Black-Body Radiation

Goal: Test Planck’s law and the Stephan-Boltzmann law

Equipment: Tungsten lamp with power supply, monochromator, pyrometer

1. Introduction

The Planck distribution law states that a “black” body at temperature T emits radiation with a spectral distribution $f(\lambda)$ given by

$$f(\lambda) = \frac{8\pi hc\lambda^{-5}}{e^{\frac{hc}{\lambda kT}} - 1}.$$  

where $\lambda$ is the wavelength, T the temperature in K, $h=6.626\times10^{-34}$ Js is Planck’s constant, $k=1.381\times10^{-23}$ J/K is the Boltzmann constant, $c=2.998\times10^8$ m/s, and $\varepsilon(\lambda,T)<1$, called the emissivity, is a correction function that takes into account that ideal black bodies (for which $\varepsilon=1$) do not exist in reality. For visible light, the exponential term in eq.1 is much larger than 1, so we can replace the denominator of eq.1 by $\exp(-hc/\lambda kT)$. We will use a monochromator equipped with a photomultiplier to measure $f(\lambda)$. The anode current $i(\lambda,T)$ of the photomultiplier is then given by

$$i(\lambda,T) = b(\lambda) \varepsilon(\lambda,T) \cdot 8\pi \cdot h c \lambda^{-5} e^{-\frac{h}{\lambda kT}},$$

where $b(\lambda)$ is the response of the photomultiplier-monochromator combination to light of wavelength $\lambda$.

When one integrates the Planck distribution to get the total power radiated, $P_R$, one obtains the law of Stephan-Boltzmann,

$$P_R = \varepsilon_{eff} \cdot \sigma \cdot A \cdot T^4,$$  

where $\sigma=5.64\times10^{-8}$ J/m²K⁴ is the Stephan-Boltzmann constant, $A$ is the surface area, and $\varepsilon_{eff}$ is the weighted average of the wavelength dependent emissivity, $\varepsilon(\lambda,T)$.

**TASK 1:** read chapter 3-4 on black-body radiation in ref. [TIP78]

**TASK 2:** Using eq.2 with $\varepsilon(\lambda,T)=1$, find the wavelength $\lambda_{max}$ for which $f(\lambda)$ has a maximum.

**TASK 3:** If you would be given the function $\varepsilon(\lambda,T)$, how would you determine $\varepsilon_{eff}$ in eq.3 ?

2. Monochromator

The monochromator is a spectrograph with a diffraction grating. Only light within a certain (small) range of wavelengths between $\lambda$ and $\lambda + \Delta\lambda$ is transmitted. The wavelength is given by the angle of the grating which is adjustable. Its position is read out on the front panel in units of Å (1 Ångström = 0.1 nm). Entrance and exit slits are also adjustable and determine the resolution $\Delta\lambda$ of the instrument. The monochromator is calibrated with a source of light that has a spectral line for which the wavelength is known. It is convenient to use a sodium lamp, and to observe the Na-D doublet with wavelengths $\lambda_{D1}=589.59$ nm and $\lambda_{D2}=589.00$ nm. The operating voltage of the photomultiplier should be chosen such that the anode current is a linear function of the intensity.

**TASK 4:** set up a sodium lamp at the entrance of the monochromator. With a wide slit setting,
search for the Na-D doublet. With a narrow slit, establish the calibration of the monochromator.

**TASK 5:** Sitting on one of the peaks, measure the PM current as a function of the slit width. Establish the range over which the PM response is linear. Do this for two or three values of the PM high voltage.

3. Tungsten Filament as a “Black” Body

The “black” body in this experiment is the filament of a projection lamp. The filament is made from tungsten (W), which is not an ideal black body. Hence, the emissivity $\varepsilon(\lambda, T)$ is less than one. The value of $\varepsilon(\lambda, T)$ for W for different wavelengths and temperatures is listed in tab.1.

The W filament is heated by a steady current $I$, and the voltage across the filament is $U$. The resistance $R=U/I$ arises partly from the wire connection to the hot filament, and partly from the filament itself. The resistivity of tungsten depends strongly on the temperature. Table 2 lists the resistivity $\rho$, and the ratio $\frac{R(T)}{R(300K)}$ of the resistance at temperature $T$ divided by the resistance at room temperature ($T=300K$).

