

P310/510 Test 1
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Equations and Equivalencies:**Mechanics:**

$$PE = mgh, \quad g = 9.8 \text{ N/kg (Potential Energy)}$$

$$W = \mathbf{F} \cdot \mathbf{d} = Fd\cos(\theta) \quad (\text{Work})$$

$$KE = \frac{1}{2} mv^2 \quad (\text{Kinetic Energy})$$

$$P = W/t = E/t \quad (\text{Power})$$

$$I = P/A = E/(At) \quad (\text{Intensity})$$

$$U = kQ_1Q_2/r, \quad k = 9.0 \times 10^9 \text{ J m Coul}^{-2}$$

Thermodynamics:

$$K = ^\circ\text{C} + 273 \text{ (degrees absolute)}$$

$$dQ = dU + dW \text{ (First Law of Thermo)}$$

$$\Delta Q = c_v n \Delta T, \quad c_v = \text{specific heat at const volume}$$

$$\Delta Q = c_p n \Delta T, \quad c_p = \text{specific heat at const press.}$$

$$\Delta U = c_v n \Delta T \text{ and } \Delta U = 3/2 n R \Delta T \text{ (monatomic)}$$

$$\langle KE \rangle_{\text{molecule}} = 3/2 kT, \quad k = 1.38 \times 10^{-23} \text{ J/K}$$

$$1 \text{ Btu} = 1055 \text{ Joules}$$

$$1 \text{ Kcal} = 4186 \text{ Joules}$$

$$1 \text{ cal} = 4.186 \text{ Joules}$$

$$pV = nRT \quad (\text{Ideal Gas Law})$$

$$R = 8.31 \text{ J/mole/K} = 2 \text{ cal/mole/K}$$

$$R = c_p - c_v, \quad \gamma = c_p/c_v$$

$$pV^\gamma = \text{constant}, \quad TV^{\gamma-1} = \text{constant (adiabatic)}$$

$$\eta = \varepsilon = \text{efficiency} = \text{good stuff/what you pay}$$

$$\text{COP} = \text{coeff of perf} = \text{good stuff/what you pay}$$

$$dS = dQ/T, \quad dS > 0 \text{ (Second Law of Thermo)}$$

Conduction and Convection:

$$Q/t = kA(\Delta T)/(\Delta x) = A(\Delta T)/R$$

$$R = \Delta x/k$$

$$R_{\text{air}} = 1/h, \text{ where}$$

$$h_{\text{wall}} = 1.8(\Delta T)^{1/4} \text{ Wm}^{-2}\text{C}^{-1}$$

$$h_{\text{floor}} = 2.5(\Delta T)^{1/4} \text{ Wm}^{-2}\text{C}^{-1}$$

$$h_{\text{ceiling}} = 1.3(\Delta T)^{1/4} \text{ Wm}^{-2}\text{C}^{-1}$$

Resources and Production:

$$Q_T = \text{area under production curve}$$

$$dN/dt = kN \rightarrow N(t) = N(t_0) \exp(k(t-t_0))$$

$$T_2 = \ln(2)/k = (0.693)/k$$

$$\text{Hubbert: } N(t) = N_M \exp[-(t-T_M)^2/2\sigma^2],$$

$$\text{where } N_M = Q_T/(\sigma\sqrt{2\pi})$$

$$\text{and } \sigma \text{ is the standard deviation,}$$

$$\text{or, } N(t) = N_M \exp[-1/2 * z^2], \quad z = (t-T_M)/\sigma$$

Radiation:

$$\lambda f = c, \quad c = 3 \times 10^8 \text{ m/s} = \text{speed of light}$$

$$P = \sigma AT^4, \quad \sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4,$$

$$[\text{Stefan's Law for black bodies}]$$

$$I_0 = 1367 \text{ W/m}^2 \text{ (solar constant)}$$

$$\alpha = \text{albedo} = \text{fraction of light reflected}$$

$$\lambda_{\text{max}} T = 2.9 \times 10^{-3} \text{ m K [Wien's Law]}$$



I. (5 points) “Carbon neutral” fuels are ones that don’t add net carbon dioxide to the atmosphere when burned. The process of growing biomass (wood, switch grass, etc.) and burning it to generate electricity is viewed to be largely carbon neutral while burning coal is not. Please explain.

II.) The sun's power output is approximately 3.8×10^{26} watts.

a.) (10 points) What solar intensity, I_0 , do you expect at the Earth some 1.495×10^8 kilometers away from the sun?

b.) (5 points) The intensity of light from the sun peaks at a wavelength of 550 nanometers (550×10^{-9} m), corresponding to yellow light, falling off at both shorter and longer wavelengths. Please calculate the surface temperature of the sun.

