

Problem Set 13

(Due: February 5, 2009)

1) Jackson, problem 7.1.

For each set of Stokes parameters given below deduce the amplitude of the electric field, up to an overall phase, in both linear polarization and circular polarization bases and make an accurate drawing similar to Fig. 7.4 showing the lengths of the axes on one of the ellipses and its orientation.

$$(a) \quad s_0 = 3, \quad s_1 = -1, \quad s_2 = 2, \quad s_3 = -2 ;$$

$$(b) \quad s_0 = 25, \quad s_1 = 0, \quad s_2 = 24, \quad s_3 = 7 .$$

2) Jackson, problem 7.2.

A plane wave is incident on a layered interface as shown in the figure on page 340 of Jackson. The indices of refraction of the three nonpermeable media are n_1 , n_2 , n_3 . The thickness of the intermediate layer is d . Each of the other media is semi-infinite.

(a) Calculate the transmission and reflection coefficients (ratios of transmitted and reflected Poynting's flux to the incident flux), and sketch their behavior as a function of frequency for $n_1 = 1, n_2 = 2, n_3 = 3$; $n_1 = 3, n_2 = 2, n_3 = 1$; and $n_1 = 2, n_2 = 4, n_3 = 1$.

(b) The medium n_1 is part of an optical system (e.g., a lens); medium n_3 is air ($n_3 = 1$). It is desired to put an optical coating (medium n_2) on the surface so that there is no reflected wave for frequency ω_0 . What thickness d and index of refraction n_2 are necessary?

3) Jackson, problem 7.12.

The time dependence of electrical disturbances in good conductors is governed by the frequency-dependent conductivity (7.58). Consider longitudinal electrical fields in a conductor, using Ohm's law, the continuity equation, and the differential form of Coulomb's law.

(a) Show that the time-Fourier-transformed charge density satisfies the equation

$$[\sigma(\omega) - i\omega\epsilon_0]\rho(\mathbf{x}, \omega) = 0$$

(b) Using the representation $\sigma(\omega) = \sigma_0/(1 - i\omega\tau)$, where $\sigma_0 = \epsilon_0\omega_p^2\tau$ and τ is a damping time, show that in the approximation $\omega_p\tau \gg 1$ any initial disturbance will oscillate with the

plasma frequency and decay in amplitude with a decay constant $\lambda = 1/2\tau$. Note that if you use $\sigma(\omega) \simeq \sigma(0) = \sigma_0$ in part a, you will find no oscillations and extremely rapid damping with the (wrong) decay constant $\lambda_w = \sigma_0/\epsilon_0$.