The moving capacitor

Today in lecture we worked out both the magnetic and electric pressures on the plate of a capacitor due to the other plate. At the end of class I tried to qualitatively draw a connection to relativity. If we had just a little more time I would have made this connection a little more quantitative. Let’s finish off the problem.

We showed that the magnetic pressure due to the upper plate on the lower plate is repulsive and has a magnitude

$$|F| = \mu_0 \sigma^2 v^2.$$  

(1)

here $\sigma$ is the charged density and $v$ is the velocity of the plates. At the same time the electric pressure due to the lower plate on the upper plate is attractive and has a magnitude

$$|F| = \frac{\sigma^2}{2\epsilon_0}.$$  

(2)

Let’s assume that the plates have equal but opposite charges $\pm q$. Then, we can write the net upward force on the lower plate as

$$F = \frac{q^2}{2\epsilon_0} - \frac{q^2 v^2 \mu_0}{2} = \frac{q^2}{2\epsilon_0} \left(1 - v^2 \mu_0 \epsilon_0\right).$$  

(3)

Now, suppose that we have two observers. One observer is standing on the upper plate as the plates move by. That observer is at rest with respect to the plates. The other observer is at rest in the lab watching the plates move by with a speed $v$. The two observers decide to both measure how long it takes the lower plate to move upward some distance $d$ and then compare notes.

We can easily compute the time it takes the lower plate to move a distance $d$. Note that the force is independent of the separation so we have constant acceleration motion, and

$$d = \frac{1}{2} at^2,$$  

(4)

or

$$t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2dm}{F}},$$  

(5)

where $F$ is the net force on the plate and $m$ is the mass of the plate.

In the reference frame of the observer standing on the upper plate (the moving reference frame), the plates are at rest, $v = 0$, and we have

$$t' = \sqrt{\frac{2dm}{q^2/2\epsilon_0}}.$$  

(6)

Meanwhile, the observer standing in the lab watching the plates move by measures a time

$$t = \sqrt{\frac{2dm}{q^2/2\epsilon_0(1 - v^2 \mu_0 \epsilon_0)}}.$$  

(7)

Comparing the two times we see that

$$\frac{t}{t'} = \sqrt{\frac{1}{1 - v^2 \epsilon_0 \mu_0}} = \sqrt{\frac{1}{1 - v^2/c^2}} = \gamma.$$  

(8)

In other words, the observer at rest in the lab watching the plates move measures a relatively longer time $t$ for the lower plate to move some distance $d$. This time $t$ is a factor of $\gamma$ longer than the proper time $t'$ as measured by the observer with respect to the plates. From the observer watching the plates move by, the clock of the moving observer appears to be running relatively slower.