P506

Problem Set 4

(Due: October 2, 2008)

1) (a) Formulate the Green's function appropriate for Dirichlet boundary conditions for the region between two infinite parallel planes at $z = 0$ and $z = d$.

(b) Recall that the solution for the potential of a *single* plane at $z = 0$ with the potential specified to be $\Phi = V$ inside a circle of radius a and zero elsewhere on the plane is

$$\Phi = V\left(1 - \frac{z}{\sqrt{a^2 + z^2}}\right) ,$$

along the axis of the circle (cf. Jackson problem 2.7). Consider introducing the second plane held at zero potential at $z = d$ with $d >> a$. What is the leading order perturbation on the potential for points near the first plane, i.e. points with $z << d$?

2) Jackson, problem 2.12.

Starting with the series solution (2.71) for the two-dimensional potential problem with the potential specified on the surface of a cylinder of radius b, evaluate the coefficients formally, substitute them into the series, and sum it to obtain the potential *inside* the cylinder in the form of Poisson's integral:

$$\Phi(\rho, \phi) = \frac{1}{2\pi} \int_0^{2\pi} \Phi(b, \phi') \frac{b^2 - \rho^2}{b^2 + \rho^2 - 2b\rho\cos(\phi' - \phi)} d\phi'$$

What modification is necessary if the potential is desired in the region of space bounded by the cylinder and infinity?

3) Jackson, problem 2.15.

(a) Show that the Green function $G(x, y; x', y')$ appropriate for Dirichlet boundary conditions for a square two-dimensional region, $0 \leq x \leq 1$, $0 \leq y \leq 1$, has an expansion

$$G(x, y; x', y') = 2\sum_{n=1}^{\infty} g_n(y, y') \sin(n\pi x) \sin(n\pi x')$$

where $g_n(y, y')$ satisfies

$$\left(\frac{\partial^2}{\partial y'^2} - n^2\pi^2\right) g_n(y, y') = -4\pi\delta(y - y')$$

and

$$g_n(y, 0) = g_n(y, 1) = 0 \ .$$

(b) Taking for $g_n(y, y')$ appropriate linear combinations of $\sinh(n\pi y')$ and $\cosh(n\pi y')$ in the two regions, $y' < y$ and $y' > y$, in accord with the boundary conditions and the discontinuity in slope required by the source delta function, show that the explicit form of G is

$$G(x, y; x', y') = 8 \sum_{n=1}^{\infty} \frac{1}{n \sinh(n\pi)} \sin(n\pi x) \sin(n\pi x') \sinh(n\pi y_<) \sinh[n\pi(1 - y_>)]$$

where $y_<(y_>)$ is the smaller (larger) of y and y'.