7.5

\[ K_F = \frac{1}{2} m_F (v_F^i)^2 = \frac{1}{2} m_S (v_S^i)^2 = \frac{1}{2} m_S (v_S^i)^2 = \frac{1}{2} m_F (v_F^i)^2 \]

(2)

\[ K_F = \frac{1}{2} m_F (v_F^i + 1)^2 = \frac{1}{2} m_S (v_S^i)^2 = \frac{1}{2} m_F (v_S^i)^2 \]

\[ \Rightarrow \quad \sqrt{v_S^i} = 2 \sqrt{v_F^i} \quad \text{and} \quad (v_F^i + 1)^2 = \frac{1}{2} (v_S^i)^2 - 2(v_F^i)^2 \]

\[ (v_F^i)^2 + 2v_F^i - 1 = 0 \quad \Rightarrow \quad v_F^i = (1 + \sqrt{2}) \text{m/s} \]

\[ = 2.4 \text{ m/s} \]

The initial speeds of the father and son are 2.4 m/s and twice that:

\[ v_F^i = 2.4 \text{ m/s} \quad \text{and} \quad v_S^i = 4.8 \text{ m/s} \]

7-18a

\[ W_f = F \cdot d \]

\[ = 360 \text{ kN} \cdot 0.10 \text{m} \]

\[ W_f = 36 \text{ kJ} \]

\[ \text{as positive} \]

b) \[ W_M = 4000 \text{ N} \cdot 0.05 \text{m} \]

\[ = 200 \text{ J} \]

\[ W_M = 200 \text{ J} \]

\[ \text{also positive} \]
\[ mg = 0.25 \text{ kg} \cdot 9.8 \text{ m/s}^2 = 2.45 \text{ N} \]

\[ a = \frac{3.0 \text{ N} - 2.45 \text{ N}}{0.25 \text{ kg}} = +2.2 \text{ m/s}^2 \]

The elevator cab is accelerating upward at a rate of +2.2 m/s²

\[ T - 900 \text{ kg} \cdot 9.8 \text{ m/s}^2 - 0.25 \text{ kg} \cdot 9.8 \text{ m/s}^2 = 900 \text{ kg} \cdot 2.2 \text{ m/s} \]

\[ \Rightarrow T = 900 \text{ kg} \left( 9.8 \text{ m/s}^2 + 2.2 \text{ m/s}^2 \right) + 0.25 \text{ kg} \left( 9.8 \text{ m/s}^2 + 2.2 \text{ m/s}^2 \right) \]

\[ = 10.8 \text{ kN} \]

\[ \Rightarrow W_a = 10.8 \text{ kN} \cdot 2.40 \text{ m} = 25.9 \text{ kJ} \]

b) \[ W_b = 92.61 \text{ kJ} = T_b \cdot 10.5 \text{ m} \Rightarrow T_b = 8820 \text{ N} \]

Looking at the FBD for the cab, we find

\[ 8820 \text{ N} - 900 \text{ kg} \cdot 9.8 \text{ m/s}^2 = 900 \text{ kg} \cdot \alpha_b \]

\[ \Rightarrow \alpha_b = 0.0 \text{ m/s}^2 \]

\[ \Rightarrow N_c - m_c g = m_c \cdot 0 \text{ m/s}^2 \Rightarrow N_c = m_c g \]

\[ = 0.25 \text{ kg} \cdot 9.8 \text{ m/s}^2 \]

\[ N_c = 2.45 \text{ N} \]
\[ F_x = -k (x - x_0) \Rightarrow |F_x| = 80 N \text{ for } |x - x_0| = 2.0 \text{ cm} \]

(assume the box is 2.0 cm, convert to m)

Let's have \( x_0 = 0 \), then from the first condition we have:

\[ 80 N = -k (-0.020 \text{ m}) \Rightarrow k = 4000 \text{ N/m} \]

