**Labortory #19: Viscosity**

**Goal:** Measure the temperature dependence of the viscosity of water.

**Equipment:** Coffeepot viscometer, assorted capillaries, thermometer, plastic trays, graduated glass cylinder.

**(A) Physics:**

When, in the flow of a real fluid, adjacent "layers" move with different velocities, there are frictional forces that oppose this motion. Thus, some of the flow energy is dissipated. The quantity that describes to what extent this happens in a certain fluid is called viscosity. The viscosity depends on temperature, density, and pressure (in so-called non-Newtonian fluids, the viscosity also depends on other factors (for instance, so-called thixotropic liquids such as ketchup and concrete have a lower viscosity when they are agitated).

Attempts to calculate viscosities from the statistical properties of the fluid are successful only for simple fluids (such as noble gases at low pressure). For these simple fluids, one finds that the viscosity increases with increasing temperature. For most liquids, however, the temperature dependence is opposite, and the viscosity decreases with temperature (see fig.1). This is not yet understood. Good sources to learn more about viscosity are [REI87] and [MAR72]. The viscosity is defined as the shear force (in N/m²) divided by the velocity gradient (in 1/s) perpendicular to the motion. Thus, the SI unit for the viscosity $\eta$ is kg m⁻¹ s⁻¹, but one frequently encounters also the historic unit of a Poise, where 1 Poise = 0.1 kg m⁻¹ s⁻¹.

There are some cases where the effect of the viscosity on the flow of the liquid can be calculated easily. One such case is the flow through a cylindrical tube (see, e.g., [KAU63], p.66-72). The result that is also known as Poiseuille’s Law is an expression for the flow rate $Q$ (in m³/s) in terms of the viscosity $\eta$, the radius $R$ (m) of the pipe, its length $L$ (m), and the pressure difference $\Delta p$ (Pa) over the length of the flow:

$$Q = \frac{\pi R^4 \Delta p}{8 \eta L} \quad (1)$$

When the fluid enters the pipe it has to be accelerated, and energy conservation requires that this is associated with a pressure drop. Thus, only a fraction of the total pressure difference $\Delta p$ between intake and outlet of the pipe is available for $\Delta p_{tube}$ in eq.1. From Bernoulli’s law and the fact that the flow rate is the area of the cross section times the average flow velocity $v$, or $Q = \pi R^2 v$, one derives easily that
\[ \Delta p_{\text{tube}} = \Delta p - \frac{\rho Q^2}{2\pi^2 R^4}, \quad C \]  

where \( \rho \) (in kg/m\(^3\)) is the density of the fluid, and the constant \( C \) is a fudge factor that takes into account the exact flow at the mouth of the tube, and that has been determined empirically to have the value \( C=2.25 \) (see [STR61]). Inserting eq.2 into eq.1 and solving for \( \eta \), yields the equation we need to analyze our experiment

\[ \eta = \frac{\pi R^4}{8Q L} \left[ \Delta p - \frac{\rho Q^2 C}{2\pi^2 R^4} \right]. \]  

**Experiment:**

Usually, the principle of operation of a viscosimeter is to actually measure the flow through a tube. In our case, a coffee pot has been commandeered to serve as a reservoir for the liquid (tap water). The pressure difference is given by the height \( h \) (m) of the water column in the pot. Thus, the pressure difference across the tube is given by \( \Delta p = \rho gh \) (\( g=9.81 \text{ m/s}^2 \)). The density of water is \( \rho = 10^3 \text{ kg/m}^3 \), a value that is accurate to within 2\% over the whole temperature range that we have to consider.

An assortment of pipes (actually, glass capillaries) is provided. These pipes are installed from the inside by reaching into the pot. Three different bore sizes are available, with diameters 1.36 mm (small), 1.96 mm (medium), and 3.99 mm (large). This was measured with an accuracy of ±0.03 mm by inserting drill bits of varying size.

In this experiment you determine the flow rate as a function of \( L \) for all three different sizes of capillaries. This is to be done at room temperature. Deduce the viscosity of water at this temperature and check for consistency (different pipes should yield the same \( \eta \)) and reproducibility (repeating a measurement should give the same result).

Then decide on a capillary size and repeat the experiment at 10°C and at 50°C to obtain the temperature dependence of \( \eta \). Discuss the uncertainties of the measurement and compare with the accepted values (fig.1).
Fig. 1: Viscosity of water as a function of the temperature. The data are from the CRC Handbook of Chemistry and Physics, 55th edition, p. F-49