

Problem Set 6

(Due: October 23, 2008)

1) Jackson, problem 2.13.

(a) Two halves of a long hollow conducting cylinder of inner radius b are separated by small lengthwise gaps on each side, and are kept at different potentials V_1 and V_2 . Show that the potential inside is given by

$$\Phi(\rho, \phi) = \frac{V_1 + V_2}{2} + \frac{V_1 - V_2}{\pi} \tan^{-1} \left(\frac{2b\rho}{b^2 - \rho^2} \cos \phi \right)$$

where ϕ is measured from a plane perpendicular to the plane through the gap.

(b) Calculate the surface-charge density on each half of the cylinder.

2) Note that all branches of $\log z$ have the same real component which satisfies Laplace's equation everywhere except the origin. Using $w = \log z$, show that the electrostatic potential $\Phi(x, y)$ in the space between two coaxial conducting cylindrical surfaces $x^2 + y^2 = 1$ and $x^2 + y^2 = r_0^2$ ($r_0 > 1$) when $\Phi = 0$ on the inner surface and $\Phi = V$ on the outer surface is

$$\Phi = \frac{\ln(x^2 + y^2)}{2 \ln r_0}.$$

Note: This problem is immediately solved using Eqn. (2.71) from Jackson. I want you to use the log function on the complex plane instead.

3) The two-dimensional region, $\rho \geq a$, $0 \leq \phi \leq \beta = \frac{\pi}{2}$ is bounded by conducting surfaces at $\phi = 0$, $\rho = a$, and $\phi = \beta = \frac{\pi}{2}$ held at potential V (there is an appropriate figure on page 93 of Jackson). At large ρ the potential is determined by some configuration of charges and/or conductors at fixed potentials.

(a) Using a conformal mapping, write down a solution for the potential $\Phi(\rho, \phi)$ that satisfies the boundary conditions for finite $\rho > a$. [Hint: Consider the map $F(z) = z^2 + 1/z^2$.]

(b) What is the electric field for finite $\rho > a$?

(c) Now consider a cylinder of radius a attached to the end of a half-plane, i.e. the situation described in part (a) but with $\beta = 2\pi$. What conformal mapping would allow you to solve this problem? *Just give the map; you don't need to find an expression for $\Phi(\rho, \phi)$.*