Electricity and Magnetism I (P331)  
Homework 2  
Due: Wednesday, September 16, 4:00 PM

1. Find the magnitude of the electric field at a distance \( z \) above a circular loop of charge \( Q \) and radius \( a \). Compute the electric field by directly integrating over the charge distribution 
\[
\frac{1}{4\pi\epsilon_0} \int \frac{Q}{r^2} \, dl \hat{r}.
\]
(Here, \( \hat{r} \) is the separation vector, “script r” in the text.)

2. The electric field in some region of space is given by \( \mathbf{E} = \alpha(r^4 - \frac{1}{r})\hat{r} \) in spherical coordinates.
   (a) Find the charge density \( \rho \).
   (b) Use your answer to (a) to find the total charge contained in a sphere of radius \( R \) centered at the origin.
   (c) Check your answer to (b) by using the integral form of Gauss’ Law to find the total charge contained in a sphere of radius \( R \) centered at the origin.

3. Assume there is a ribbon of electric charge in the \( x, y \) plane. It extends from \(-a \leq y \leq a\) in the \( y \) direction and infinitely long in the \( x \) direction. If the surface charge density is \( \sigma \), calculate the electric field at the position \((0, 0, d)\). (A distance \( d \) above the origin on the \( z \) axis.)

4. Consider a spherical shell with inner radius \( a \) and outer radius \( b \). In the region \( a < r < b \) the charge density is given by \( \rho = kr^3 \). (Elsewhere \( \rho = 0 \).)
   (a) Find the electric field in three regions: (i) \( r < a \), (ii) \( a \leq r \leq b \), and (iii) \( r > b \).
   (b) Plot \( |\mathbf{E}| \) as a function of \( r \).

5. A long coaxial cable carries a uniform \textit{volume} charge density \( \rho \) on the inner cylinder (radius \( a \)), and a uniform \textit{surface} charge density on the outer cylindrical shell (radius \( b \)). This surface charge is negative and of just the right magnitude so that the cable as a whole is electrically neutral.
   (a) Find the electric field in three regions: (i) \( s < a \), (ii) \( a \leq s \leq b \), and (iii) \( s > b \).
   (b) Plot \( |\mathbf{E}| \) as a function of \( s \).

6. Two solid spheres each with radius \( R \) carrying charge densities \( \rho \) and \(-\rho \) are placed so that the partially overlap with each other. Assume that the vector from the center of one sphere to the center of the other sphere is \( \mathbf{d} \) (where, because the spheres overlap, \( |\mathbf{d}| < 2R \)). Show that the field in the region of the overlap is constant, and find its value.

7. Repeat Problem 1, but this time compute \( V \) first and then compute the electric field from the the gradient of \( V \).

8. Given the charge distribution Problem 4 above, compute the potential at the center of the spherical shell.

9. Consider a hollow cylindrical shell (imagine an empty can with no ends) with surface charge density \( \sigma \), length \( \ell \), and radius \( a \). The shell is aligned so that the center is at the origin and the axis is parallel to the \( z \) axis (so the shell stretches from \( z = -\ell/2 \) to \( z = \ell/2 \)).
   (a) Write an expression for the potential \( V \) as a function of \( z \) along the \( z \) axis.
   (b) Show that as \( z \to \infty \) the electric field resembles that produced by a point charge with total charge equal to the charge of the cylindrical shell.