1. (a) Show that the electric field of a pure dipole can be written in a coordinate-free form as
\[ E(r) = \frac{1}{4\pi\epsilon_0} \frac{1}{r^3} [3(p \cdot \hat{r})\hat{r} - p] . \]  
(b) Show that the energy of an ideal dipole \( p \) in an electric field \( E \) is given by
\[ U = -p \cdot E. \] 
To do this consider the amount of work done to move a dipole from infinity through an electric field and then orient the dipole in some specific direction with respect to the electric field.
(c) Use your answers to (a) and (b) to show that the interaction energy of two dipoles separated by a distance \( r \) is
\[ U = \frac{1}{4\pi\epsilon_0} \frac{1}{r^3} [p_1 \cdot p_2 - 3(p_1 \cdot \hat{r})(p_2 \cdot \hat{r})] . \]

2. Imagine three charged rods with length \( \ell \) and charge \( Q \). The rods are arranged so that one end of each rod is at the origin and the other end is a distance \( \ell \) away from the origin along the \( x, y, \) and \( z \) axis.
(a) Find the dipole moment of this charge charge distribution \( p \) as measured about the origin. Be sure to express \( p \) as a vector.
(b) Write an approximate expression for the potential along the \( z \) axis \( V(0,0,z) \) for \( z \gg \ell \) that includes monopole and dipole terms.
(c) What are the monopole and dipole contributions to the electric field along the \( z \) axis \( E(0,0,z) \) for \( z \gg \ell \). (For the latter, the expression in Problem 1(a) might be useful.)

3. A linear quadrupole consists of three charges: \( q, -2q, \) and \( q \), on the \( z \) axis. The positive charges are at \( z = \pm d \) and the negative charge is at the origin.
(a) Show that this system is the same as two dipoles, with dipole moments \( +qd\hat{z} \) and \( -qd\hat{z} \), centered at \( z = +d/2 \) and \( z = -d/2 \) respectively and that the total dipole moment is zero.
(b) Show that the potential \( V(r,\theta) \) in spherical coordinates for \( r \gg d \) can be approximated by the expression
\[ V(r,\theta) = \frac{qd^2}{4\pi\epsilon_0} \frac{3\cos^2 \theta - 1}{r^3}. \]

4. A pure dipole \( p \) is placed a distance \( z_0 \) from a grounded conducting plane, taken to be the \( x, y \) plane. The direction of \( p \) is at an angle \( \theta \) with respect to the normal of the plane (\( \hat{z} \)). Find the torque on \( p \). What are the equilibrium values of \( \theta \). (Hint: you’ll want to use the method of images.)

5. Find the force on a dipole in the field of a point charge. Let the dipole moment be \( p = p_x\hat{x} + p_y\hat{y} + p_z\hat{z} \) and the separation vector between the charge and the dipole be \( r \). Use the equation \( F = (p \cdot \nabla)E \) and work in rectangular coordinates. Show that this result is what you expect from Newton’s Third Law and the answer to Problem 1(a) above.

6. A point charge \( q \) is situated a large distance \( r \) from a neutral atom of polarizability \( \alpha \). Find the force of attraction between them.

7. A short cylinder, of radius \( a \) and length \( L \), carries a “frozen-in” uniform polarization \( P \), parallel to its axis. (Such an object is referred to as a “bar electret,” the electrostatic analog of a bar magnet.) Find the bound charge, and sketch the electric field (i) for \( L \gg a \), (ii) for \( L \ll a \), and (iii) for \( L \approx a \).