

Problem Set 12

(Due: January 29, 2009)

1) Jackson, problem 6.5.

A localized electric charge distribution produces an electrostatic field, $\mathbf{E} = -\nabla\Phi$. Into this field is placed a small localized time-independent current density $\mathbf{J}(\mathbf{x})$, which generates a magnetic field \mathbf{H} .

(a) Show that the momentum of these electromagnetic fields, (6.117), can be transformed to

$$P_{\text{field}} = \frac{1}{c^2} \int \Phi \mathbf{J} d^3x$$

provided the product $\Phi\mathbf{H}$ falls off rapidly enough at large distances. How rapidly is “rapidly enough”?

(b) Assuming that the current distribution is localized to a region small compared to the scale of variation of the electric field, expand the electrostatic potential in a Taylor series and show that

$$P_{\text{field}} = \frac{1}{c^2} \mathbf{E}(0) \times \mathbf{m}$$

where $\mathbf{E}(0)$ is the electric field at the current distribution and \mathbf{m} is the magnetic moment, (5.54), caused by the current.

(c) Suppose the current distribution is placed instead in a *uniform* electric field \mathbf{E}_0 (filling all space). Show that, no matter how complicated is the localized \mathbf{J} , the result in part (a) is augmented by a surface integral contribution from infinity equal to minus one-third of the result of part (b), yielding

$$P_{\text{field}} = \frac{2}{3c^2} \mathbf{E}_0 \times \mathbf{m}$$

2) Jackson, problem 6.8.

A dielectric sphere of dielectric constant ϵ and radius a is located at the origin. There is a uniform applied electric field E_0 in the x direction. The sphere rotates with an angular velocity ω about the z axis. Show that there is a magnetic field $\mathbf{H} = -\nabla\Phi_M$, where

$$\Phi_M = \frac{3}{5} \left(\frac{\epsilon - \epsilon_0}{2\epsilon + \epsilon_0} \right) \epsilon_0 E_0 \omega \left(\frac{a}{r_{>}} \right)^5 \cdot xz$$

where $r_>$ is the larger of r and a . The motion is nonrelativistic.

You may use the results of Section 4.4 for the dielectric sphere in an applied field.

3) Jackson, problem 6.11.

A transverse plane wave is incident normally in vacuum on a perfectly absorbing flat screen.

(a) From the law of conservation of linear momentum, show that the pressure (called radiation pressure) exerted on the screen is equal to the field energy per unit volume in the wave.

(b) In the neighborhood of the earth the flux of electromagnetic energy from the sun is approximately 1.4 kw/m^2 . If an interplanetary "sailplane" had a sail of mass 1 g/m^2 of area and negligible other weight, what would be its maximum acceleration in meters per second squared due to the solar radiation pressure? How does this compare with the acceleration due to the solar "wind" (corpuscular radiation)?