16b) Transverse Wave \( y(x,t) = 6.0 \sin(0.02\pi x + 4\pi t) \)

a) Amplitude \( A = 6.0 \text{ cm} \)

b) The standard form tells us that \( k = 0.02\pi \text{ cm}^{-1} \)
\[
\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.02\pi \text{ cm}^{-1}} = 100 \text{ cm} = 1 \]

c) Again, from comparison to the standard form
\[
\omega = 4\pi \text{ s}^{-1} \Rightarrow f = \frac{\omega}{2\pi} = 2 \text{ Hz}
\]

d) \( v = \omega/k = \frac{4\pi \text{ s}^{-1}}{0.02\pi \text{ cm}^{-1}} = 200 \text{ cm/s} \)

e) The wave is progressing to the right, negative \( x \).
As \( t \) increases, \( x \) must decrease or go more negative to keep the values off the phase angle constant.

\( V < 0 \)

f) Maximum transverse speed is \( 2 \omega A \) off the medium. Thus \( V_{f,max} = 2\omega A \), since \( V_f = -\omega A \sin(\lambda x + \phi) \)
\[
V_{f,max} = 4\pi \text{ cm/s} \cdot 6.0 \text{ cm} = 75 \text{ cm/s}
\]

g) \( y(3.5 \text{ cm}, 0.26 \text{ s}) = 6.0 \text{ cm} \cdot \sin(0.719\pi \text{ rad} + 3.167 \text{ rad}) \)
\( = -2.0 \text{ cm} \)
\[ V_w = \sqrt{\frac{\rho}{\mu}} \quad \text{where } \mu \text{ is frictional shear stress, } \rho \text{ is linear mass density.} \]

Assuming that \( \rho \) does not change appreciably when \( \mu \) is increased, then

\[ V_1 = \sqrt{\frac{\rho}{\mu}} \quad V_2 = \sqrt{\frac{\rho}{\mu}} \]

\[ \Rightarrow \frac{V_2}{V_1} = \sqrt{\frac{\rho}{\mu}} \quad = \frac{180 \text{ m/s}}{170 \text{ m/s}} \]

\[ \Rightarrow \mu_2 = 120 N \cdot \left( \frac{180 \text{ m/s}}{170 \text{ m/s}} \right)^2 \]

\[ \mu_2 = 135 N \]
We use the general form for a harmonic wave:

\[ y = y_m \sin(\kappa x - \omega t + \phi_0) = 5.0 \text{ cm} \sin(1.0 - 4.0 \text{ cm}^{-1} t) \]

a) From looking at the above we see immediately

\[ \omega = 4.0 \text{ rad s}^{-1} \quad \Rightarrow \quad f = \frac{\omega}{2\pi} = 0.637 \text{ Hz} \]

\[ f \approx 0.64 \text{ Hz} \]

b) \( f \tau = u \omega = 40 \text{ cm/s} \Rightarrow \tau = \frac{40 \text{ cm/s}}{0.637 \text{ Hz}} = 62.8 \text{ cm} \]

\[ \tau = 63 \text{ cm} \]

c) \[ y_m = 5.0 \text{ cm} \]

d) \[ \kappa = \frac{u}{\omega} = \frac{62.8 \text{ cm}}{0.1 \text{ cm}^{-1}} = 0.1 \text{ cm}^{-1} = 10 \text{ rad/m} \]

Note we also have \( kx + \phi_0 = 1.0 \text{ rad} \), but we could determine \( k \) from this only if we knew that \( \phi_0 = 0 \). In this case it appears that it is.

e) \( u = 4.0 \text{ m/s} \) as discussed in a.

f) The sign is -. Since the problem says nothing about the direction of propagation, we must assume the sign given is the one they want.

g) \[ 40 \text{ cm/s} = 0.40 \text{ m/s} = \sqrt{\frac{k}{\mu}} = \sqrt{\frac{0.4 \text{ kg/m}}{1}} \]

\[ \Rightarrow \quad T = 6.4 \times 10^{-2} \text{ N} \]

Note the numbers for \( T \) and \( \mu \) in this problem are not very realistic, but they are what were given!
16-24) We are told (or could figure out from the relevant force body diagram) that \( T = \frac{1}{2} Mg = 2.45 \text{ N} \)

