1) (a) Formulate the Green’s function appropriate for Dirichlet boundary conditions for the region between two infinite parallel planes at \( z = 0 \) and \( z = d \).

(b) Recall that the solution for the potential of a single plane at \( z = 0 \) with the potential specified to be \( \Phi = V \) inside a circle of radius \( a \) and zero elsewhere on the plane is

\[
\Phi = V \left( 1 - \frac{z}{\sqrt{a^2 + z^2}} \right),
\]

along the axis of the circle (cf. Jackson problem 2.7). Consider introducing the second plane held at zero potential at \( z = d \) with \( d >> a \). What is the leading order perturbation on the potential for points near the first plane, i.e. points with \( z << d \)?

2) Jackson, problem 2.12.

Starting with the series solution (2.71) for the two-dimensional potential problem with the potential specified on the surface of a cylinder of radius \( b \), evaluate the coefficients formally, substitute them into the series, and sum it to obtain the potential inside the cylinder in the form of Poisson’s integral:

\[
\Phi(\rho, \phi) = \frac{1}{2\pi} \int_0^{2\pi} \Phi(b, \phi') \frac{b^2 - \rho^2}{b^2 + \rho^2 - 2b\rho \cos(\phi' - \phi)} d\phi'
\]

What modification is necessary if the potential is desired in the region of space bounded by the cylinder and infinity?

3) Jackson, problem 2.15.

(a) Show that the Green function \( G(x, y; x', y') \) appropriate for Dirichlet boundary conditions for a square two-dimensional region, \( 0 \leq x \leq 1, 0 \leq y \leq 1 \), has an expansion

\[
G(x, y; x', y') = 2 \sum_{n=1}^{\infty} g_n(y, y') \sin(n\pi x) \sin(n\pi x')
\]

where \( g_n(y, y') \) satisfies

\[
\left( \frac{\partial^2}{\partial y'^2} - n^2 \pi^2 \right) g_n(y, y') = -4\pi \delta(y - y')
\]
and

\[ g_n(y, 0) = g_n(y, 1) = 0. \]

(b) Taking for \( g_n(y, y') \) appropriate linear combinations of \( \sinh(n\pi y') \) and \( \cosh(n\pi y') \) in the two regions, \( y' < y \) and \( y' > y \), in accord with the boundary conditions and the discontinuity in slope required by the source delta function, show that the explicit form of \( G \) is

\[
G(x, y; x', y') = 8 \sum_{n=1}^{\infty} \frac{1}{n \sinh(n\pi)} \sin(n\pi x) \sin(n\pi x') \sinh(n\pi y_<) \sinh[n\pi(1 - y_>)]
\]

where \( y_< (y_> \) is the smaller (larger) of \( y \) and \( y' \).