Experiments in Modern Physics
Introductory Lab: Poisson Statistics

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Goal
In this experiment you will explore the statistics of random events both by physical measurements and by computer simulations. The random events used in this study will be pulses from a scintillation detector exposed to gamma rays from a radioactive source.

Introduction
A sequence of independent random events is one in which the occurrence of any event has no effect on the occurrence of any other. One example is simple radioactive decay such as the emission of 663 KeV photons by a sample of $^{137}$Cs. In contrast, the fissions of nuclei in a critical mass of $^{235}$U are correlated events in a "chain reaction" in which the outcome of each event, the number of neutrons released, affects the outcome of subsequent events.

A continuous random process is said to be "steady state with mean rate $\mu$" if

$$\lim_{T \to \infty} \left( \frac{X}{T} \right) = \mu$$

where $X$ is the number of events accumulated in time $T$.

How can one judge whether a certain process does, indeed, have a rate that is steady on time scales of the experiment itself? The only way is to make repeated measurements of the number of counts $x_i$ in time intervals $t_i$ and determine whether there is a trend in the successive values of $x_i/t_i$. Since these ratios are certain to fluctuate, the question arises as to whether the observed fluctuations are within reasonable bounds for a fixed rate. Clearly, one needs to know the probability distribution of the numbers of counts in a fixed interval of time if the process does indeed have a steady rate. That distribution is known as the Poisson distribution and is defined

$$P(n; \mu) = \frac{\mu^n e^{-\mu}}{n!}$$

which is the probability of recording $n$ counts (always an integer) when for a mean $\mu$ (generally not an integer). It is simple to show that the standard deviation of the Poisson distribution is simply $\sqrt{\mu}$, that is, the square root of the mean.

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1Revised from MIT Junior Lab: "Poisson Statistics" [1].
**Equipment**
- Scintillation counter (PMT attached to a Thallium-doped NaI crystal).
- High Voltage Supply (Bertan)
- Amplifier (Ortec 460)
- Counter/Timer (Ortec 776)
- Oscilloscope
- Coaxial cables and connectors

**Preparation**
- Read about scintillation counters and associated electronics in Refs. [2,3].
- Read about the Poisson Distribution in Ref. [3].

**Setup**
- Connect the scintillation counter to the HV supply and set to +1500V (watch the polarity!, 2000V maximum!)
- Observe the PMT signals on the oscilloscope and record.
- Obtain a $^{60}$Co and place in the holder near the scintillation counter. Observe the signals.
- Connect the PMT anode to the amplifier ("negative in") and the output (unipolar) of the amplifier to the counter/timer. The counter/timer displays the number of counts in a selectable interval. It also has a tunable discriminator to allow the counting of signals only above a certain voltage level ("discrimination").
- Set the amplifier gain to ~100, the discriminator level to ~3V.
- You can adjust the count rate by varying the source-counter distance, the PMT HV, amplifier gain, and discriminator level.
- Adjust these settings with the source in and out of place to get a feel for how the equipment works and the count rate with and without the source.

**Experiment**
- Adjust the settings as described above to get three different mean count rates of approximately 1 Hz, 10 Hz, and 100 Hz.
- Record the number of counts obtained in a 1 second interval for 100 trials at each of the three rates.

**Analysis**
The following analysis requires the use of a spreadsheet and plotting program such as Sigmaplot (or equivalent).

- Tabulate the 100 data points of the three sets and calculate the mean and standard deviation for each set.
- Make a plot (histogram) of "number of trials" (x-axis) vs "count rate" (y-axis). Be sure to include the vertical error bars.
- Plot the "theoretical" distribution on the same graph and compare.
How do the data compare to the theory? Pick an "outlier" bin of one of your histograms with a number of counts that looks improbable. How improbable is it? Does the data look Poisson-distributed? Compare the 100 Hz data to a normal (Gaussian) distribution.

References