1) The energy of each subsystem is such that there are no macroscopic quantities that are vastly more microstates or sub-systems. This energy is distributed in such a way as to affect the temperature. When two subsystems are microstates with different energy, allocation tends to lead to the number of microstates with a small number of energy units. That, in turn, affects the variance of small measure.
b) Consider the Three Legs:

\[ W_{2\rightarrow 3} = 0 \quad W = 0 \]

\[ W_{1\rightarrow 2} = -P_2 \Delta V = -0.1 \text{ MPa} \cdot (6.4 \text{ m}^3 - 8 \text{ m}^3) = -5.6 \text{ MN/m}^2 \cdot \text{m}^3 = -5.6 \text{ MJ} \]

\[ W_{3\rightarrow 1} = \int PdV = C \int_{6.4 \text{ m}^3}^{8 \text{ m}^3} \frac{dV}{V^{5/3}} = 102.4 \text{ MPa} \cdot \text{m}^{5/3} \]

\[ = C \left[ -\frac{3}{2} \frac{1}{V^{2/3}} \right]_{6.4}^{8} = 153.6 \text{ MPa} \cdot \text{m}^{5/3} \left[ \frac{1}{(8 \text{ m}^3)^{2/3}} - \frac{1}{(6.4 \text{ m}^3)^{2/3}} \right] = 28.8 \text{ MJ} \]

\[ W_{\text{Tot}} = (28.8 - 5.6) \text{ MJ} + 0 \text{ J} \]

\[ W_{\text{Tot}} = 23.2 \text{ MJ} \]
32. \( T_h = 350 \degree C = 623 \, K \)

- \( C = 3.4 \times 10^4 \, J/\, K \)
- \( T_0 = 10 \degree C = 283 \, K \)
- \( T_f = 350 \degree C = 623 \, K \)

a) \[ Q = C \Delta T \]
   \[ = 3.4 \times 10^4 \, J/\, K \times (623 \, K - 283 \, K) \]
   \[ = 1.156 \times 10^7 \, J \]
   \[ Q = 1.16 \times 10^7 \, J \]  \( \text{Total heat rejected into the cold reservoir} \)

b) \[ |\Delta S_c| = \int_{283}^{623} \frac{C}{T} \, dT \]
   \[ = 3.4 \times 10^4 \, J/\, K \times \ln \left( \frac{623}{283} \right) \]
   \[ = 2.68 \times 10^4 \, J/\, K \]
   \[ \Delta S_c = 2.7 \times 10^4 \, J/\, K \]  \( \text{(NOT} \frac{350 \degree C}{10 \degree C}) \)

The sink absorbs heat so its entropy must increase. The change is positive.
c) The engine is reversible so $\Delta S_{\text{univ}} = 0$ but the only change in entropy takes place in the heat reservoir & the heat sink (the engine itself goes in a cycle and has no entropy change). Hence

$$\Delta S_H = \Delta S_c \quad \Delta S_H = -\Delta S_c$$

$$\Delta S_H = -2.68 \times 10^4 \text{ J/K} = \frac{Q_H}{T_H}$$

$$Q_H = -2.68 \times 10^4 \text{ J/K} \cdot 623 \text{ K}$$

$$Q_H = -1.67 \times 10^7 \text{ J}$$

(Not this is more than we supplied into the piston so some work could have been done)
4) \( Z_N = (Z_1)^{3N} = (Z_x)^N \)

where \( Z_x \) is the partition function for a single direction in space. Each atom is effectively 3 1-0 harmonic oscillators.

\[
Z_x = \sum_{n=0}^{\infty} e^{-\beta \omega_x (n + \frac{1}{2}) / \hbar} = \sum_{n=0}^{\infty} e^{-\beta \omega_x (n + \frac{1}{2}) / \hbar} = e^{-\beta \omega_x / 2} \sum_{n=0}^{\infty} e^{-\beta \omega_x (n + \frac{1}{2}) / \hbar} = e^{-\beta \omega_x / 2} \sum_{n=0}^{\infty} e^{-\beta \omega_x n / \hbar}
\]

Let \( x = e^{\beta \omega_x / \hbar} \) and we can use the result given in the question:

\[
Z_x = e^{-\beta \omega_x / 2} \frac{1}{1 - e^{-\beta \omega_x / \hbar}}
\]

\[\Rightarrow Z_N = \left[ \frac{e^{-\beta \omega_x / \hbar}}{1 - e^{-\beta \omega_x / \hbar}} \right]^{3N}\]
b) \( \langle E \rangle = k_B T^2 \left( \frac{2 \ln 2}{T} \right) \mu \nu \)

\[ = k_B T^2 \cdot 3N \cdot \left( \frac{3}{T} \ln 2 \right) \]

\[ = k_B T^2 \cdot 3N \frac{3}{2T} \left[ -\frac{\theta_E}{2T} - \ln \left( 1 - e^{-\theta_E T} \right) \right] \]

\[ = 3Nk_B T^2 \left[ \frac{\theta_E}{2T^2} + \frac{e^{\theta_E T} \theta_E T^2}{1 - e^{-\theta_E T}} \right] \]

\[ = 3Nk_B \theta_E \left[ \frac{1}{2} + \frac{e^{-\theta_E T}}{1 - e^{-\theta_E T}} \right] \]

\[ \langle E \rangle = 3Nk_B \theta_E \left[ \frac{1}{2} + \frac{1}{e^{\theta_E T} - 1} \right] \]