1) Jackson, problem 10.1.

(a) Show that for arbitrary initial polarization, the scattering cross section of a perfectly conducting sphere of radius \(a\), summed over outgoing polarizations, is given in the long-wavelength limit by

\[
\frac{d\sigma}{d\Omega}(\epsilon_0, n_0, n) = k^4 a^6 \left[ \frac{5}{4} - |\epsilon_0 \cdot n|^2 - \frac{1}{4} |n \cdot (n_0 \times \epsilon_0)|^2 - n_0 \cdot n \right]
\]

where \(n_0\) and \(n\) are the directions of the incident and scattered radiations, respectively, while \(\epsilon_0\) is the (perhaps complex) unit polarization vector of the incident radiation \((\epsilon_0^* \cdot \epsilon_0 = 1; n_0 \cdot \epsilon_0 = 0)\).

(b) If the incident radiation is linearly polarized, show that the cross section is

\[
\frac{d\sigma}{d\Omega}(\epsilon_0, n_0, n) = k^4 a^6 \left[ \frac{5}{8} (1 + \cos^2 \theta) - \cos \theta - \frac{3}{8} \sin^2 \theta \cos 2\phi \right]
\]

where \(n \cdot n_0 = \cos \theta\) and the azimuthal angle \(\phi\) is measured from the direction of the linear polarization.

(c) What is the ratio of scattered intensities at \(\theta = \pi/2, \phi = 0\) and \(\theta = \pi/2, \phi = \pi/2\)? Explain physically in terms of the induced multipoles and their radiation patterns.

2) Diffraction from a rectangular aperture: Show that the intensity pattern for a rectangular aperture, \(|x'| < a\) and \(|y'| < b\), has the form

\[
|\psi(x)|^2 = I_0 \left( \frac{\sin(q_x a)}{q_x a} \right)^2 \left( \frac{\sin(q_y b)}{q_y b} \right)^2
\]

where \(q = k(n_0 - n)\) is the momentum transfer and \(I_0\) is some constant.

3) Jackson, problem 11.3.

Show explicitly that two successive Lorentz transformations in the same direction are equivalent to a single Lorentz transformation with a velocity

\[
v = \frac{v_1 + v_2}{1 + (v_1 v_2/c^2)}
\]

This is an alternative way to derive the parallel-velocity addition law.