III.) (10 points) In class Brabson carried out a demonstration of a diesel engine where air (diatomic gas with $c_v = 5/2R$, $c_p = 7/2R$) was rapidly compressed from a volume of about 15 cm^3 to a volume of about 1 cm^3 . Assuming that the air started at room temperature of 20°C (293 K), what was its temperature when compressed to 1 cm^3 ? You can safely assume that the compression was adiabatic. That is, the compression was rapid enough that essentially no heat escaped during the compression.

IV.) World oil production seems to be following the Hubbert model of resource consumption, with a total oil resource from its discovery to its exhaustion of $2,600 \times 10^9$ bbl, and a standard deviation, σ , of 30 years.

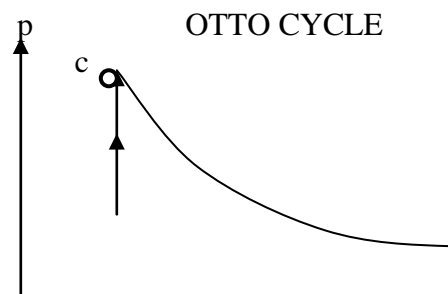
a.) (10 points.) Following this model, calculate the rate of oil production at the peak of the production curve.

b.) (10 points.) From the World Resources Institute 2000-2001 publication, we find that in 1997, the World oil production was 25.7×10^9 bbls/year. From this information calculate when the world will reach the maximum production rate calculated in part **a.**

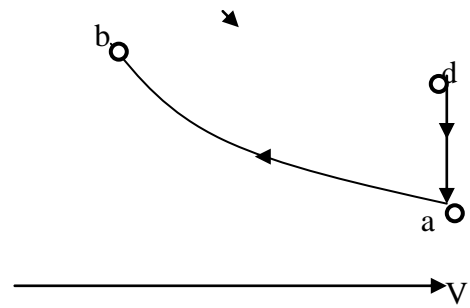
c.) (5 points) Make a sketch of the Hubbert model for world oil production, labeling the axes, putting a scale on each axis, and marking both the peak production and peak year calculated in parts a. and b. Also, mark the 1997 production on the sketch.

V. The p-V diagram of a real internal combustion engine may be approximated by the Otto Cycle, shown here. The Otto Cycle begins with an adiabatic transition, a \rightarrow b, a constant volume part, b \rightarrow c, a second adiabatic transition, c \rightarrow d, and a second constant volume part, d \rightarrow a. The working fluid of the engine is 0.10 moles of an ideal monatomic gas with a specific heat at constant volume, $c_v = 3/2R = 3/2(8.31 \text{ J/K/mole})$.

a.) (10 points) The temperatures at each point on the diagram are $T_a = 100 \text{ }^\circ\text{C}$, $T_b = 400 \text{ }^\circ\text{C}$, $T_c = 2100 \text{ }^\circ\text{C}$, and $T_d = 900 \text{ }^\circ\text{C}$. Using these temperatures, determine the work done when



going from a to b. Also, find the work done in going from c to d, and finally calculate the total work done by the engine in a full cycle.



b.) (5 points) Calculate the heat input during the b \rightarrow c part of the cycle.

c.) (5 points) Calculate the overall engine efficiency.

VI. (10 points) During the early stages of the development of nuclear power in the U.S., the production of electricity from nuclear power grew exponentially. The production rate in 1961 was 1.7×10^9 kWh/yr and by 1971 it had risen to 38.1×10^9 kWh/yr. Find k , the growth constant of the exponential, during this 10-year period.

VII. (10 points) In Israel a heat engine uses the highly salty Dead Sea in the generation of electricity. Solar energy warms the upper layers of the sea to 27°C , while the lower layers stay at 7°C . There is very little convection or mixing of the layers because of the large density difference between the really salty water layers below and the less salty layers above. Therefore, the surface water serves as the hot reservoir for the heat engine and the deep water as the cold reservoir. Calculate the maximum efficiency of such a heat engine.

VIII. (5 points) You are standing outside on a still day when the air temperature is 20°C (68°F). Please calculate the heat energy per second (in watts) transmitted through the dead air layer next to your skin by conduction while you are standing still. You may estimate your skin temperature to be 35°C (95°F) and the area of your body to be about 2.0 m^2 and a vertical surface. You may also ignore your clothes.

EXTRA CREDIT:

(3 points) Let's assume you consume 2500 Kcal of food energy each day, and that you neither gain nor lose weight. Your energy output comes both in the form of mechanical work done and in the form of heat energy. Adding these two outputs together, what is your average power output in watts?

(2 points) The input food energy per second calculated in the extra credit part above is considerably bigger than the heat loss per second calculated in problem VII. Can you explain this? You might consider other ways that your body loses heat besides conduction/convection to the air.