8A. ANALYSIS OF COMPLEX SOUNDS

Amplitude, loudness, and decibels

Last week we found that we could synthesize complex sounds with a particular frequency, $f$, by adding together sine waves from the harmonic series with a fundamental equal to the frequency ($f, 2f, 3f, 4f,...$). We can reverse the process: a complex sound with particular frequency can be analyzed and quantified by the amplitude spectrum: the relative amplitudes of the harmonics.

Our goal today is to understand how the pitch, loudness, and timbre of a sound are represented in a spectrum. However, rather than use an analog Band Filter, we will use the PRAAT program, which has powerful analysis software built in to calculate and graph the spectrum of a recorded sound.

For the last two weeks we have used the amplitudes to represent the different waves. We can represent these in a table (the choice for the amplitude of the fundamental wave to be 2V is completely arbitrary):

**Table 1: The spectrum of a square wave**

<table>
<thead>
<tr>
<th>Harmonic number=fn/f1</th>
<th>Harmonic frequency (Hz)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplitude (Volt)</td>
<td></td>
<td>2.000</td>
<td>0.000</td>
<td>0.667</td>
<td>0.000</td>
<td>0.400</td>
<td>0.000</td>
<td>0.286</td>
<td>0.000</td>
<td>0.222</td>
</tr>
<tr>
<td>Relative amplitude=An/A1</td>
<td></td>
<td>1.000</td>
<td>0.000</td>
<td>0.333</td>
<td>0.000</td>
<td>0.200</td>
<td>0.000</td>
<td>0.143</td>
<td>0.000</td>
<td>0.111</td>
</tr>
<tr>
<td>Relative Power=(An/A1)^2</td>
<td></td>
<td>1.0000</td>
<td>0.0000</td>
<td>0.1111</td>
<td>0.0000</td>
<td>0.0400</td>
<td>0.0000</td>
<td>0.0204</td>
<td>0.0000</td>
<td>0.0123</td>
</tr>
<tr>
<td>Relative Power (dB)</td>
<td></td>
<td>0.0</td>
<td>Error</td>
<td>-9.5</td>
<td>Error</td>
<td>-14.0</td>
<td>Error</td>
<td>-16.9</td>
<td>Error</td>
<td>-19.1</td>
</tr>
</tbody>
</table>

Another way to represent the relative amplitudes, which is used by PRAAT, is to graph the power for each harmonic. Since the sound power is proportional to the square of the amplitude for each harmonic, these numbers often get quite small for the higher harmonics. For this reason, the power is often expressed as a decibel. The power for each harmonic in **decibels** (dB) is:

$$\text{Relative Power (dB)} = P_H = 10 \log_{10} ([A_H/A_1]^{2}) = 20 \log_{10} ([A_H/A_1])$$

Since the logarithm of zero is negative infinity, the Relative Power (dB) for the even harmonics is "Error".
We can display the **harmonic power spectrum** as a bar graph:

![Harmonic power spectrum graph]

Note that the bars for the even harmonics are missing since these harmonic are "missing" from a square wave.

### Analysis of a Triangle Wave

#### A. Predictions based on known relative amplitudes

Try to make a table similar to Table 1 for the **triangle wave** you synthesized several weeks ago. Assume the amplitude of the fundamental wave to be 2V. Remember that the even harmonics had zero amplitude and the amplitudes for the odd harmonics were given by: $A_n = A_1/n^2$.

**Table 2: The spectrum of a 440 Hz triangle wave:**

<table>
<thead>
<tr>
<th>Harmonic</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>frequency (Hz)</td>
<td>440</td>
<td>880</td>
<td>1320</td>
<td>1760</td>
<td>2200</td>
<td>2640</td>
<td>3080</td>
<td>3520</td>
<td>3960</td>
</tr>
<tr>
<td>Harmonic number fn/f1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>Amplitude (Volt)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative amplitude = An/A1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative Power = (An/A1)^2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative Power (dB)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Now graph your results for the relative power in dB for each harmonic. Note that the fundamental is 0 dB by definition:

![Graph showing relative power in dB for each harmonic](image_url)

B. "Warm up": Analysis of a measured sine wave

Before using **PRAAT** to analyze a more complex wave, first analyze a simple sine wave to better understand frequency spectra.

