

P506

Problem Set 5

(Due: October 16, 2008)

1) Jackson, problem 3.1.

Two concentric spheres have radii a, b ($b > a$) and each is divided into two hemispheres by the same horizontal plane. The upper hemisphere of the inner sphere and the lower hemisphere of the other sphere are maintained at potential V . The other hemispheres are at zero potential.

Determine the potential in the region $a \leq r \leq b$ as a series in Legendre polynomials. Include terms at least up to $\ell = 4$. Check your solution against known results in the limiting cases $b \rightarrow \infty$, and $a \rightarrow 0$.

2) Jackson, problem 3.2.

A spherical surface of radius R has charge uniformly distributed over its surface with a density $Q/4\pi R^2$, except for a spherical cap at the north pole, defined by the cone $\theta = \alpha$.

(a) Show that the potential inside the spherical surface can be expressed as

$$\Phi = \frac{Q}{8\pi\epsilon_0} \sum_{\ell=0}^{\infty} \frac{1}{2\ell+1} [P_{\ell+1}(\cos \alpha) - P_{\ell-1}(\cos \alpha)] \frac{r^\ell}{R^{\ell+1}} P_\ell(\cos \theta)$$

where, for $\ell = 0$, $P_{\ell-1}(\cos \alpha) = -1$. What is the potential outside?

(b) Find the magnitude and the direction of the electric field at the origin.

(c) Discuss the limiting forms of the potential (part a) and electric field (part b) as the spherical cap becomes (1) very small, and (2) so large that the area with charge on it becomes a very small cap at the south pole.

3) Jackson, problem 3.3.

A thin, flat, conducting, circular disc of radius R is located in the $x - y$ plane with its center at the origin, and is maintained at a fixed potential V . With the information that the charge

density on a disc at fixed potential is proportional to $(R^2 - \rho^2)^{-1/2}$, where ρ is the distance out from the center of the disc,

(a) show that for $r > R$ the potential is

$$\Phi(r, \theta, \phi) = \frac{2V}{\pi} \frac{R}{r} \sum_{\ell=0}^{\infty} \frac{(-1)^\ell}{2\ell + 1} \left(\frac{R}{r}\right)^{2\ell} P_{2\ell}(\cos \theta)$$

(b) find the potential for $r < R$.

(c) What is the capacitance of the disc?