P507

Problem Set 19

(Due: April 16, 2009)

1) Jackson, problem 11.9.

An infinitesimal Lorentz transformation and its inverse can be written as

$$x'^{\alpha} = (g^{\alpha\beta} + \epsilon^{\alpha\beta})x_{\beta}$$

$$x^{\alpha} = (g^{\alpha\beta} + \epsilon'^{\alpha\beta})x'_{\beta}$$

(a) Show from the definition of the inverse that $\epsilon'^{\alpha\beta} = -\epsilon^{\alpha\beta}$.

(b) Show from the preservation of the norm that $\epsilon^{\alpha\beta} = -\epsilon^{\beta\alpha}$.

(c) By writing the transformation in terms of contravariant components on both sides of the equation, show that $\epsilon^{\alpha\beta}$ is equivalent to the matrix L (11.93).

2) Jackson, problem 11.13.

An infinitely long straight wire of negligible cross-sectional area is at rest and has a uniform linear charge density q_0 in the inertial frame K'. The frame K' (and the wire) move with a velocity $\boldsymbol{v}$ parallel to the direction of the wire with respect to the laboratory frame K.

(a) Write down the electric and magnetic fields in cylindrical coordinates in the rest frame of the wire. Using the Lorentz transformation properties of the fields, find the components of the electric and magnetic fields in the laboratory.

(b) What are the charge and current densities associated with the wire in its rest frame? In the laboratory?

(c) From the laboratory charge and current densities, calculate directly the electric and magnetic fields in the laboratory. Compare with the results of part a.

3) Jackson, problem 11.15.

In a certain reference frame a static, uniform, electric field E_0 is parallel to the x axis, and a static, uniform, magnetic induction $B_0 = 2E_0$ lies in the x-y plane, making an angle θ with the axis. Determine the relative velocity of a reference frame in which the electric and magnetic fields are parallel. What are the fields in that frame for $\theta << 1$ and $\theta \to (\pi/2)$?