Take the output from the frequency synthesizer ground (black socket) to the headphone box ground input (also black socket). Use another banana plug cable from the top red 8 Ω output socket of the synthesizer to the other input on the headphone box.

Use the synthesizer to generate a sine wave (use oscillator 1-Left, change the function using the switches at the top left corner of the synthesizer box; make sure to switch all other oscillators out).

1. Send the signal from the headphone box to the microphone input of the computer using the cable supplied. When you start the program, two windows should have appeared: a **Praat objects window** and a **Praat picture window**. Recall how you record a sound with **PRAAT**: choose **Record mono Sound...** from the **New menu** in the Praat objects window. A **SoundRecorder** window will appear on your screen.

2. Use the **Record** and **Stop** buttons to record a few seconds of your sine wave signal. You can repeat this several times until you are satisfied with your levels. Adjust the gain of the synthesizer output so that the level meter of the SoundRecorder window registers and is "green", or occasionally "yellow". Type in a name for your sound file (e.g., "sine") in the text box below the **Save to list:**
button. hit the **Save to list** button and the text string "sound sine" should appear in the **Praat objects window** that indicates the file where your sound is recorded.

3. With the name of the file highlighted in the Praat objects window, hit the **Edit** button to see the waveform that you just recorded. To simplify the display (if it is not already done), go across the menu of the Edit window (that should be titled "Sound sine"),

   - **Spectrum** menu, pull down, **Show spectrogram** deselect (i.e., if a check mark is shown, select it. If no check mark, exit without doing anything).
   
   - **Pitch** menu, pull down, deselect **Show pitch**
   
   - **Intensity** menu, pull down, deselect **Show intensity**
   
   - **Formants** menu, pull down, deselect **Show formants**
   
   - **Pulses** menu, pull down, deselect **Show pulses**.

4. Set up by going to the **Spectrum** menu, going to **Advanced Spectrum settings**..., and in **Pre-Emphasis** (dB/out) type "0.0" if it is "6.0". This latter option will be used when we analyze speech in the next lab.

5. Check that you have a clean sine wave by using the **View menu** and combinations of **Zoom in** (<CTRL>-i), **Zoom out** (<CTRL>-o), or selecting and **Zoom to selection**.

6. Now generate a frequency spectrum of this input waveform. From the **Spectrum menu**, choose **View spectral slice**. A **Spectrum Slice window** should appear as shown below.
This frequency spectrum shows frequency in Hz along the horizontal axis and power of that frequency component on the vertical axis.

Use the cursor to click at the tip of the first frequency peak (the fundamental). The frequency of the peak is given as a red number at the top of the display (in this case, 442 Hz, very close to the 440 Hz sine wave expected from the synthesizer). Its power level is given as a number at the left hand side. Note that the frequency spectrum of a pure sine wave is dominated by the power of the frequency of the sine wave. The other harmonics simply show that this is not a perfect sine wave; an ideal one would have only the first peak.

C. Analysis of a measured triangle wave

Now use the synthesizer to generate a triangle wave (use oscillator 1-Left, change the function using the switches at the top left corner of the synthesizer box; switch other oscillators out).

a) Follow the same procedure as with the sine wave to record the wave with PRAAT.
In the window obtained from using Edit, use the **Spectrogram menu, View spectral slice**, to generate a power spectrum of the triangle wave. It should appear similar to the example below.

Frequency Spectrum of a triangle wave.

b) Notice that the frequency power spectrum is made up of a series of peaks. Each peak represents one of the harmonics. You can use the mouse to click at the tip of each peak. The number above the display shows the frequency and the number to the left the power (in decibels) for that frequency. In the example above,
the frequency peak of 1329 Hz has a power of 31.5 dB. Record the frequency and power for each peak in the table below. To obtain the last line of the table, you need to subtract the power level of the fundamental ($P_1$) from each harmonic power level ($P_n$).

