$t = 0 \quad U_s(t = 0) = U_{\text{max}} \cdot 0.8
\quad = -A \omega \sin (\omega t + \phi) \bigg|_{t = 0}$

$\Rightarrow \sin \phi = -0.8$

$\Rightarrow \phi = -53.1^\circ \quad \text{or} \quad -0.927 \text{rad}$

$\quad \phi = 233^\circ \quad \text{or} \quad 4.07 \text{rad}$

(\text{or } -127^\circ)$

How do we distinguish between these cases?

The velocity is getting smaller as time moves forward, therefore only the 1st pair of answers will work. If $\phi = 127^\circ$ then at later times $v$ would increase, not decrease!
\[ X_L = 1.0 \text{cm} \cos(\omega t + \pi/2) \Rightarrow V_x = -W \cdot 1.0 \text{cm} \sin(\omega t + \pi/2) \]
\[ \omega = \frac{2\pi}{0.02 \text{s}} = 314.2 \text{ rad/s} \Rightarrow V_{Lx} = -3.14 \text{ m/s} \cdot \sin(\pi) = 0 \]

The question suggests that we can neglect the spring during the collision, so momentum will be conserved at \( V_x = 0 \) initially. Since the two blocks stick together,

\[ m_1 V_1 = (m_1 + m_2) V_f \]
\[ 4 \text{ kg} \cdot 6.0 \text{ m/s} = (4 + 2) \text{ kg} \cdot V_f \]
\( \Rightarrow \]
\[ V_f = \frac{4 \text{ kg}}{(4 + 2) \text{ kg}} \cdot 6.0 \text{ m/s} = 4.0 \text{ m/s} \]

\( \Rightarrow \) After the collision, the system has potential energy

\[ \omega^2 = \frac{k}{m} \Rightarrow k = 2.0 \text{ kg} \cdot (314.2 \text{ rad/s})^2 = 197.4 \text{ kN/m} \]

\[ U_{el} = \frac{1}{2} \cdot 197.4 (0.01 \text{ m})^2 \times 10^3 \]
\[ H = \frac{1}{2} m V_2^2 = \frac{1}{2} (4 + 2) \text{ kg} \cdot (4 \text{ m/s})^2 = 48 \text{ J} \]

\( \Rightarrow \) \[ E_{tot} = 57.87 \text{ J} = \frac{1}{2} k A^2 \]

\[ A = 2.42 \text{ cm} \quad \Rightarrow A = 2.4 \text{ cm} \]
By the parallel axis theorem, we have

\[ I = mx^2 + \frac{1}{2} ml^2 \]  

(see Table 10-2 p 253)

\[ T = \frac{2\pi}{\sqrt{g}} \sqrt{x + \frac{1}{2} L^2} \]

we can find the extreme of \( T \) by setting its derivative with respect to \( x \) to zero

\[ \frac{dT}{dx} = \frac{\pi}{\sqrt{g}} \frac{y_1 (1 - y_1 L^2/x^2)}{\sqrt{x + y_1 L^2/x}} = 0 \]

\[ \Rightarrow \frac{1}{x} \frac{L^2}{x^2} = 1 \]

\[ \Rightarrow x = \sqrt{y_1} L = \frac{1.85m}{3.485} \]

\[ x = 0.53 \text{ m} \]

b) \[ T = \frac{2\pi}{\sqrt{g}} \left( 0.53 + \frac{1}{y_1} \left( \frac{1.85m}{0.53} \right)^2 \right)^{y_1} \]

\[ T = 2.1 \text{ s} \]
\[ x = A \cos(\omega t + \phi) \quad \phi = \frac{\pi}{5} \]

@ \( t = 0 \) \quad x = A \cos(\frac{\pi}{5}) = 0.8090 A

The total energy is proportional to \( A^2 \).

The potential energy is proportional to \( x^2 \). Hence,

\[ U(1) = \frac{1}{2} k (0.8090 A)^2 = 0.655 \cdot \frac{1}{2} k A^2 \]

\[ U(1) = 0.655 \cdot A \cdot E_{\text{tor}} \]

\( \Rightarrow 65.5\% \) of the energy is potential at this point.
\[ \mathbf{T} = F_r = (k \cdot r \theta) \mathbf{r} \]

Since \( r \theta \) is approximately equal to the displacement from equilibrium, we have:

\[ \ddot{r} = \frac{1}{r^2} \frac{d}{dt} \frac{\dot{r}}{r} = \frac{k r^2}{m r^3} \]

\[ \alpha = \frac{T}{I} = \frac{k r^2}{m r^2} \theta = \frac{d^2 \theta}{dt^2} \]

We see that the second derivative of \( \theta \) w.r.t. \( t \) is proportional to \( \theta \), so the motion is indeed simple harmonic. The proportionality constant to \( \omega^2 \) hence is:

\[ \omega = \sqrt{\frac{k}{m}} \frac{r^2}{R^2} \]

(Note: The dimensions clearly work out correctly.)

b) \( r = R \Rightarrow \omega = \sqrt{\frac{k}{m}} \)

c) \( r \to 0 \Rightarrow \omega \to 0 \) (i.e. There is no oscillation in this case since the restoring torque goes to zero).
a) Since $V = 0$ at this point, the spring is stretched to its maximum extent in the oscillation.

\[ F_{sp} = 100 \text{N/m} \times 0.3 \text{m} = 30 \text{ N} \quad \text{upward} \quad \text{and gravity is down} \]

\[ \Rightarrow F_{net} = 10 \text{ N up} \]

b) \( A \) is not simply 0.3 m, since with the weight attached, \( x = 0 \) will not be the equilibrium position. That will actually be when \( |F_{el}| = mg \) (i.e. \( F_{net} = 0 \))

\[ 20 \text{ N} = 100 \text{N/m} \times x_{eq} \]

\[ \Rightarrow x_{eq} = 0.2 \text{m} \]

\[ A = 0.1 \text{m} \]

c) \[ \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{100 \text{N/m}}{2.04 \text{kg}}} = 7.00 \text{ rad/s} \]

\[ \Rightarrow T = \frac{2\pi}{\omega} = 0.90 \text{ sec} \]

d) \[ KE_{max} = \frac{1}{2} m v_{max}^2 = \frac{1}{2} k A^2 \]

\[ KE_{max} = \frac{1}{2} \cdot 100 \text{N/m} \cdot (0.10 \text{m})^2 \]

\[ KE_{max} = 0.50 \text{J} \]