3: PROPERTIES OF WAVES

Definition of Wave

A wave is a disturbance traveling in a medium.

A. SMALL GROUP ACTIVITIES WITH SLINKIES

Several basic properties of wave behavior can be demonstrated with long springs and slinkies. Quite a bit of work will be done in this class with springs and strings because you can see the waves on a spring, which you can’t do with sound waves. You study of sound will be easier for you if you can learn to make analogies to other kinds of waves which you can visualize.

1. Wave Propagation and Reflection

Have one person hold one end of the spring and another person hold the other end. Stretch the spring until it is 15 ft long. One person should send pulses down the spring by jerking the spring to the right and back to center. Do this very fast, to make the pulse as short as possible, but try not to overshoot when you bring your hand back to the center. The pulse should be only on one side:

This

Not This

Be sure that you understand why this is a wave which you are creating.

What is the medium in this case? What is the disturbance? Is the wave transverse or longitudinal?

All waves have the property that they can be reflected from boundaries. Waves are reflected different ways from different types of boundaries.

What happens to the pulses when they are reflected from the fixed end?

2. Speed of a Wave
Many important properties of sound depend on the speed at which it travels through the air, so it is important that you know what speed is and how to calculate it.

Speed is the distance something moves in one unit of time. The easiest way to measure speed is to measure the time it takes to travel a known distance and then divide the distance by the time. Speed = distance/time.

Have a third person use a stopwatch to measure the amount of time it takes for a pulse to go down and back once. It may take several tries to get a good value for the time. Calculate the speed of the wave. Remember, distance = 30 ft! (15 down, 15 back.)

How far does this wave travel in one second? How far would it travel in 15 seconds?

3. Wave Speed Depends on the Medium

Stretch the spring out to 20 ft., and once again send pulses down the spring. From casual observation, does the pulse travel faster or slower than before? Measure the speed with the watch.

4. Is there Reflection from a Free End?

Tie a long string (10 ft. or more) to the end of the spring.

Before you send pulses down the spring, predict what will happen to the pulses when they hit the string. Will they be reflected, or disappear, or what? (The spring/string junction is called a “free end” because the string allows the spring back and forth freely. Since this is a transverse wave, only the back-and-forth direction matters as far as the wave is concerned.)
Now try it. **What happens? Draw sketches** showing how this is the same or different from when the wave hit the fixed end.

5. **Longitudinal Waves on a Slinky**

Coil the long spring neatly in the box and get out the slinky. Most demonstrations work better with the slinky if you only use half of its length, so have the person who holds the fixed end keep half of the slinky all bunched up in his or her hands. Stretch the remaining half of the slinky until it is about 10 ft long.

The slinky is useful because you can use it to make both transverse and longitudinal waves. First, make a couple of transverse waves in the same way you did with the long spring. Now try making longitudinal waves (waves in which the disturbance is in the same direction as the direction of propagation): thrust your hand suddenly toward the person holding the fixed end and then back to its starting point. Do this as quickly as possible, but try not to get any side to side motion. Watch the **compression** pulse move down the slinky. This is analogous to a region of high pressure moving as a sound pulse in the air; sound waves are longitudinal.

Longitudinal wave are also called “compression-rarefaction waves.” Places where the medium is all bunched together are called regions of compression, whereas places where the medium is a spread out are called regions of rarefaction. In air this corresponds to regions of high and low pressure.

If you are careful, you can see a pure rarefaction pulse moving along the slinky. To do this, let out the entire slinky and stretch it to that it covers only 8 ft. Now suddenly jerk your hand away from the person holding the fixed end **but do not return it to its original position** until the wave has died out. If you push the slinky back and forth you have alternating regions of high and low pressure, like a continuous sound wave in the air.

**B. STANDING WAVES ON THE LONG SPRING**

Vibrating strings are important sound sources, particularly for musical instruments. Violins, guitars, pianos, and a whole host of instruments make sounds which begin with vibrating strings. The vocal cords are similar to vibrating strings. Furthermore, strings provide an exact visual analogy to air columns, which are used in many other instruments and which allow humans to control the timbre of the sounds that they make in speech.

