P507

Problem Set 14

(Due: February 12, 2009)

1) Jackson, problem 7.16a,b.

Plane waves propagate in a homogeneous, nonpermeable, but *anisotropic* dielectric. The dielectric is characterized by a tensor ϵ_{ij}, but if the coordinate axes are chosen as the prinicple axes, the components of displacement along these axes are related to the electric-field components by $D_i = \epsilon_i E_i$ $(i = 1, 2, 3)$, where ϵ_i are the eigenvalues of the matrix ϵ_{ij}.

(a) Show that plane waves with frequency ω and wave vector $\boldsymbol{k}$ must satisfy

$$\boldsymbol{k}\times(\boldsymbol{k}\times\boldsymbol{E}) + \mu_0\omega^2\boldsymbol{D} = 0$$

(b) Show that for a given wave vector $\boldsymbol{k} = k\boldsymbol{n}$ there are two distinct modes of propagation with different phase velocities $v = \omega/k$ that satisfy the Fresnel equation

$$\sum_{i=1}^{3} \frac{n_i^2}{v^2 - v_i^2} = 0$$

where $v_i = 1/\sqrt{\mu_0\epsilon_i}$ is called a principal velocity, and n_i is the component of $\boldsymbol{n}$ along the ith principal axis.

2) Jackson, problem 7.19.

An approximately monochromatic plane wave packet in one dimension has the instantaneous form, $u(x, 0) = f(x)e^{ik_0x}$, with $f(x)$ the modulation envelope. For each of the forms $f(x)$ below, calculate the wave-number spectrum $|A(k)|^2$ of the packet, sketch $|u(x, 0)|^2$ and $|A(k)|^2$, evaluate explicitly the rms deviations from the means Δx and Δk (defined in terms of the intensities $|u(x, 0)|^2$ and $|A(k)|^2$), and test inequality (7.82).

(a) $f(x) = Ne^{-\alpha|x|/2}$

(b) $f(x) = Ne^{-\alpha^2x^2/4}$

(c) $f(x) = N(1 - \alpha|x|)$ for $\alpha|x| < 1$ and zero otherwise

(d) $f(x) = N$ for $|x| < a$ and zero otherwise

3) Jackson, problem 7.22.

Use the Kramers-Kronig relation (7.120) to calculate the real part of $\epsilon(\omega)$, given the imaginary part of $\epsilon(\omega)$ for positive ω as

(a)

$$\mathrm{Im}\,\epsilon/\epsilon_0 = \lambda[\theta(\omega-\omega_1)-\theta(\omega-\omega_2)], \qquad \omega_2>\omega_1>0$$

(b)

$$\mathrm{Im}\,\epsilon/\epsilon_0 = \frac{\lambda\gamma\omega}{(\omega_0^2-\omega^2)^2-\gamma^2\omega^2}$$

In each case sketch the behavior of $\mathrm{Im}\,\epsilon(\omega)$ and the result for $\mathrm{Re}\,\epsilon(\omega)$ as functions of ω. Comment on the reasons for similarities or differences of your results as compared with the curves in Fig. 7.8. The step function is $\theta(x)=0$, $x<0$ and $\theta(x)=1$, $x>0$.