

## Problem Set 10

(Due: December 4, 2008)

1) Jackson, problem 5.1.

Starting with the differential equation

$$d\mathbf{B} = \frac{\mu_0 I}{4\pi} d\mathbf{l}' \times \frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^3}$$

for the magnetic induction at the point  $P$  with coordinate  $\mathbf{x}$  produced by an increment of current  $I d\mathbf{l}'$  at  $\mathbf{x}'$ , show explicitly that for a closed loop carrying a current  $I$  the magnetic induction at  $P$  is

$$\mathbf{B} = \frac{\mu_0 I}{4\pi} \nabla \Omega$$

where  $\Omega$  is the solid angle subtended by the loop at the point  $P$ . This corresponds to a magnetic scalar potential,  $\Phi_M = -\mu_0 I \Omega / 4\pi$ . The sign convention of the solid angle is that  $\Omega$  is positive if the point  $P$  views the “inner” side of the surface spanning the loop, that is, if a unit normal  $\mathbf{n}$  to the surface is defined by the direction of current flow via the right-hand rule,  $\Omega$  is positive if  $\mathbf{n}$  points it away from the point  $P$ , and negative otherwise. This is the same convention as in Section 1.6 for the electric dipole layer.

2) Jackson, problem 5.2.

A long, right cylindrical, ideal solenoid of arbitrary cross section is created by stacking a large number of identical current-carrying loops one above the other, with  $N$  coils per unit length and each loop carrying a current  $I$ . [In practice such a solenoid could be wound on a mandrel machined to the arbitrary cross section. After the coil was made rigid (e.g. with epoxy), the mandrel would be withdrawn.]

(a) In the approximation that the solenoidal coil is an ideal current sheet and infinitely long, use Problem 5.1 to establish that at any point inside the coil the magnetic field is axial and equal to

$$H = NI$$

and that  $H = 0$  for any point outside the coil.

(b) For a realistic solenoid of circular cross section of radius  $a$  ( $Na \gg 1$ ), but still infinite in length, show that the “smoothed” magnetic field just outside the solenoid (averaged axially

over several turns) is not zero, but is the same in magnitude and direction as that of a single wire on the axis carrying a current  $I$ , even if  $Na \rightarrow \infty$ . Compare fields inside and out.

3) Jackson, problem 5.3.

A right-circular solenoid of finite length  $L$  and radius  $a$  has  $N$  turns per unit length and carries a current  $I$ . Show that the magnetic induction on the cylinder axis in the limit  $NL \rightarrow \infty$  is

$$B_z = \frac{\mu_0 N I}{2} (\cos \theta_1 + \cos \theta_2)$$

where the angles are defined in the figure on page 225 of Jackson.