The best way to “get a feel” for standing waves is to make some yourself on a long rope or spring.
1. Standing Waves on a Spring with Fixed Ends

With the spring still at 15 ft, oscillate your arm back and forth, sending regularly spaced pulses down the spring. Vary the frequency of oscillation until you set up a standing wave that looks like this:

Each hump is one half of a wavelength, so this spring now has one and a half wavelengths on it. Since the spring is 15 ft long, the wavelength of the standing wave is 10 ft. Wavelength is the length of two humps, and is symbolized by the lower case Greek letter \( \lambda \) (lambda).

a. With 3 humps on the spring, measure the amount of time required to make 20 complete oscillations. This is twenty periods. **What is one period of oscillation of this standing wave? What is the frequency of oscillation?**

b. Create a standing wave with one wavelength on the spring (two humps). **What is \( \lambda \)?** (It is not 2. If you think the wavelength is 2, ask for help.) Measure the amount of time required to make 20 complete oscillations. This is \( 20T \). What is the period of oscillation of this standing wave? Put this and the previous data into the appropriate spaces in the table below. Then complete the table by setting up standing waves with two complete wavelengths and with one half of a wavelength, measuring \( 20T \) for each one. Include units. Leave the last column empty.

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### Data from standing waves on a spring fixed at both ends

<table>
<thead>
<tr>
<th>looks like</th>
<th># humps</th>
<th>( \lambda )</th>
<th>( 20T )</th>
<th>( T )</th>
<th>( f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( f_1 ) =</td>
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<td>2</td>
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<tr>
<td>3</td>
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<td>10 ft</td>
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<td>4</td>
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</tbody>
</table>

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c. The frequencies should form a harmonic series. To find out if they do, divide each frequency by the fundamental frequency \( f_1 \). Label the last column \( \frac{f}{f_1} \) and fill it in. **Are the other frequencies two, three, and four times the frequency of the fundamental?**

Do not be discouraged if your results are not perfect. It is difficult to time the periods of the long spring accurately. Unfortunately, the fundamental is usually the least accurate of the measurements. If you find, for example, that the harmonics are 1.6, 2.5, and 3.4 times the fundamental, the problem is probably that the \( f_1 \) you calculated is too large. You might try assuming that the upper harmonics are correct and try to determine the expected fundamental.
d. Compare the periods to the travel time you measured previously: one of them should be close. This is no accident: the relationship between the speed, \( v \), wavelength, \( \lambda \), and period, \( T \), of a wave is:

\[
v = \frac{\lambda}{T}
\]

The relationship between the speed, the distance traveled, \( D \), and the time traveled, \( t \), for a wave is:

\[
v = \frac{D}{t}
\]

Therefore, the time it takes a wave to travel down the spring and back \((D = 2L)\) will be the same as the period of a wave that has a wavelength \( \lambda = 2L \).

2. **Standing Waves on a Spring with a Free End (optional)**

Different kinds of standing waves can be created on a spring on which one end is free to move back and forth. To study these types of standing waves, have one person hold the end of the string which is attached to the end of the spring. The spring itself should still be stretched to 15 ft; the string just serves as a way to hold the spring without restricting its side-to-side movement. If the spring is stretched to the same length as before, then the speed of waves on the spring will also be the same as before.

Set up a standing wave that looks like this:

![Standing Wave Diagram](image)

This is *one and a quarter wavelengths*. Each bulge in the spring is \((1/2)\lambda\), so the part left over at the end, half a bulge, is \((1/4)\lambda\). Since the spring is 15 ft long, the wavelength is \((15 \text{ ft})/1.25 = 12 \text{ ft}\). You should be able to check this by counting the number of floor tiles that are taken up by the full wavelength.

a. Vary the frequency to generate different patterns. Notice that every pattern you make has a quarter of a wavelength (half of a hump) at the end held by the string. The lowest frequency pattern should have only half a hump (one-quarter wavelength).

b. Measure the period of the four lowest frequency standing waves you can make, filling in the table below in the same way as you did for the fixed-end spring. Remember that the string is not part of the standing wave pattern, and should not be shown in the picture you put in the “looks like” column.
It is more difficult to determine \( \lambda \) for the free-end standing waves. The easiest way is to divide the length of the spring (15 ft) by the number of wavelengths on the spring. For this reason, an extra column for “number of wavelengths” has been added to the table.

