

P506

Problem Set 7

(Due: October 30, 2008)

1) Jackson, problem 3.6.

Two point charges q and $-q$ are located on the z axis at $z = +a$ and $z = -a$, respectively.

(a) Find the electrostatic potential as an expansion in spherical harmonics and powers of both r for both $r > a$ and $r < a$.

(b) Keeping the product $qa \equiv p/2$ constant, take the limit of $a \rightarrow 0$ and find the potential for $r \neq 0$. This is by definition a dipole along the z axis and its potential.

(c) Suppose now that the dipole of part b is surrounded by a *grounded* spherical shell of radius b concentric with the origin. By linear superposition find the potential everywhere inside the shell.

2) Jackson, problem 3.7.

Three point charges $(q, -2q, q)$ are located in a straight line with separation a and with the middle charge $(-2q)$ at the origin of a grounded conducting spherical shell of radius b , as indicated in the sketch on p. 137 of Jackson.

(a) Write down the potential of the three charges in the absence of the grounded sphere. Find the limiting form of the potential as $a \rightarrow 0$, but the product $qa^2 = Q$ remains finite. Write this latter answer in spherical coordinates.

(b) The presence of the grounded sphere of radius b alters the potential for $r < b$. The added potential can be viewed as caused by the surface-charge density induced on the inner surface at $r = b$ or by image charges located at $r > b$. Use linear superposition to satisfy the boundary conditions and find the potential everywhere inside the sphere for $r < a$ and $r > a$. Show that in the limit $a \rightarrow 0$,

$$\Phi(r, \theta, \phi) \rightarrow \frac{Q}{2\pi\epsilon_0 r^3} \left(1 - \frac{r^5}{b^5} \right) P_2(\cos \theta) .$$

3) Jackson, problem 3.13.

Solve for the potential in Problem 3.1, using the appropriate Green function obtained in the text, and verify that the answer obtained in this way agrees with the direct solution from the differential equation.