1. Two semi-infinite conducting planes meet at at a 60° angle. A charge +q is placed in the narrow gap between the planes along the bisector of the 60° angle. Assuming you want to compute the potential in this region sketch the location and magnitude of all of the necessary image charges. (You do not need to compute the potential.)

2. Two semi-infinite conducting planes meet at a 90° angle. A charge +q is placed a distance d from each plane.
   (a) Compute the potential in the region in between the planes.
   (b) How much work is required to move the charge from infinity to its location in between the planes?

3. Study the image charge problem discussed in Example 3.2 in your text (p. 124).
   (a) Show that Equation 3.17 can be written as
   \[ V(r, \theta) = \frac{1}{4\pi\varepsilon_0} \left( \frac{q}{\sqrt{r^2 + a^2 - 2ra \cos \theta}} - \frac{q}{\sqrt{R^2 + (ra/R)^2 - 2ra \cos \theta}} \right). \]  
   This form should make it clear that when \( r = R \), \( V = 0 \).
   (b) Find the induced surface charge density on the sphere as a function of \( \theta \).
   (c) Integrate this charge distribution to find the total induced charge.

4. (a) Show that the Fourier series of a square wave \( f(x) \) that has wavelength \( 2\ell_2 \) and amplitude 1, with \( f = 1 \) for \( 0 < x < \ell_2 \) and \( f = -1 \) for \( -\ell_2 < x < 0 \), is
   \[ f(x) = \frac{4}{\pi} \sum_{n \text{ odd}} \frac{1}{n} \sin \left( \frac{n\pi x}{\ell_2} \right). \]  
   (b) Use a computer program to plot the first term and the sum of the first five terms of the series. Plot over the range \( -\ell_2 < x < \ell_2 \).
   (c) Notice that when the function in (a) is evaluated at \( x = \ell_2/2 \) (in the region where \( f = 1 \)) the result is the following infinite series
   \[ \frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \ldots. \]  
   This series, discovered by Leibniz in 1673, shows that \( \pi \) can be expressed using every odd integer exactly once! Prove Leibniz’s series using the Taylor series of \( \tan^{-1}x \). (Now you know at least one method to calculate \( \pi \) to arbitrary precision.)

5. Study Example 3.3 in the text. Now, suppose that boundary at \( x = 0 \) is composed of two metal strips: one from \( y = 0 \) to \( y = a/2 \) that is held at constant potential \( V_0 \), and another from \( y = a/2 \) to \( y = a \) that is held at constant potential \( -V_0 \). Show that the expression for potential in the slot is
   \[ V(x, y) = \frac{8V_0}{\pi} \sum_{n=0}^{\infty} \frac{1}{(4n+2)} e^{-(4n+2)\pi x/a} \sin \left( \frac{(4n+2)\pi y}{a} \right). \]

6. (a) Consider long rectangular conducting pipe as shown in Figure 3.20 of your text and discussed in Example 3.4. Suppose that the sides at \( y = a \) and \( y = 0 \) are grounded and the sides at \( x = \pm b \) have a potential given by \( V_0 \sin(\pi y/a) \). Write an expression for \( V(x, y) \) inside the pipe.
(b) Check your answer using Excel by creating a grid (pick \(a = b = 1\)) and let each cell be a step of, say, 0.05. Populate the cells on the boundary according to the boundary condition (pick \(V_0 = 1\)) and compute the potential in the center using the relaxation technique we discussed in class. Then compute the potential on the grid directly using your solution. Do they look similar? Make a plot of the difference of the two computations: \(V_{\text{relaxation}} - V_{\text{analytical}}\) over the \(x, y\) plane and turn it in. (The relaxation technique is an iterative solution. Excel stops iterating when the change in the values with each iteration is below some threshold. By adjusting this threshold or by directly setting the number of iterations, you can change the precision of the result obtained by the relaxation technique.)

7. Again, consider the arrangement in Example 3.4 (Figure 3.20). Suppose that the sides at \(y = a\) and \(y = 0\) are grounded, and the side at \(x = b\) has potential \(V_0\), while the side at \(x = -b\) has potential \(-V_0\).

(a) Write an expression for the potential \(V(x, y)\) inside of the conductor.

(b) Write down an expression for the magnitude of the electric field at the point \((0, a/2)\), in the center of the pipe. (This expression will be an infinite series.)