P507

Problem Set 17

(Due: March 26, 2009)

1) Jackson, problem 10.1.

(a) Show that for arbitrary initial polarization, the scattering cross section of a perfectly conducting sphere of radius a, summed over outgoing polarizations, is given in the long-wavelength limit by

$$\frac{d\sigma}{d\Omega}(\boldsymbol{\epsilon}_0, \boldsymbol{n}_0, \boldsymbol{n}) = k^4 a^6 \left[\frac{5}{4} - |\boldsymbol{\epsilon}_0 \cdot \boldsymbol{n}|^2 - \frac{1}{4}|\boldsymbol{n} \cdot (\boldsymbol{n}_0 \times \boldsymbol{\epsilon}_0)|^2 - \boldsymbol{n}_0 \cdot \boldsymbol{n}\right]$$

where $\boldsymbol{n}_0$ and $\boldsymbol{n}$ are the directions of the incident and scattered radiations, respectively, while $\boldsymbol{\epsilon}_0$ is the (perhaps complex) unit polarization vector of the incident radiation ($\boldsymbol{\epsilon}_0^* \cdot \boldsymbol{\epsilon}_0 = 1$; $\boldsymbol{n}_0 \cdot \boldsymbol{\epsilon}_0 = 0$).

(b) If the incident radiation is linearly polarized, show that the cross section is

$$\frac{d\sigma}{d\Omega}(\boldsymbol{\epsilon}_0, \boldsymbol{n}_0, \boldsymbol{n}) = k^4 a^6 \left[\frac{5}{8}(1 + \cos^2\theta) - \cos\theta - \frac{3}{8}\sin^2\theta \cos 2\phi\right]$$

where $\boldsymbol{n} \cdot \boldsymbol{n}_0 = \cos\theta$ and the azimuthal angle ϕ is measured from the direction of the linear polarization.

(c) What is the ratio of scattered intensities at $\theta = \pi/2$, $\phi = 0$ and $\theta = \pi/2$, $\phi = \pi/2$? Explain physically in terms of the induced multipoles and their radiation patterns.

2) Diffraction from a rectangular aperture: Show that the intensity pattern for a rectangular aperture, $|x'| < a$ and $|y'| < b$, has the form

$$|\psi(x)|^2 = I_0 \left(\frac{\sin(q_x a)}{q_x a}\right)^2 \left(\frac{\sin(q_y b)}{q_y b}\right)^2$$

where $\boldsymbol{q} = k(\boldsymbol{n}_0 - \boldsymbol{n})$ is the momentum transfer and I_0 is some constant.