<table>
<thead>
<tr>
<th>looks like</th>
<th># humps</th>
<th># wavelengths</th>
<th>( \lambda )</th>
<th>20T</th>
<th>T</th>
<th>f</th>
<th>( \text{f}/f_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>f_i =</td>
<td></td>
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</tr>
<tr>
<td>2 1/2</td>
<td>1 1/4</td>
<td>12 ft</td>
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</table>

c. How does the fundamental frequency compare to the fundamental for the fixed-end spring? Why?

d. Do the frequencies form a harmonic series?
C. THE STANDING WAVE APPARATUS

It is difficult to get accurate results with the spring and stopwatch. In contrast, very accurate results can be achieved using the apparatus provided in the lab room. A string is driven by a speaker which is controlled by the function generator. Measurements are made on the frequency counter.

1. Turn on the counter and function generator. Set the frequency to 90 Hz and turn the amplitude all the way up. Move the clear plastic “bridge” until you get a fundamental standing wave. Play with it until you get the largest possible amplitude. If the hanging mass which provides the tension is swinging around, it will make it hard to adjust, so you may want to stop the mass if it is swinging. Once you get the best standing wave, do not move the bridge until you see you should do so in the manual. Record the length of the string.

\[ L = \]

2. It is important to know the frequency of the fundamental as accurately as possible. Switch the GATE knob to the 10 s position, wait 20 seconds, and record the frequency (to the nearest 0.1 Hz) in the table below. Then switch the counter back to the 1 second gate for future measurements.

3. Turn the dial on the function generator to increase the frequency until you get the next standing wave, which should have two humps. Record the frequency in the table below.

4. Continue in this way until you get to the eighth harmonic. Then turn down the amplitude while you do the calculations in the next part.

<table>
<thead>
<tr>
<th># humps</th>
<th>f (Hz)</th>
<th>f/f_1</th>
<th>Harmonic number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
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<td>7</td>
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<td>8</td>
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</table>

Calculate f/f_1. You should find that these values are very close to being whole numbers. Round these values to whole numbers and put the result under “Harmonic Number.” It should be obvious that the number of humps is the number of the harmonic for a string fixed at both ends. Use this knowledge to predict the frequency which will be necessary to create the 11th harmonic. Then try it.


**D. Speed and Harmonics**

The speed of a wave on a string depends on the stiffness, density, and tension in the string. It does not depend on the wavelength of the wave sent down the string. In the same way, the speed of sound in air does not depend on the wavelength of the sound.

When standing waves are created on a string, the wavelength, frequency, and speed are related according to the formula \( v = \lambda f \).

In first part of the lab, you set up standing waves on the long spring. Copy your data from lab for the long spring into the table below. Then calculate \( c \) for each standing wave.

<table>
<thead>
<tr>
<th># humps</th>
<th>( \lambda )</th>
<th>( f )</th>
<th>( v )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
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<td>4</td>
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</table>

1. Is \( v \) roughly constant?

2. Are these \( v \)'s similar to the \( v \) which you measured directly by timing a pulse going down the spring (recall that this was the first thing you did with the spring)?

The data you took with the standing-wave apparatus should be much more accurate. Determine the velocity of waves on the string for the four harmonics indicated in the table.

<table>
<thead>
<tr>
<th># humps</th>
<th>( \lambda )</th>
<th>( f )</th>
<th>( v )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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</table>

Is the velocity constant?
E. LENGTH AND FREQUENCY

You have seen that a vibrating string can vibrate at a lowest frequency, called the fundamental, and also in an entire harmonic series of frequencies above the fundamental. The frequency of the fundamental can be increased either by making the string shorter or by making the speed higher. The speed can be increased either by increasing the tension in the string or by making the string lighter (for a given length).

Vocal chords are not really chords, but are more like flaps. Nonetheless, the factors of length, weight, and tension affect vocal chords and other vibrating objects in the same way that they affect strings. Thus, women’s voices are generally higher in pitch than men’s because they are both shorter and lighter. People can change the pitches of their voices by increasing the tension in the chords with muscles.

Speed, frequency, and wavelength are related by the equation

\[ v = \lambda f. \]

For a string of fixed length, only certain wavelengths will fit onto the string, resulting in a harmonic series. The least number of wavelengths which will fit onto a fixed string is one-half wavelength, so for this situation \( \lambda = 2L \). Therefore, for a string fixed at both ends,

\[ v = 2Lf_1, \]

where \( f_1 \) is the fundamental frequency. Since you have already studied the harmonic series in some detail, the standing waves referred to in this lab will always be fundamental standing waves.

