P506
Problem Set 8
(Due: November 6, 2008)

1) Jackson, problem 3.12.

An infinite, thin, plane sheet of conducting material has a circular hole of radius a cut in it. A thin, flat disc of the same material and slightly smaller radius lies in the plane, filling the hole, but separated from the sheet by a very narrow insulating ring. The disc is maintained at a fixed potential V, while the infinite sheet is kept at zero potential.

(a) Using appropriate cylindrical coordinates, find an integral expression involving Bessel functions for the potential at any point above the plane.

(b) Show that the potential a perpendicular distance z above the *center* of the disc is

$$\Phi_0(z) = V\left(1 - \frac{z}{\sqrt{a^2 + z^2}}\right)$$

(c) Show that the potential a perpendicular distance z above the *edge* of the disc is

$$\Phi_a(z) = \frac{V}{2}\left[1 - \frac{kz}{\pi a}K(k)\right]$$

where $k = 2a/(z^2 + 4a^2)^{1/2}$, and $K(k)$ is the complete elliptic integral of the first kind.

2) Jackson, problem 4.2.

A point dipole with dipole moment $\boldsymbol{p}$ is located at point $\boldsymbol{x}_0$. From the properties of the derivative of a Dirac delta function, show that for calculation of the potential Φ or the energy of a dipole in an external field, the dipole can be described by an effective charge density

$$\rho_{\text{eff}}(\boldsymbol{x}) = -\boldsymbol{p} \cdot \nabla\delta(\boldsymbol{x} - \boldsymbol{x}_0)$$

3) Jackson, problem 4.7a-b.

A localized distribution of charge has a charge density

$$\rho(\boldsymbol{r}) = \frac{1}{64\pi} r^2 e^{-r} \sin^2\theta$$

(a) Make a multipole expansion of the potential due to this charge density and determine all the nonvanishing multipole moments. Write down the potential at large distances as a finite expansion in Legendre polynomials.

(b) Determine the potential explicitly at any point in space, and show that near the origin, correct to r^2 inclusive,

$$\Phi(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0}\left[\frac{1}{4} - \frac{r^2}{120} P_2(\cos\theta)\right]$$