Laboratory #31: Debye-Sears: Speed of Sound in Water

Goal: study the diffraction of light by density fluctuations in a liquid; determine the speed of sound by measuring the wavelength of ultrasound. Measure the compressibility of water

Equipment: Cuvette, ultrasonic generator, laser diode, system of mirrors, frequency counter, thermometer.

A. Physics: the Debye-Sears Effect

A.1. Interaction between light and sound waves in a liquid

A sound wave is characterized by regions of density that are higher or lower than that of the undisturbed medium *M*. These regions alternate in the direction of propagation, and are separated by the wavelength λ_S of the sound in the medium. If f_S is the frequency of the sound, the speed of sound, v_S, is given by

$$v_S = f_S \cdot \lambda_S \tag{1}$$

In a transparent liquid medium, variations in density correspond to a deviation δn_M of the index of refraction from its average value n_M.

A light ray that travels perpendicular to the sound direction is refracted. The angle of refraction is zero if the ray coincides with a density maximum or minimum, and largest if the ray is halfway between. The range of refraction angles depends on the sound intensity.

A.2. Diffraction by a phase grating

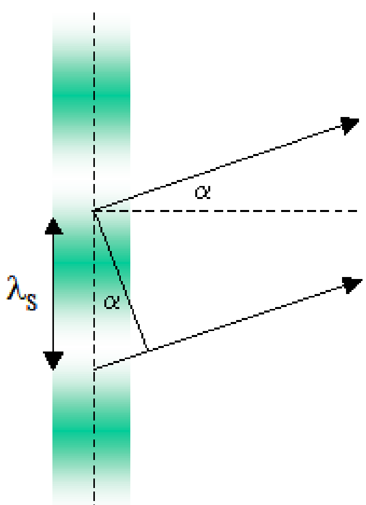


Fig. 1

The transmitted light rays that are refracted by an angle α are enhanced if the path difference of rays originating from two points, separated by a sound wave length, equals an integer *k* times the wavelength λ_{LM} of the light in the medium (see fig.1). Thus, we expect a bright spot in the direction α_k if

$$\lambda_S \cdot \sin\alpha_k = k \cdot \lambda_{LM} \tag{2}$$

Note, that the wavelengths of the light in vacuum, λ_{L0}, and in the medium, λ_{LM} are related by

$$\lambda_{L0} = \lambda_{LM} \cdot n_M \tag{3}$$

When the light rays exit the cuvette, they are refracted at the interface between the medium and air (away from the normal to the surface), and Snell's law tells us that

$$\sin\alpha'_k = n_M \sin\alpha_k \tag{4}$$

Diffraction of light by ultrasonic waves has been first studied by Debye and Sears [DEB32], and Raman and Nath have given a general theoretical treatment.

A.3. Compressibility of water

We often assume that a liquid is incompressible, but if that were really true, a sound wave could not propagate. In fact, the compressibility

$$\kappa = -\frac{1}{V}\frac{\partial V}{\partial p} \tag{5}$$

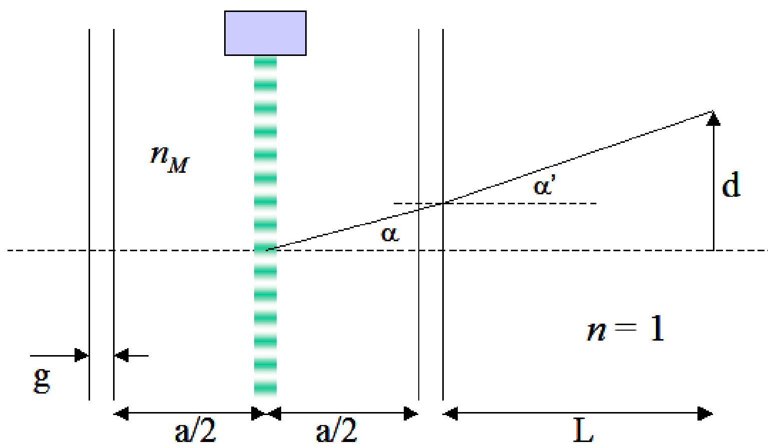
is related to the speed of sound v_S by

$$\kappa = \frac{1}{v_S^2 \cdot \rho} \tag{6}$$

Where ρ is the density of the liquid. The compressibility that we are determining this way is called the 'adiabatic' compressibility because no energy is exchanged with the region next to the sound wave. This should be distinguished from the 'isothermal' compressibility where the derivative in eq.5 is evaluated at constant temperature.