The electrical power delivered to the filament is $P_{in}=IU$. On the other hand, the power leaving the filament is due to conduction and radiation, or

$$P_{out} = c_c(T-T_0) + \sigma A(T^4-T_0^4), \quad (4)$$

where $c_c$ is the coefficient of conduction, $T$ is the filament temperature and $T_0$ is the temperature of the surroundings (lamp housing). Since in a steady state, the input power must equal the output power, the electrical parameters of the filament are related to the temperature.

<table>
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<th>Temperature (K)</th>
<th>$\rho$ (µΩcm)</th>
<th>$\frac{R}{R(300)}$</th>
<th>Temperature (K)</th>
<th>$\rho$ (µΩcm)</th>
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<th>Temperature (K)</th>
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Table 1: Emissivity of tungsten as a function of $T$ and $\lambda$ (in µm) From ref. [CRC95].

Table 2: resistivity for W and resistance ratio $R(T)/R(300K)$ vs $T(K)$
4. Optical Pyrometer

At temperatures above about 1200 K the definition of temperature is based on Planck’s law, and the method to measure it uses the emitted radiation. In an Optical Pyrometer a ‘standard lamp’ that is incorporated in the instrument is compared by eye with the brightness of the subject. Usually, a filter is used, and only the radiance in a narrow wavelength band around some wavelength $\lambda_{\text{instr}}$ is considered. In our case, $\lambda_{\text{instr}} = 650$ nm. After matching the standard brightness with the subject the temperature $T_{\text{instr}}$ can be read from the instrument. If the body under study is not a blackbody, its emissivity $\varepsilon$ will be less than unity and its true temperature $T$ will be higher than $T_{\text{instr}}$. The following relation holds

$$\frac{1}{T} - \frac{1}{T_{\text{instr}}} = \frac{k}{hc} \cdot \lambda_{\text{instr}} \cdot \ln \varepsilon = -3.71 \cdot 10^{-5} \text{ K}^{-1}$$.

Here, we have used $\varepsilon=0.44$ for the emissivity of tungsten at $\lambda=650$ nm. The temperatures are given in units of K (Kelvin), where $T(\text{K}) = T(\text{oC}) + 273.15$.

**TASK 6:** derive eq.5

**TASK 7:** using the pyrometer, measure the filament temperature $T_{\text{instr}}$ and the filament current and voltage. Deduce the true temperature $T$, the resistance $R$, and the electrical power $P_{\text{in}}$. Plot the temperature $T$ vs filament current $I$. This allows you to deduce $T$ from the measured current for future measurements. Plot the resistance $R$ vs temperature $T$, and try to explain the measurement using the data in table 2. Plot the input power $P_{\text{in}}$ vs temperature $T$ and try to explain the measurement using eq. 4.

5. Planck’s Law

You are now ready to test Planck’s law. After setting up the lamp in front of the monochromator, choose a wavelength (500 nm) and measure the PM current $i_{\text{PM}}$ as a function of the filament temperature $T$. Plot $\log i_{\text{PM}}$ vs $1/T$ and determine the slope of the straight line. Determine $hc/k$ and its uncertainty and compare with the expected value. Repeat for a monochromator setting of 400 nm and 300 nm

References

Modern Physics Lab: 
Black-Body Radiation: Supplementary Notes

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Misc. Hints:
- Read Preston and Dietz, Exp. 8, for more info (and help with the tasks) on this experiment.
- Use plastic spectrometer to get a feel for spectrum of Na-D.
- Reasonable PMT voltages: 300-500 V.
- Monochrometer is sensitive to vibration!
- PMT current is read via a circuit inside the PMT housing which converts to a voltage signal (i.e. multimeter should be reading voltage).
- PMT is a Hamamatsu R3896.
- Allow lamps to warm up before doing measurements.

Task Notes:
- Task 4: Find the Na-D doublet with a wide slit setting (~50µm), determine the calibration (and error) with and narrow slit setting (~10µm).
- Task 6: The idea here is that the temperature of the tungsten filament is higher than the reading from the pyrometer due to the emissivity.
- Task 7: Measure the filament temperature, current, and voltage for 5-6 current settings of the bulb. Note that the current and voltage for the bulb are AC measurements.