\( a) \quad V_1 = \sqrt{\frac{T}{m_1}} = \sqrt{\frac{2.45 \text{ N}}{0.003 \text{ kg/m}}} = 28.6 \text{ m/s} \)

\( b) \quad V_2 = \sqrt{\frac{T}{m_2}} = \sqrt{\frac{2.45 \text{ N}}{0.005 \text{ kg/m}}} = 22.1 \text{ m/s} \)

\( c) \quad \text{For this case we have } T_1 + T_2 = 4.90 \text{ N, but they no longer need be equal. However, we want the wave speeds to be equal, i.e.:} \)

\[ \sqrt{\frac{T}{0.003 \text{ kg/m}}} = \sqrt{\frac{T}{0.005 \text{ kg/m}}} \]

\[ T_1 = \frac{3}{5} T_2 \]

\[ \frac{3}{5} T_2 + T_2 = 4.90 \text{ N} \]

\[ T_2 = 4.90 \text{ N} \cdot \frac{5}{8} \]

\[ T_1 = \frac{3}{8} \cdot 4.90 \text{ N} \]

\( d) \quad T_2 = 3.062 \text{ N} \rightarrow m_2 = 0.312 \text{ kg} \)

\( e) \quad T_1 = 1.838 \text{ N} \rightarrow m_1 = 0.188 \text{ kg} \)
According to 16.37, the rate at which energy is transferred is

\[ P = \frac{1}{2} \mu v W^2 y_m^2 \]

\[ v = \sqrt{\frac{1200 N}{0.002 \text{ kg/m}}} = 774.6 \text{ m/s} \]

\[ y_m = 0.003 \text{ m} \]

\[ P = \frac{1}{2} \cdot (0.002 \text{ kg/m}) \cdot (774.6 \text{ m/s})^2 \cdot (0.003 \text{ m})^2 \]

\[ = 10.0 \text{ W} \]

b) Two sinusoidal waves, propagating independently, would carry energy at a rate of twice that of a

\[ P_2 = 2P_1 = 20.0 \text{ W} \]

c) \( \phi = 0 \Rightarrow \text{perfect constructive interference,} \; \mu \text{ and wall stay the same, but} \; y_m \text{ decreases} \)

\[ P_3 = 40 \text{ W} \]

d) \( \phi = 0, 0.4 \pi \text{ rad} \)

\[ y = 2y_m \cos(\phi) \left[ \sin(kx - \omega t + \phi) \right] / A \]

The destructive interference is when \( 4.85 \text{ mm} = 1.618 \cdot y_m \)

\[ \Rightarrow \] \( P_4 = 2.62 \cdot P_1 = 26.2 \text{ W} \)

e) Now interference is perfectly destructive

\[ \Rightarrow A = 0 \quad P = 0 \]
L = 1.20 m so the wavelength of any standing wave must be an integer fraction of 2L:

\[
\begin{align*}
&\frac{2L}{1} = \lambda = 2L \quad \frac{2L}{2} = \lambda = L \\
&\quad \lambda = \frac{3}{2} L \text{ etc}
\end{align*}
\]

a) For the case \( \lambda = \frac{L}{2} \Rightarrow \omega = 120 \text{ Hz} \cdot \frac{1.20 \text{ m}}{2} = 72.0 \text{ m/s} \)

\[
\nu = \sqrt{\frac{\omega}{\mu}} \Rightarrow T = (720 \text{ m/s})^2 \cdot 0.0016 \text{ kg/m} \]

\[
T = 8.274 \Rightarrow m = 0.846 \text{ kg}
\]

\[
\sin \theta = m g
\]

b) \( m = 1.0 \text{ kg} \Rightarrow T = 9.80 \text{ N} \Rightarrow \omega = \sqrt{\frac{9.80 \text{ N}}{0.0016 \text{ kg/m}}} = 7812 \text{ m/s} \)

\[
7812 \text{ m/s} = f \lambda \Rightarrow \lambda = \frac{7812 \text{ m/s}}{120 \text{ Hz}} = 0.652 \text{ m}
\]

This wavelength is not an integer fraction of 2L so no wave cannot set up a standing wave at this frequency set up with this weight.

\[
\frac{0.652 \text{ m}}{1.20 \text{ m}} = 0.54
\]