We have that at \( t = 0, x = 0 \) the wave produces a transverse velocity of -4.0 m/s and this is decreasing in magnitude.

\[
U(x,t) = -y_m \omega \cos(kx - \omega t + \phi_0)
\]

\[
U(0,0) = -y_m \omega \cos(\phi_0)
\]

The graph tells us that \( y_m \omega = 1.25u_s \)

\[
u = \frac{-40 \text{ m/s}}{5.0 \text{ m/s}} = 0.8
\]

\[
\phi_0 = \pm 0.6435 \text{ rad}
\]

Hence we find that \( \cos(\phi_0) = \frac{-40 \text{ m/s}}{5.0 \text{ m/s}} = 0.8 \)

\[
\alpha(x,t) = -y_m \omega^2 \sin(kx - \omega t + \phi_0)
\]

(Note, you get two minus signs, one from \( \sin(\phi_0) \) and another from \( \frac{\partial}{\partial t}(-\omega t) \), so the sign remains unchanged)

\[
\alpha(0,0) = -y_m \omega^2 \sin(\phi_0) > 0
\]

\[
\Rightarrow \phi_0 = -0.64 \text{ rad}
\]
\[ y(x, t) = 15.0 \text{ cm} \cos \left( \frac{\pi x}{15} - 15\pi t \right) \]

\[ v_y(x, t) = +15 \text{ rad/s} \cdot 15.0 \text{ cm} \sin \left( \frac{\pi x}{15} - 15\pi t \right) \]

a) \[ y = 12.0 \text{ cm} = 15.0 \text{ cm} \cos \left( \frac{\pi x}{15} - 15\pi t \right) \]

\[ \theta = \frac{\pi x - 15\pi t}{15.0 \text{ cm}} = 0.644 \text{ rad} \]

\[ v_y = 225 \text{ cm/s} \sin \left( 0.644 \text{ rad} \right) \]

\[ v = 4.24 \text{ m/s} \]
WE are told (or could figure out from the relevant free body diagram) that \( T = \frac{1}{2} M g = 2.45 N \)

\[
a) \quad v_1 = \sqrt{\frac{T}{m_1}} = \sqrt{\frac{2.45 N}{0.009 kg/m}} = 28.6 \text{ m/s}
\]
\[
b) \quad v_2 = \sqrt{\frac{T}{m_2}} = \sqrt{\frac{2.45 N}{0.005 kg/m}} = 22.1 \text{ m/s}
\]

\(\text{c) For this case we have } T_1 + T_2 = 4.90 N \text{ but they no longer need be equal.} \) Hence we want the wave speeds to be equal, \( v_1 \), \( v_2 \)

\[
\sqrt{\frac{T_1}{0.003 kg/m}} = \sqrt{\frac{T_2}{0.005 kg/m}}
\]

\[
\Rightarrow T_1 = \frac{3}{5} T_2
\]

\[
\frac{3}{5} T_2 + T_2 = 4.90 N \quad T_2 = 4.90 N \cdot \frac{5}{8}
\]

\[
T_1 = \frac{3}{5} \cdot 4.90 N
\]

\[d) \quad T_2 = 3.062 N \Rightarrow m_2 = 0.312 \text{ kg}
\]
\[e) \quad T_1 = 1.838 N \Rightarrow m_1 = 0.188 \text{ kg}
\]
Standing wave (x + t do not appear together in the argument of the sine/cosine)

\[ y(x,t) = (0.10 \text{m}) \sin \left( \frac{\pi}{3} x \right) \sin(12\pi t) \]

a) Fixed at both ends and second harmonic
   \( \Rightarrow \text{One internal node} \)
   \[ L = \lambda \]
   but we have \( k = \frac{\pi}{2} \Rightarrow L = \frac{2\pi}{\frac{\pi}{2}} = 4.0 \text{ m} \]

b) From the given \( y(x,t) \) we have \( \omega = 12\pi \text{ rad/s} \)
   \( v_w = \frac{\omega}{k} = \frac{12\pi \text{ rad/s}}{\frac{\pi}{2} \text{ rad/m}} = 24 \text{ m/s} = v_w \)

c) We know the tension is \( T = 200 \text{ N} \) hence
   \[ M = \frac{T}{v_w^2} = \frac{200 \text{ N}}{(24 \text{ m/s})^2} = 0.3472 \text{ kg/m} \]
   \[ M = \mu L = 0.3472 \text{ kg/m} \times 4.0 \text{ m} = 1.4 \text{ kg} = M \]

d) \( v_w \) would remain the same, but the wavelength would change noticeably \( \frac{\lambda}{3} = \lambda_3 \)
   \[ f = \frac{v_w}{\lambda_3} = \frac{24 \text{ m/s}}{\frac{\lambda}{3} \times 4.0 \text{ m/s}} \]
   \[ f = 9.0 \text{ Hz} \]
   \[ \Rightarrow T = 0.112 \text{ s} \]
a) \( h(3) = h_y = 2 \) This point moves at a constant rate as time passes.

\[
3 = x - 5t = 0 \quad \Rightarrow \quad x = 5t + 3
\]

\( v = 5\text{cm/s} \)

b) \( x \) is moving toward positive \( x \).

c) \( 2.0\text{cm} \)

\[
\begin{array}{c|ccccc}
\hline
& 11 & 12 & 13 & 14 & x \\
\hline
\phi & 3 & 4 & & & \\
\hline
\end{array}
\]

\( x = 13\text{cm} \) \( \phi = 3 \)

\( x = 14\text{cm} \) \( \phi = 4 \)

\( t = 2.0\text{sec} \)

\( \phi = 3 \) \( \phi = 4 \)

\( \phi = 1 \) \( 1.8\text{sec} = t \)

\( \phi = 3 \) \( t = 1.4\text{sec} \)

\( \phi = 4 \) \( t = 1.2\text{sec} \)
fundamental mode ➞ No internal nodes

\[ f = 5.0 \text{ Hz} \Rightarrow \omega = 2 \pi f = 31.4 \text{ rad/s} \]

\[ y(x,t) = 2y_m \sin (kx) \cos (\omega t) \quad (16.60) \]

\[ u(x,t) = -2y_m \omega \sin (kx) \sin (\omega t) \]

at the midpoint \( kx = \frac{\pi}{2} \) (maximum amplitude)

\[ v(x,t) = -2y_m \omega \sin (\omega t) \]

1. \[ 2y_m \cdot 31.4 \text{ rad/s} = 5.0 \text{ m/s} \]
   (at zero displacement, the velocity has its greatest magnitude)

   \[ 2y_m = 0.16 \text{ m} = 0.16 \text{ m} \]

   Note: The amplitude of the standing wave is \( 2y_m \), \( y_m \) is the amplitude of one of the two traveling waves. Thus, the standing wave can be considered to be composed of.

2. \[ \mu = 1.2 \text{ kg/m} \cdot 3.0 \text{ m} = 0.6 \text{ kg/m} \]
   \[ \nu_w = \frac{4.0 \text{ m}}{2 \times 10^{-2} \text{ s}} = 20 \text{ m/s} \]

   \[ \tau = \mu \nu_w^2 = 240 \text{ N} \]

3. \[ v(x,t) = 0.16 \text{ m} \cdot \sin \left( \frac{\pi}{2} \text{ m} x \right) \cos (10\pi t) \]