We are also told that \( U(x) - U(2 \text{ cm}) = +4.0 \text{ J} \)

\[ \frac{1}{2} k x^2 - \frac{1}{2} k (0.02 \text{ m})^2 = +4.0 \text{ J} \]

\[ x^2 = \frac{+8.0 \text{ J}}{4000 \text{ N/m}} \cdot (0.02 \text{ m})^2 \]

\[ x = \pm 4.90 \text{ cm} \]

The difference between these two is that the final position is either at the same side of the equilibrium position or the opposite side.
a) If the rope does 900 J of work pulling the skier up at 2.0 m/s, it does exactly the same amount of work while pulling the skier up at 2.0 m/s.

\[ 900 \text{ J} \]

b) The rate at which the work is done for \( U = 1.0 \text{ m/s} \) can be found by realizing that the time to travel 8.0 m up the slope is \( 8.0 \text{ m} / (1.0 \text{ m/s}) = 8.0 \text{ sec} \)

\[ P_1 = \frac{900 \text{ J}}{8.0 \text{ sec}} = 112.5 \text{ W} = 110 \text{ W} \]

c) At \( U = 2.0 \text{ m/s} \), the time to travel 8.0 m would be \( (8.0 \text{ sec} ) \) itself.

\[ P_2 = \frac{900 \text{ J}}{4.0 \text{ sec}} = 225 \text{ W} = 220 \text{ W} \]
\[ M = 0.250 \text{ kg} \]
\[ \Delta y = 0.12 \text{ m} \] (compression distance) 
\[ k = 250 \text{ N/m} \] (for the spring)

a) \[ F_y = mg = 0.25 \text{ kg} \cdot 9.8 \text{ m/s}^2 = 2.45 \text{ N} \]
\[ W_y = F_y \Delta y \]
\[ = 2.45 \text{ N} \cdot 0.12 \text{ m} \]
\[ = 0.294 \text{ J} \]

\[ W_y = 0.294 \text{ J} \]

b) \[ W_{sp} = -\frac{1}{2} k (\Delta y)^2 \] (-since spring force is upwards)
\[ = -\frac{1}{2} \cdot 250 \text{ N/m} \cdot (0.12 \text{ m})^2 \]
\[ = -1.8 \text{ J} \]

\[ W_{sp} = -1.8 \text{ J} \]

c) The work performed by the combined forces of the spring and gravity must equal the change in the block's kinetic energy (which goes from \( V_1 \) to 0 as the spring is compressed)

\[ 0 = 1.8 \text{ J} + 0.294 \text{ J} = 0 - \frac{1}{2} m v_0^2 \]

\[ \Rightarrow \]
\[ v_0 = \sqrt{\frac{2(1.8 \text{ J} - 0.294 \text{ J})}{m}} \]

\[ v_0 = 3.47 \text{ m/s} = 3.5 \text{ m/s} \]

d) \[ v_1 \text{ in general becomes} \]
\[ \frac{1}{2} m v_0^2 = -mgd + \frac{1}{2} kd^2 \]

where \( d \) is the spring compression distance. We can then relate \( d \) to \( v_0 \) through the quadratic formula:

\[ d = \frac{mg \pm \sqrt{(mg)^2 + 4 \cdot \frac{1}{2} k \cdot \frac{1}{2} m v_0^2}}{k} \]

We are only interested in the + sign in the
above because a negative value for d would correspond to the spring being stretched, not compressed.

\[ d = \frac{0.25 \text{ kg} \cdot 9.8 \text{ m/s}^2 + \sqrt{(0.15 \text{ kg} \cdot 9.8 \text{ m/s}^2)^2 + 250 \text{ N/m} \cdot 0.15 \text{ kg} \cdot (6.94 \text{ m/s})^2}}{250 \text{ N/m}} \]

\[ = 0.229 \text{ m} \]

\[ d_{2V_0} = 23 \text{ cm} \]