max}$, and then carefully scan the resonance in fine steps.*

2. Insert a 100 Ω resistor R in the circuit (see Fig. 1). Measure its value beforehand. Measure again V_{out} and j as a function of the frequency. Determine f_0, Df, and Q. Is f_0 it different from experiment (1)? Explain!

3. Repeat experiment (2) for a 4.7 kΩ resistor.

4. Replace the 4.7 kΩ resistor with a 50 kΩ variable resistor (potentiometer). Select a square wave of low frequency as input to the *RLC* circuit. Observe the decaying oscillations. Find the resistance at which the oscillations disappear. When the oscillations disappear, we have the case of *critical damping*.