1. Hang 500 g (50 g weight holder + 450 grams) from the end of the string. Put the plastic bridge at 90.0 cm (0.900 m).

2. Set the amplitude of the function generator to maximum. Find the frequency that gives the best fundamental standing wave. Best, in this case, means largest amplitude. Measure the frequency with the green frequency counter. Record your data in the first row of the data table on the next page.

3. Repeat for \( L = 45.0 \) cm, 30.0 cm, and 22.5 cm. Look at your data. Use the space here to describe the pattern which you see.
Frequency versus Length
for a string with a fixed tension

<table>
<thead>
<tr>
<th>$L$ (m)</th>
<th>$f$ (Hz)</th>
<th>prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.900</td>
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</table>

4. Use your data to predict what frequency will resonate for a string 50.0 cm long. Show your reasoning and calculations in the space below. Then determine the frequency experimentally and put it in the table above. Also put your prediction in the third column.

5. Repeat for a length of 72.0 cm.

5. Use you data to determine what length of string that will vibrate at 280 Hz, showing your work below. Then set the function generator a close as you can to 280 Hz, which will take a little patience. Then move the bridge until you get the best standing wave. Fill in the table.

6. How well do your predictions compare with the actual values?
7. In the last column of the table put the heading \( v = 2Lf \) (m/s). Then calculate the the speed of the wave for each row in the table. Is the speed constant?

8. You may have answered questions 3, 4, and 5 above using ratios. This is a good approach. However, if you have to answer several questions about the same spring, the table suggests a faster approach. Since the speed of the wave on the string is constant, the best approach is to use the first set of data to calculate the speed of the wave using \( v = 2Lf \). Then use that speed to calculate \( L \) or \( f \) for each new situation. Try it. Use the known speed of the string to calculate the resonant frequency for a length of 43 cm. Then test your result.

**F. TENSION AND FREQUENCY**

For a string of fixed length it is possible to raise the fundamental frequency by increasing the tension in the string. This is because increasing the tension increases the wave speed. On your apparatus, tension is provided by the weight of a hanging mass; by changing the mass you can change the tension. The tension in the string is the mass multiplied by the earth gravitational constant \( g = 9.8 \text{ m/s}^2 \). In your data table, however, it will be easier to see what is going on if you record the total hanging mass \( m \). You must always remember that the mass hanger already has a mass of 50 g.

1. Put the bridge at 90 cm. Hang 125 g on the string (50 g + 75 g). Find the best fundamental standing wave, which will be quite low for such a small tension. Record the frequency in the table below (next page).

2. Double the mass. That is, put on a mass of 250 g (50 g + 200 g). Find the new frequency. Record your data in the table. Does doubling the tension double the frequency?
3. Measure the frequency for masses of 500 g and 1000 g; put the data in the table.

**Frequency versus Tension**
for a string with a fixed length

<table>
<thead>
<tr>
<th>( m ) (g)</th>
<th>( f ) (Hz)</th>
<th>( v_T ) (m/s)</th>
<th>( v_f ) (m/s)</th>
</tr>
</thead>
<tbody>
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</table>

Theoretically, the speed of a wave on a string is given by

\[
v = \left( \frac{T}{\mu} \right)^{1/2}
\]

where \( T = (9.8 \text{ m/s}^2) \times m \) is the tension and \( \mu \) is the mass density of the string (measured in g/m).

4. You now have two independent ways to calculate the speed of the wave on the string: from the theoretical equation above, and from \( v = 2Lf \). These speeds should be the same.

Complete the table by calculating the speed both ways:

a. Use the equation with \( T \) and \( \mu \) to calculate \( v_T \), the subscript \( T \) standing for Tension. The density of the string has been measured with an accurate balance and determined to be 0.34 grams per meter. If you use mass in grams, \( g \) as 9.8 m/s\(^2\), and \( \mu \) in g/m, you will get the speed in meters per second (m/s).

b. Use the standard equation \( v_f = 2Lf \) to determine \( v_f \), where the subscript \( f \) stands for frequency.

Are the speeds the same? (Of course, experimental uncertainties in the measurement mean that they will never be exactly the same. However, they should be within a few percent of each other.)

5. Predict the frequency which will resonate for a mass of 1125 g. That is, calculate the speed for a mass of 1125 g using the known density. Then use the speed and the length of the string to calculate the resonant frequency. After you have made your prediction, try it to see if you were right. Put your results below.