B. Measurement

B.1. Diffraction method



When we view the bright spots on a screen at a distance *L* from the wall of the cuvette we find that they are separated by

d_k = L·tg α_k' + (a/2+g) tg α_k

from the center spot that belongs to the undeflected beam. Since α_k is small, we may use sin $\alpha \approx$ tg α, and by combining eqs. 2 – 4, we find

$$d_k = k \cdot \frac{\lambda_{L0}}{\lambda_S} \cdot \left(L + \frac{1}{n_M}\left[\frac{a}{2} + g\right]\right) \tag{7}$$

Eq. 7 tells us that the spots are equidistant. From the measured spot separation, d_k / k, we can now determine the wavelength of the sound, and from eq.1 its speed v_S in the medium. This should be done for all frequencies between 1 and 8 MHz. Adjusting the transducer position with the three leveling screws helps in making more diffraction spots visible.

Determine the error of your measurement of the speed of sound and compare the result with the data in the literature (see fig.3, note that you need to know the temperature of the water).

Deduce the *adiabatic compressibility* of water from eq.6.

Here are some measurements that are hard to get (you may want to verify them):

Wave length of the laser: λ_{L0} = (650 ± 5) nm

Width of the cuvette: a = 9.6 cm

Thickness of the glass wall: g = 0.5 cm

Distance from table surface to mirror on the ceiling (needed to get L): (239 ± 0.5) cm

Index of refraction of water: n_M = 1.33

Index of refraction of the glass wall: n_G = 1.46

B.2. Projection method

It is possible to directly project the pattern formed by the sound wave. For this experiment we insert a lens (focal length in air, f_L = 10.0 cm) between the laser and the wall of the cuvette. This generates a focus somewhere between the sound wave and the downstream wall of the cuvette. An object in the center of the cuvette is then projected onto the screen, magnified a lot, since L is much larger than the distance from the object to the focus.

The projection needs a still pattern, which is obtained by establishing a standing sound wave inside the cavity. The standing wave is caused by interference between the direct sound wave and the one reflected from the bottom of the cuvette. To get a visible pattern, one has to select 4 MHz sound frequency and carefully adjust the transducer position with the three leveling screws. The pattern on the screen then shows bright zones, and we can measure their separation.

It is important to realize that the bright spots are caused by rays that traverse the sound wave region unperturbed. This is the case if the rays coincide with either a pressure maximum *or* minimum. Thus the bright zones correspond to *half* a wavelength, ½ λ_S.

Convince yourself that the projection magnification μ is given by

$$\mu = \frac{L - \left(f_L - a/n_M - 2g/n_G\right)}{f_L - \frac{1}{2}a/n_M - g/n_G} \qquad (6)$$

and determine the wavelength of the sound. Compare with the corresponding result obtained with the diffraction method.

C. References

[DEB32] R. Debye and F.W. Sears, Proc. Nat. Acad. Sci. **18**, 410 (1932).

[BOR75] M. Born and E. Wolf, *Principles of Optics*, Pergamon Press, 5th ed. (1975), chapter XII, p.593-596.

[KOH68] F. Kohlrausch, *Praktische Physik*, Teubner, Stuttgart, 22nd ed. (1968)

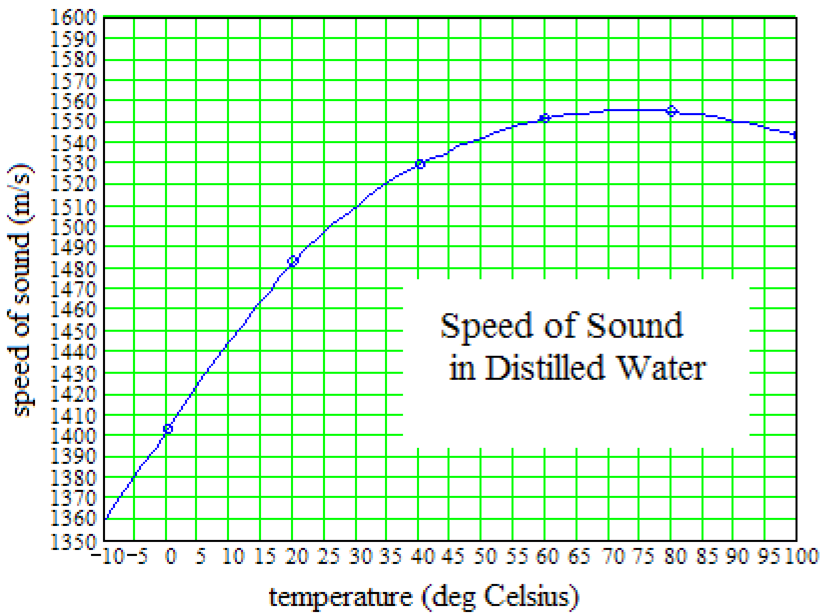


Fig.3: Speed of sound in water (from [KOH68])