

Problem Set 18

(Due: April 2, 2009)

1) Jackson, problem 11.3.

Show explicitly that two successive Lorentz transformations in the same direction are equivalent to a single Lorentz transformation with a velocity

$$v = \frac{v_1 + v_2}{1 + (v_1 v_2 / c^2)}$$

This is an alternative way to derive the parallel-velocity addition law.

2) Jackson, problem 11.5.

A coordinate system K' move with a velocity \mathbf{v} relative to another system K . In K' a particle has a velocity \mathbf{u}' and an acceleration \mathbf{a}' . Find the Lorentz transformation law for accelerations, and show that in the system K the components of acceleration parallel and perpendicular to \mathbf{v} are

$$\mathbf{a}_{\parallel} = \frac{\left(1 - \frac{v^2}{c^2}\right)^{3/2}}{\left(1 + \frac{\mathbf{v} \cdot \mathbf{u}'}{c^2}\right)^3} \mathbf{a}'_{\parallel}$$

$$\mathbf{a}_{\perp} = \frac{\left(1 - \frac{v^2}{c^2}\right)}{\left(1 + \frac{\mathbf{v} \cdot \mathbf{u}'}{c^2}\right)^3} \left(\mathbf{a}'_{\perp} + \frac{\mathbf{v}}{c^2} \times (\mathbf{a}' \times \mathbf{u}') \right)$$

3) Jackson, problem 11.6.

Assume that a rocket ship leaves the earth in the year 2100. One of a set of twins born in 2080 remains on earth; the other rides in the rocket. The rocket ship is so constructed that it has an acceleration g in its own rest frame (this makes the occupants feel at home). It accelerates in a straight-line path for 5 years (by its own clocks), decelerates at the same rate for 5 more years, turns around, accelerates for 5 years, decelerates for 5 years, and lands on earth. The twin in the rocket is 40 years old.

(a) What year is it on earth?

(b) How far away from the earth did the rocket ship travel?