**Table 3: Measured power spectrum for a triangle wave**

<table>
<thead>
<tr>
<th>Harmonic</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>frequency (Hz) = $f_n$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Harmonic number = $f_n / f_1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Power (dB) = $P_n$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative Power (dB) = $P_n - P_1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c) How do your **relative** power numbers compare to what you calculated for Table 2?

d) Return to the initial **Sound xxx window** obtained from the **Edit** button. Zoom in to a few cycles of the triangle wave. Use the mouse click on a peak and an adjacent valley in order to measure the peak-to-peak amplitude of the triangle wave.

________________________________ Pascals.

e) Now reduce the amplitude of the oscillator; record the sound again using **PRAAT**. Measure the peak-to-peak amplitude of the sound signal:

________________________________ Pascals.

Record the power levels of all of the harmonics in the table below.
Table 4: Measured power spectrum for a triangle wave, new amplitude

<table>
<thead>
<tr>
<th>Harmonic</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>frequency (Hz) = fn</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Harmonic number = fn/f1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Power (dB) = Pn</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative Power (dB) = Pn - P1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

f) Compare your results to those from Table 3:

Do all of the power levels change?

By how much do they change?

Do they change by different amounts or the same amount?

Do the relative power levels change?

g) The change in the power levels for each harmonics present should be the same as the change in the overall power level, which you can calculate from your measurements of the new and old amplitudes.

\[
\text{Change in Power (dB)} = 10 \log_{10} \left( \frac{A_{\text{new}}}{A_{\text{old}}} \right)^2
\]

Are your measurements consistent with this observation?

C. Analysis of a superposition of a triangle wave and another harmonic, not present in the triangle wave
Now switch in oscillator 4 (summing it to the triangle wave). Turn its amplitude halfway up of the oscillator 1 level (make this adjustment by recording signal for oscillator 4 first, before summing the two signals. You will probably have to go through several cycles of recording, adjusting amplitude, recording, adjusting amplitude. etc.). Note that this harmonic was not present in the triangle wave signal.

a) Record the sound signal. How do the sound signal and the power spectrum change?

b) Change the phase of oscillator 4 by 180° and record the signal again. Does the sound change? Qualitatively, how do the sound signal and power spectrum change?

D. Analysis of a superposition of a triangle wave and another harmonic, already present in the triangle wave

Now switch out oscillator 4 and switch in oscillator 3. Adjust its amplitude (by recording the signal separately first) to be approximately half of that from oscillator 1-Left. Note that this harmonic was present in the original triangle wave.

a) Record the sound signal. Qualitatively, how do the sound signal and the power spectrum change?

b) Change the phase of oscillator 3. Does the sound change? Record the signal again. Qualitatively, how do the sound signal and power spectrum change?
E. Discussion of expected effects.

If the Power Spectrum of the sound changes, you can probably hear the difference. If the Power Spectrum does not change, you can probably not hear the difference. For example, you should have found that changing the phase of the fourth harmonic did not change the power spectrum, but changing the phase of the third harmonic did matter. Why? The third harmonic was present in the original sound. Phase may matter when you add sounds that are close together (or the same) in frequency.

Switch Oscillator 1-Left back to a sine wave.

a) Now switch in or out additional oscillators, one at a time, listen to the differences and record the signal after each switch. The sound should change every time, and new peaks should appear or peaks should disappear in the Power Spectrum. If you see no change, the amplitude for that oscillator is probably close to zero.

b) Changing the phases of any of the oscillators should not change the sound or the power spectrum. The lesson is that the shape of the power spectrum determines the timbre.

We can now give very precise ways to define how to measure sounds made up of a harmonic series using the Power Spectrum:

**Pitch:** the frequency of the fundamental of the harmonic series

**Timbre:** the relative heights of the peaks in the power spectrum (compared to the fundamental)

**Loudness:** the overall heights of the peaks in the power spectrum