Problem Set 6
(Due: October 23, 2008)

1) Jackson, problem 2.13.
   (a) Two halves of a long hollow conducting cylinder of inner radius \( b \) are separated by small lengthwise gaps on each side, and are kept at different potentials \( V_1 \) and \( V_2 \). Show that the potential inside is given by
   \[
   \Phi(\rho, \phi) = \frac{V_1 + V_2}{2} + \frac{V_1 - V_2}{\pi} \tan^{-1}\left(\frac{2b\rho}{b^2 - \rho^2 \cos \phi}\right)
   \]
   where \( \phi \) is measured from a plane perpendicular to the plane through the gap.
   (b) Calculate the surface-charge density on each half of the cylinder.

2) Note that all branches of \( \log z \) have the same real component which satisfies Laplace’s equation everywhere except the origin. Using \( w = \log z \), show that the electrostatic potential \( \Phi(x, y) \) in the space between two coaxial conducting cylindrical surfaces \( x^2 + y^2 = 1 \) and \( x^2 + y^2 = r_0^2 \) (\( r_0 > 1 \)) when \( \Phi = 0 \) on the inner surface and \( \Phi = V \) on the outer surface is
   \[
   \Phi = \frac{\ln(x^2 + y^2)}{2 \ln r_0}.
   \]
   Note: This problem is immediately solved using Eqn. (2.71) from Jackson. I want you to use the \( \log \) function on the complex plane instead.

3) The two-dimensional region, \( \rho \geq a \), \( 0 \leq \phi \leq \beta = \frac{\pi}{2} \) is bounded by conducting surfaces at \( \phi = 0 \), \( \rho = a \), and \( \phi = \beta = \frac{\pi}{2} \) held at potential \( V \) (there is an appropriate figure on page 93 of Jackson). At large \( \rho \) the potential is determined by some configuration of charges and/or conductors at fixed potentials.
   (a) Using a conformal mapping, write down a solution for the potential \( \Phi(\rho, \phi) \) that satisfies the boundary conditions for finite \( \rho > a \). [Hint: Consider the map \( F(z) = z^2 + 1/z^2 \).]
   (b) What is the electric field for finite \( \rho > a \)?
   (c) Now consider a cylinder of radius \( a \) attached to the end of a half-plane, i.e. the situation described in part (a) but with \( \beta = 2\pi \). What conformal mapping would allow you to solve this problem? Just give the map; you don’t need to find an expression for \( \Phi(\rho, \phi) \).