1. Determine the angle between the diagonal of a cube and an adjacent edge. *Hint:* use the unit cube – the diagonal is given by $\hat{x} + \hat{y} + \hat{z}$.

2. (a) Prove that $|\mathbf{A} \times \mathbf{B}|$ is equal to the area of the parallelogram defined by $\mathbf{A}$ and $\mathbf{B}$.
(b) Show that $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$ is equal to the volume of the parallelepiped defined by $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$. From this result prove that $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}$.

3. Prove the following identities:
   (a) $\nabla \cdot (g\mathbf{F}) = g\nabla \cdot \mathbf{F} + \nabla g \cdot \mathbf{F}$
   (b) $\nabla \times (g\mathbf{F}) = g\nabla \times \mathbf{F} + \nabla g \times \mathbf{F}$
   (c) $\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot (\nabla \times \mathbf{F}) - \mathbf{F} \cdot (\nabla \times \mathbf{G})$
   (d) $\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2 \mathbf{F}$

4. (a) Prove that the divergence of a curl is always zero, and check it for the function $\mathbf{v} = xz^2\hat{x} + 2xy\hat{y} + zy\hat{z}$.
(b) Prove that the curl of a gradient is always zero, and check it for the function $f = x^3 + zx + y^2z$.

5. Consider the function $g = x^3 + 3xy^2$. Check the theorem for gradients by showing that the integral of $\nabla g$ from $(0,0,0)$ to $(1,1,0)$ is the same for the following three paths:
   (a) the path from $(0,0,0) \rightarrow (0,1,0) \rightarrow (1,1,0)$
   (b) the path from $(0,0,0) \rightarrow (1,0,0) \rightarrow (1,1,0)$
   (c) the path from $(0,0,0) \rightarrow (1,1,0)$ along the parabola $y = x^2$

6. (a) Sketch a picture of the vector field $\mathbf{F} = x\hat{x}$.
   (b) Calculate directly the flux of $\mathbf{F}$ outward through the surface of the unit cube defined by $0 \leq x, y, z \leq 1$.
   (c) Calculate the flux using the divergence theorem.

7. For the following problem work in *rectangular coordinates*.
   (a) Sketch a picture of the vector field $\mathbf{F} = \hat{z} \times x\hat{x}$
   (b) Calculate directly the line integral of $\mathbf{F}$ around a circle in the $xy$ plane, centered at 0 with a radius $a$.
   (c) Calculate the line integral by using Stokes’ theorem.

8. Rewrite $\mathbf{F}$ from the problem above in *cylindrical coordinates* and rework the problem in cylindrical coordinates.